

# THE HIT PROBLEM OF RANK FIVE IN A GENERIC DEGREE

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## Abstract

Let  $E^k$  be an elementary abelian 2-group of rank  $k$  and let  $BE^k$  be the classifying space of  $E^k$ . Then,  $P_k := H^*(BE^k) \cong \mathbb{F}_2[x_1, x_2, \dots, x_k]$ , a polynomial algebra in  $k$  generators  $x_1, x_2, \dots, x_k$ , with the degree of each  $x_i$  being 1. This algebra is regarded as a module over the mod-2 Steenrod algebra,  $\mathcal{A}$ .

We study the *Peterson hit problem* of finding a minimal set of generators for  $\mathcal{A}$ -module  $P_k$ . It is an open problem in Algebraic Topology. In this paper, we explicitly determine a minimal set of  $\mathcal{A}$ -generators for  $P_5$  in terms of the admissible monomials for the case of the generic degree  $m = 2^{d+2} + 2^{d+1} - 3$  with  $d \geq 6$ .

## 1 Introduction

Let  $E^k$  be an elementary abelian 2-group of rank  $k$  and let  $BE^k$  be the classifying space of  $E^k$ . Then,

$$P_k := H^*(BE^k) \cong \mathbb{F}_2[x_1, x_2, \dots, x_k],$$

a polynomial algebra in  $k$  generators  $x_1, x_2, \dots, x_k$ , each of degree 1. Here, the cohomology is taken with coefficients in the prime field  $\mathbb{F}_2$  of two elements. The algebra  $P_k$  is a module over the mod-2 Steenrod algebra,  $\mathcal{A}$ . The action

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of  $\mathcal{A}$  on  $P_k$  is determined by the elementary properties of the Steenrod squares  $Sq^i$  and subject to the Cartan formula (see Steenrod and Epstein [18]).

An element  $g$  in  $P_k$  is called *hit* if it can be written as a finite sum  $g = \sum_{u \geq 0} Sq^{2^u}(g_u)$  for suitable polynomials  $g_u \in P_k$ . That means  $g$  belongs to  $\mathcal{A}^+P_k$ , where  $\mathcal{A}^+$  is the augmentation ideal of  $\mathcal{A}$ .

We study the problem, set up by Frank Peterson, of determining a minimal set of generators for the polynomial algebra  $P_k$  as a module over the Steenrod algebra. It is called the *Peterson hit problem of rank  $k$* . In other words, we want to determine a basis of the  $\mathbb{F}_2$ -vector space

$$QP_k := P_k/\mathcal{A}^+P_k = \mathbb{F}_2 \otimes_{\mathcal{A}} P_k.$$

The hit problem was studied by Peterson [7], Wood [30], Singer [16], and Priddy [11], who showed its relation to many problems in the homotopy theory respectively in cobordism theory, modular representation theory, Adams spectral sequence for the stable homotopy of spheres, and stable homotopy type of classifying spaces of finite groups. Then, this problem was studied by Carlisle-Wood [1], Crabb-Hubbuck [2], Janfada-Wood [3], Nam [6], Kameko [4], Mothebe [5], Repka-Selick [12], Singer [17], Silverman [13, 14], Silverman-Singer [15], Sum-Tin [24], Tin [26], Walker and Wood [27], Wood [31], the present author [19, 20, 22] and others.

The space  $QP_k$  was explicitly computed by Peterson [7] for  $k = 1, 2$ , by Kameko [4] for  $k = 3$  and by the present author [20] for  $k = 4$ . The problem is still open for any  $k > 4$ . Recently, Walker and Wood presented the results on the hit problem and its applications to representations of general linear groups in the monographs [28, 29].

Denote by  $(P_k)_m$  and  $(QP_k)_m$  the subspaces of degree  $m$  homogeneous polynomials in the spaces  $P_k$  and  $QP_k$  respectively, where  $m$  is an any nonnegative integer. Set  $\mu(m) = \min\{u \in \mathbb{Z} : \alpha(m+u) \leq u\}$ , where  $\alpha(a)$  denotes the number of one in dyadic expansion of a positive integer  $a$ .

An early result for the hit problem was established by Wood.

**Theorem 1.1** (See Wood [30]). *If  $\mu(m) > k$ , then  $(QP_k)_m = 0$ .*

Note that this theorem is originally Peterson's conjecture in [7].

Kameko's homomorphism  $\widetilde{Sq}_*^0 : QP_k \rightarrow QP_k$  is one of important tools in the study of the hit problem. This homomorphism is induced by the  $\mathbb{F}_2$ -linear map  $\phi : P_k \rightarrow P_k$  given by

$$\phi(x) = \begin{cases} z, & \text{if } x = x_1x_2 \dots x_kz^2, \\ 0, & \text{otherwise,} \end{cases}$$

for any monomial  $x \in P_k$ . Note that  $\phi$  is not an  $\mathcal{A}$ -homomorphism. However, for any non-negative integer  $s$ ,  $\phi Sq^{2s} = Sq^s \phi$ , and  $\phi Sq^{2s+1} = 0$ .

**Theorem 1.2** (See Kameko [4]). *If  $q$  is a positive integer such that  $\mu(2q+k) = k$ , then*

$$(\widetilde{Sq}_*)_{(k,q)}^0 := \widetilde{Sq}_*^0 : (QP_k)_{2q+k} \longrightarrow (QP_k)_q$$

*is an isomorphism of the  $GL_k$ -modules.*

From Theorems 1.1 and 1.2, the hit problem is reduced to the case of degree  $m$  such that  $\mu(m) < k$ . For  $\mu(m) = k - 1$ , the problem was studied by Crabb-Hubbuck [2], Nam [6], Repka-Selick [12], Walker-Wood [27] and the present author [19, 20]. A new tool for studying the Peterson hit problem has been presented in [22] when the author studies it for the case  $\mu(m) = k - 2$ .

Recently, many authors study the hit problem for the case  $k = 5$  and the problem has been explicitly determined for the case of degree  $m$  such that  $\alpha(m+s) = 1$  with  $s = \mu(m) < 5$  (see e.g. [23]).

In this paper, we study the hit problem for a case of  $k = 5$  and a family of degree  $m$  such that  $\alpha(m+s) = 2$ . More precisely, we explicitly compute the space  $(QP_k)_{2^{d+2}+2^{d+1}-3}$ , with  $d \geq 6$ , in terms of the admissible monomials (see Section 2). To do this, we consider Kameko's homomorphism

$$(\widetilde{Sq}_*)_{(5,2^{d+1}+2^d-4)}^0 : (QP_5)_{2^{d+2}+2^{d+1}-3} \longrightarrow (QP_5)_{2^{d+1}+2^d-4}.$$

The space  $(QP_5)_{2^{d+1}+2^d-4}$  can easily determine by using the results in [20]. So, we need to compute the space  $\text{Ker}((\widetilde{Sq}_*)_{(5,2^{d+1}+2^d-4)}^0)$ . We prove the following.

**Theorem 1.3.** *For any integer  $d \geq 4$ , there exist exactly 1085 classes of degree  $m = 2^{d+2} + 2^{d+1} - 3$  in  $\text{Ker}((\widetilde{Sq}_*)_{(5,2^{d+1}+2^d-4)}^0)$  represented by the admissible monomials in  $P_5$ . Consequently  $\dim \text{Ker}((\widetilde{Sq}_*)_{(5,2^{d+1}+2^d-4)}^0) = 1085$ .*

A simple computation using Theorems 1.3 and 1.4 in our work [20] we easily obtain  $\dim(QP_5)_{(2^{d+1}+2^d-4)} = (2^5 - 1) \dim(QP_4)_8 = 31 \times 55 = 1705$  for any  $d \geq 6$ . Since  $(\widetilde{Sq}_*)_{(5,2^{d+1}+2^d-4)}^0$  is an epimorphism, by combining this and Theorem 1.3 we get the following.

**Corollary 1.4.** *For any integer  $d \geq 6$ , we have*

$$\dim(QP_5)_{(2^{d+2}+2^{d+1}-3)} = 2790.$$

For  $d \leq 2$ , the computation is easy and the results have been published by other authors. We also study the problem for  $d = 3, 4, 5$ . We have  $1690 \leq \dim(QP_5)_{45} \leq 1739$ ,  $2466 \leq \dim(QP_5)_{93} \leq 2511$  and  $2790 \leq \dim(QP_5)_{189} \leq 2796$ . The exact dimension of  $(QP_5)_{(2^{d+2}+2^{d+1}-3)}$  for  $3 \leq d < 6$  can be checked by using a computer calculation (see Section 4).

The paper is organized as follows. In Section 2, we recall some needed results on the admissible monomials in  $P_k$ , the criteria of Singer and Silverman on the hit monomials. The detailed proof of Theorem 1.3 is presented in Section 3. In Section 4 we present some results of computation for  $(QP_5)_{(2^{d+1}+2^d-4)}$  with  $3 \leq d \leq 5$ . In Sections 5 and 6, we list the needed admissible monomials in  $P_5$  for the proof of main results.

## 2 Some preliminaries on the hit problem

In this section, we recall some needed definitions and results on the weight vector of a monomial, the admissible monomials, the criteria for hit monomials and the notion of strongly monomial from Kameko [4], Singer [17] and the present author [20, 22].

### 2.1 The weight vector of a monomial and the admissible monomials

**Definition 2.1.1.** Let  $x = x_1^{s_1} x_2^{s_2} \dots x_k^{s_k} \in P_k$ . We denote  $\nu_j(x) = s_j$ ,  $1 \leq j \leq k$ , and  $\nu(x) = \max\{\nu_j(x) : 1 \leq j \leq k\}$ . We define the sequences

$$\omega(x) = (\omega_1(x), \omega_2(x), \dots, \omega_i(x), \dots), \quad \sigma(x) = (\nu_1(x), \nu_2(x), \dots, \nu_k(x)),$$

where  $\omega_i(x) = \sum_{1 \leq j \leq k} \alpha_{i-1}(\nu_j(x))$ ,  $i \geq 1$ . Then,  $\omega(x)$  and  $\sigma(x)$  are called the weight vector and the exponent vector of  $x$  respectively.

A weight vector  $\omega$  is a sequence of non-negative integers  $(\omega_1, \omega_2, \dots, \omega_i, \dots)$  such that  $\omega_i = 0$  for  $i \gg 0$ .

The sets of weight vectors and exponent vectors respectively are ordered by the left lexicographical order.

**Definition 2.1.2.** Let  $\omega$  be a weight vector and  $f, h$  two polynomials of the same degree in  $P_k$ .

- i)  $f \equiv h$  if and only if  $f + h \in \mathcal{A}^+ P_k$ .
- ii)  $g \equiv_{\omega} h$  if and only if  $g + h \in \mathcal{A}^+ P_k + P_k^-(\omega)$ .

Note that the polynomial  $f$  is hit if and only if  $f \equiv 0$ .

It is easy to see that the relations  $\equiv$  and  $\equiv_{\omega}$  are equivalence ones. We denote

$$QP_k(\omega) = P_k(\omega) / ((\mathcal{A}^+ P_k \cap P_k(\omega)) + P_k^-(\omega)).$$

It is easy to see that for any degree  $n$ , we have

$$(QP_k)_n \cong \bigoplus_{\deg \omega = n} QP_k(\omega), \quad (\text{see e.g. [21]}). \quad (2.1)$$

**Definition 2.1.3.** Let  $v, w$  be monomials in  $P_k$  with  $\deg v = \deg w$ . We define  $v < w$  if and only if one of the following conditions holds:

- i)  $\omega(v) < \omega(w)$ ;
- ii)  $\omega(v) = \omega(w)$  and  $\sigma(v) < \sigma(w)$ .

**Definition 2.1.4.** A monomial  $x$  in  $P_k$  is said to be inadmissible if there are monomials  $w_1, w_2, \dots, w_s$  such that  $w_j < x$  for  $j = 1, 2, \dots, s$  and  $x + \sum_{j=1}^s w_j \in \mathcal{A}^+ P_k$ . A monomial  $x$  is said to be admissible if it is not inadmissible.

Obviously, the set of all the admissible monomials of degree  $m$  in  $P_k$  is a minimal set of  $\mathcal{A}$ -generators for  $P_k$  in degree  $m$ .

Denote by  $\mathcal{A}_u$  the sub-Hopf algebra of  $\mathcal{A}$  generated by  $Sq^i$  with  $0 \leq i < 2^u$ , and  $\mathcal{A}_u^+ = \mathcal{A}^+ \cap \mathcal{A}_u$ .

**Definition 2.1.5.** A monomial  $x$  in  $P_k$  is said to be strictly inadmissible if and only if there exist monomials  $w_1, w_2, \dots, w_s$  such that  $w_j < x$ , for  $j = 1, 2, \dots, s$  and  $x + \sum_{j=1}^s w_j \in \mathcal{A}_u^+ P_k$  with  $u = \max\{i : \omega_i(x) > 0\}$ .

**Theorem 2.1.6** (See Kameko [4], Sum [19]). *Let  $x, y, w$  be monomials in  $P_k$  such that  $\omega_i(x) = 0$  for  $i > t > 0$ ,  $\omega_u(w) \neq 0$  and  $\omega_i(w) = 0$  for  $i > u > 0$ .*

- i) *If  $w$  is inadmissible, then so is  $uw^{2^t}$ .*
- ii) *If  $w$  is strictly inadmissible, then so is  $wv^{2^u}$ .*

## 2.2 Some criteria for hit monomials

We recall a result of Singer [17] on the hit monomials in  $P_k$ .

**Definition 2.2.1.** A monomial  $z$  in  $P_k$  is called a spike if  $\nu_j(z) = 2^{s_j} - 1$  for  $s_j$  a non-negative integer and  $j = 1, 2, \dots, k$ . If  $s_1 > s_2 > \dots > s_{r-1} \geq s_r > 0$  and  $s_j = 0$  for  $j > r$ , then  $z$  is called the minimal spike.

The following is a criterion for hit monomials in  $P_k$ .

**Theorem 2.2.2** (See Singer [17, Theorem 1.2]). *Suppose  $f \in P_k$  is a monomial of degree  $m$  with  $\mu(m) \leq k$ . Let  $z$  be the minimal spike of degree  $m$ . If  $\omega(f) < \omega(z)$ , then  $f$  is hit.*

It can be shown that  $x_1^{31}x_2^7x_3^7$  is the minimal spike of degree 45 and the monomial  $x_1^5x_2^{13}x_3^9x_4^{13}x_5^5$  is hit, but

$$\omega(x_1^5x_2^{13}x_3^9x_4^{13}x_5^5) = (5, 0, 4, 3, 0) > (3, 3, 3, 1, 1) = \omega(x_1^{31}x_2^7x_3^7).$$

Thus, this criterion is not enough to determine all hit monomials. Hence, we need another criterion for hit polynomials in  $P_k$ .

**Theorem 2.2.3** (See Silverman [14, Theorem 1.2]). *Let  $f$  be a polynomial of the form  $ph^{2^m}$  for some homogeneous polynomials  $p$  and  $h$ . If  $\deg p < (2^m - 1)\mu(\deg h)$ , then  $f$  is hit.*

This result leads to a criterion in terms of the minimal spike which stronger version of Theorem 2.2.2.

**Theorem 2.2.4** (See Walker and Wood [27, Theorem 14.1.3]). *Let  $f \in P_k$  be a monomial of degree  $m$ , where  $\mu(m) \leq k$  and let  $z$  be the minimal spike of degree  $m$ . If there is an index  $r$  such that  $\sum_{i=1}^r 2^{i-1}\omega_i(x) < \sum_{i=1}^r 2^{i-1}\omega_i(z)$ , then  $f$  is hit.*

### 2.3 On the kernel of Kameko's homomorphism

In this subsection, we recall some notions and results from [22] for the kernel of Kameko's homomorphism which will be used in the proof of the main result.

**Definition 2.3.1.** Let  $z$  be the minimal spike of degree  $m$ . Denote by  $\mathcal{P}_{(k,m)}$  the subspace of  $P_k$  spanned by all monomials  $f$  of degree  $m$  such that

$$\sum_{1 \leq i \leq r} 2^{i-1}\omega_i(f) < \sum_{1 \leq i \leq r} 2^{i-1}\omega_i(z),$$

for some index  $r \geq 1$ .

**Definition 2.3.2.** A monomial  $f$  of degree  $m$  in  $P_k$  is said to be strongly inadmissible if there exist monomials  $y_1, y_2, \dots, y_u$  of the same weight vector  $\omega(f)$  such that  $y_t < f$ ,  $1 \leq t \leq u$ , and

$$x + y_1 + y_2 + \dots + y_u \in P_k^-(\omega(f)) + \mathcal{A}_s^+ P_k + \mathcal{P}_{(k,m)},$$

where  $s = \max\{i : \omega_i(f) > 0\}$ .

**Notation 2.3.3.** Let  $K$  be a finite sequence of positive integers. Then, there are positive integers  $d_0, d_1, \dots, d_s$  and  $r_0, r_1, \dots, r_s$  such that  $r_{i+1} \neq r_i$  and  $K = (r_0)^{|d_0|}(r_1)^{|d_1|} \dots (r_s)^{|d_s|}$ . We define the reduced length of  $K$  by  $\text{rl}(K) = d_1 + d_2 + \dots + d_s$ . For example, with  $K = (4, 4, 4, 2, 3, 3) = (4)^3|(2)|(3)|^2$ , we have  $d_0 = 3, d_1 = 1, d_2 = 2$ , hence  $\text{rl}(K) = d_1 + d_2 = 3$ .

Denote by  $\text{PSeq}_k^d$  the set of all pairs  $(\mathcal{U}, \mathcal{V})$  of sequences  $\mathcal{U} = (u_1, u_2, \dots, u_d)$ ,  $\mathcal{V} = (v_1, v_2, \dots, v_d)$ , where  $u_t, v_t$  are integers such that  $1 \leq u_t < v_t \leq k$ , for  $1 \leq t \leq d$ , and by  $\text{Plnc}_k^d$  the set of all  $(\mathcal{U}, \mathcal{V}) \in \text{PSeq}_k^d$  such that  $u_1 \leq u_2 \leq \dots \leq u_d$  and  $v_1 \leq v_2 \leq \dots \leq v_d$ . By convention,  $\text{PSeq}_k^0 = \emptyset$ . For  $(\mathcal{U}, \mathcal{V}) \in \text{PSeq}_k^d$ , we define

$$X_{(\mathcal{U}, \mathcal{V})} = \prod_{1 \leq t \leq d} X_{u_t, v_t}^{2^{d-t}} \in P_k((k-2)^{|d|}).$$

Here,  $X_{\mathbb{J}} = X_{j_1, j_2, \dots, j_s} = \prod_{j \notin \mathbb{J}} x_j$  for  $\mathbb{J} = \{j_1, j_2, \dots, j_s\} \subset \{1, 2, \dots, k\}$ .

**Definition 2.3.4.** Let  $h_0$  be a positive integer,  $h_0 > 2$ , and  $\mathcal{B}$  be a subset of  $\text{Plnc}_k^{h_0}$ . The set  $\mathcal{B}$  is said to be compatible with  $(k-2)^{|h_0|}$  if the following conditions hold:

- i) For any  $(\mathcal{U}, \mathcal{V}) \in \mathcal{B}$ ,  $\text{rl}(\mathcal{U}) \leq h_0 - 2$  and  $\text{rl}(\mathcal{V}) \leq h_0 - 2$ ,
- ii) For any  $(\mathcal{H}, \mathcal{K}) \in \text{PSeq}_k^{h_0}$ , we have

$$X_{(\mathcal{H}, \mathcal{K})} + \sum_{v=\min \mathcal{H}+1}^{\min \mathcal{K}} \sum_{(\mathcal{U}, \mathcal{V}) \in \mathcal{B}_v} X_{(\mathcal{U}, \mathcal{V})} \in P_k^-((k-1)^{|h_0|}) + \mathcal{A}_{h_0}^+ P_k + \mathcal{P}_{(k, n_{h_0})},$$

where  $\mathcal{B}_v$  is a set of some pairs  $(\mathcal{U}, \mathcal{V}) \in \mathcal{B}$  such that  $\min \mathcal{U} = \min \mathcal{H}$ ,  $\min \mathcal{V} = v$  and  $n_{h_0} = (k-2)^{(2^{h_0} - 1)}$ .

**Theorem 2.3.5** (See [22]). *Let  $h_0$  be a positive integer,  $h_0 > 2$ , and let  $k \geq 4$ ,  $m = \sum_{i=1}^{k-2} (2^{d_i} - 1)$  with  $d_i$  positive integers such that  $d_1 > d_2 > \dots > d_{k-3} \geq d_{k-2} = d \geq h_0$ . Denote  $q = \sum_{i=1}^{k-3} (2^{d_i-d} - 1)$ . Suppose the set  $\mathcal{B} \subset \text{Plnc}_k^{h_0}$  is compatible with  $(k-2)^{|h_0|}$ . Then,*

$$\bar{\mathcal{B}} := \bigcup_{(\mathcal{U}, \mathcal{V}) \in \mathcal{B}} \left\{ X_{(\mathcal{U}, \mathcal{V})} (X_{i,j})^{2^d - 2^{h_0}} (\theta_{J_{(i,j)}}(y))^{2^d} : y \in B_{k-2}(q) \right\}$$

is a set of generators for  $\text{Ker}(\widetilde{S}q_*^0)_{(k, \frac{n}{2})}$ , where  $i = \min \mathcal{U} = i_1$ ,  $j = \min \mathcal{V} = v_1$ ,  $J_{(i,j)} = (1, \dots, \hat{i}, \dots, \hat{j}, \dots, k)$  and  $B_{k-2}(q)$  is the set of all the admissible monomials of the degree  $q$  in  $P_{k-2}$ . Consequently,  $\dim \text{Ker}(\widetilde{S}q_*^0)_{(k, \frac{n}{2})} \leq |\mathcal{B}| \dim(QP_{k-2})_q$ .

We need the following for the proof of Theorem 1.3.

**Proposition 2.3.6** (See [22]). *The set*

$$\mathcal{B}_5 = \{(\mathcal{U}, \mathcal{V}) \in \text{PSeq}_5^6 : X_{(\mathcal{U}, \mathcal{V})} \in B_5((3)^6)\} \subset \text{Plnc}_5^6$$

is compatible with  $(3)^6$ .

## 2.4 Some other needed results and notations

We set

$$P_k^0 = \langle \{x \in P_k : \nu_j(x) = 0 \text{ for some } j\} \rangle,$$

$$P_k^+ = \langle \{x \in P_k : \nu_j(x) > 0, \text{ for all } j\} \rangle.$$

Then,  $P_k^0$  and  $P_k^+$  are the  $\mathcal{A}$ -submodules of  $P_k$  and  $QP_k = QP_k^0 \oplus QP_k^+$ , where  $QP_k^0 = P_k^0 / \mathcal{A}^+ P_k^0$  and  $QP_k^+ = P_k^+ / \mathcal{A}^+ P_k^+$ .

We denote

$$\mathcal{N}_k = \{(i; I) : I = (t_1, t_2, \dots, t_s), 1 \leq i < t_1 < \dots < t_s \leq k, 0 \leq s < k\}.$$

For any  $(i; I) \in \mathcal{N}_k$ , we define the homomorphism  $p_{(i; I)} : P_k \rightarrow P_{k-1}$  of  $\mathcal{A}$ -algebras by

$$p_{(i; I)}(x_u) = \begin{cases} x_u, & \text{if } 1 \leq u < i, \\ \sum_{t \in I} x_{t-1}, & \text{if } u = i, \\ x_{u-1}, & \text{if } i < u \leq k. \end{cases} \quad (2.2)$$

**Lemma 2.4.1** (See [9]). *For any monomial  $u$  in  $P_k$ , we have*

$$p_{(i; I)}(u) \in P_{k-1}(\omega(u)).$$

It is easy to see that if  $\omega$  is a weight vector of degree  $m$  and  $u \in P_k(\omega)$ , then  $p_{(i; I)}(u) \in P_{k-1}(\omega)$ . Hence,  $p_{(i; I)}$  passes to homomorphisms

$$p_{(i; I)}^{(\omega)} : QP_k(\omega) \longrightarrow QP_{k-1}(\omega), \quad p_{(i; I)}^{(m)} : (QP_k)_m \longrightarrow (QP_{k-1})_m.$$

For  $J = (s_1, s_2, \dots, s_r) : 1 \leq s_1 < \dots < s_r \leq k$ , we define a monomorphism  $\theta_J : P_r \rightarrow P_k$  of  $\mathcal{A}$ -algebras by substituting

$$\theta_J(x_v) = x_{s_v} \quad \text{for } 1 \leq v \leq r. \quad (2.3)$$

Clearly, for an arbitrary weight vector  $\omega$  of degree  $m$ ,

$$Q\theta_J(P_r^+(\omega)) \cong QP_r^+(\omega) \quad \text{and} \quad (Q\theta_J(P_r^+))_m \cong (QP_r^+)_m$$

for  $1 \leq r \leq k$ . Here,  $Q\theta_J(P_r^+) = \theta_J(P_r^+)/\mathcal{A}^+\theta_J(P_r^+)$ . Then, by a simple calculation using the result in Wood [30] and (2.1), we obtain the following.

**Proposition 2.4.2** (See Walker and Wood [28]). *Suppose  $\omega$  is a weight vector of degree  $n$ . Then, we have*

$$\begin{aligned} \dim QP_k(\omega) &= \sum_{\mu(m) \leq u \leq k} \binom{k}{u} \dim QP_u^+(\omega), \\ \dim (QP_k)_n &= \sum_{\mu(m) \leq u \leq k} \binom{k}{u} \dim (QP_u^+)_n. \end{aligned}$$

Set  $J_s = (1, \dots, \hat{s}, \dots, k)$  for  $1 \leq s \leq k$ .

**Proposition 2.4.3** (See Mothebe and Uys [5]). *Suppose  $s, h$  are integers such that  $1 \leq s \leq k$ . If  $f$  is an admissible monomial in  $P_{k-1}$  then so is  $x_s^{2^h-1}\theta_{J_s}(f)$  in  $P_k$ .*



**Notation 2.4.4.** For any nonnegative integer  $m$ , we denote  $B_k(m)$  the set of all admissible monomials of degree  $m$  in  $P_k$ . Set  $B_k^0(m) = B_k(m) \cap P_k^0$ ,  $B_k^+(m) = B_k(m) \cap P_k^+$ . For any weight vector  $\omega$  of degree  $m$ , we denote

$$B_k(\omega) = B_k(m) \cap P_k(\omega), \quad B_k^+(\omega) = B_k^+(m) \cap P_k(\omega).$$

For a homogeneous polynomial  $h \in P_k$ , we denote  $[h]$  the class in  $QP_k$  represented by  $h$ . If  $\omega$  is a weight vector and  $h \in P_k(\omega)$ , then we denote  $[h]_\omega$  the class in  $QP_k(\omega)$  represented by  $h$ . For a subset  $H$  of  $P_k$ , we denote  $[H] = \{[h] : h \in H\}$ . For  $H \subset P_k(\omega)$ , we set  $[H]_\omega = \{[h]_\omega : h \in H\}$ . Then,  $[B_k(\omega)]_\omega$  and  $[B_k^+(\omega)]_\omega$  are respectively the bases of the spaces  $QP_k(\omega)$  and  $QP_k^+(\omega) := QP_k(\omega) \cap QP_k^+$ . Denote by  $|H|$  the cardinal of  $H$ .

### 3 The indecomposables of $P_5$ in the degree $2^{d+2} + 2^{d+1} - 3$

In this section, we consider Kameko's homomorphism

$$(\widetilde{Sq}_*)_{(5,q)}^0 : (QP_5)_m \longrightarrow (QP_5)_q$$

where  $q = 2^{d+1} + 2^d - 4$  and  $m = 2q + 5 = 2^{d+2} + 2^{d+1} - 3$  with  $d$  a positive integer.

#### 3.1 The weight vector of admissible monomials in the space $\text{Ker}((\widetilde{Sq}_*)_{(5,q)}^0)$

First of all, we determine the weight vectors of the admissible monomials in  $\text{Ker}((\widetilde{Sq}_*)_{(5,q)}^0)$ .

**Lemma 3.1.1.** *Let  $x$  be an admissible monomial of degree  $m = 2^{d+2} + 2^{d+1} - 3$  in  $P_5$  with  $d$  a positive integer. If  $[x] \in \text{Ker}((\widetilde{Sq}_*)_{(5,q)}^0)$ , then either  $\omega(x) = (3)^d|(1)|^2$  or  $\omega(x) = (3)^{d+1}$ .*

*Proof.* Since  $x$  is admissible and  $[x] \in \text{Ker}((\widetilde{Sq}_*)_{(5,q)}^0)$ , using Lemma 3.1.1 in [22] we get  $\omega_i(x) = 3$  for  $1 \leq i \leq d$ . Then,  $x = \bar{x}y^{2^d}$  with  $\bar{x}$  a monomial of weight vector  $(3)^d$  and  $y$  a monomial of degree 3 in  $P_5$ . Since  $x$  is admissible, by Theorem 2.1.6,  $y$  is also admissible. Hence,  $\omega(y) = (1)^2$  or  $\omega(y) = (3)$ . The lemma is proved.  $\square$

By Lemma 3.1.1, we have

$$\text{Ker}((\widetilde{Sq}_*)_{(5,q)}^0) \cong QP_5((3)^{d+1}) \bigoplus QP_5((3)^d|(1)|^2).$$

The space  $QP_5((3)^{d+1})$  has been determined in [22, Theorem 4.2].

**Proposition 3.1.2.** *If  $d \geq 4$ , then we have  $\dim QP_5((3)|^{d+1}) = 155$ .*

The admissible monomials of weight vector  $(3)|^{d+1}$  are listed in the Appendix of the work [22].

### 3.2 Proof of Theorem 1.3

To prove Theorem 1.3, we need only to compute the space  $QP_5((3)|^d|(1)|^2)$ . We need some technical lemmas. The following follows from [20].

**Lemma 3.2.1.** *Let  $\{i, j, t, u\}$  be a subset of  $\{1, 2, 3, 4, 5\}$  such that  $i < j < t < u$ . The following monomials are strictly inadmissible:*

$$x_i^2 x_j x_t x_u, x_i^3 x_j^4 x_t^3 x_u^3, x_i^7 x_j^8 x_t^3 x_u^3, x_i^7 x_j^7 x_t^8 x_u^7, x_i^{15} x_j^{15} x_t^{15} x_u^{16}.$$

**Lemma 3.2.2.**

i) *If  $v$  is one of the monomials:*

$$\begin{array}{cccc} x_1^7 x_2^7 x_3^{15} x_4^{16} & x_1^7 x_2^{15} x_3^7 x_4^{16} & x_1^7 x_2^{15} x_3^{16} x_4^7 & x_1^7 x_2^{15} x_3^{17} x_4^6 \\ x_1^{15} x_2^7 x_3^7 x_4^{16} & x_1^{15} x_2^7 x_3^{16} x_4^7 & x_1^{15} x_2^7 x_3^{17} x_4^6 & x_1^{15} x_2^{19} x_3^5 x_4^6, \end{array}$$

then the monomial  $\theta_{J_s}(x)$ ,  $1 \leq s \leq 5$ , is strictly inadmissible.

ii) *Let  $(i, j, t, u, v)$  be an arbitrary permutation of  $(1, 2, 3, 4, 5)$ . The following monomials are strictly inadmissible:*

$$\begin{array}{l} x_i^{15} x_j^{19} x_t x_u^4 x_v^6, x_i^{15} x_j^{18} x_t x_u^4 x_v^7, x_i^{15} x_j^{18} x_t x_u^5 x_v^6, x_i^{15} x_j^{17} x_t x_u^6 x_v^6, x_i^{15} x_j^{16} x_t x_u^6 x_v^7, \\ x_i^{15} x_j^{19} x_t^5 x_u^2 x_v^4, x_i^{15} x_j^{17} x_t^2 x_u^4 x_v^7, x_i^{15} x_j^{19} x_t^3 x_u^4 x_v^4, x_i^{15} x_j^{20} x_t^3 x_u^3 x_v^4, \\ x_i x_j^{14} x_t^{16} x_u^7 x_v^7; i < u; x_i^{15} x_j^{17} x_t^3 x_u^6 x_v^4, j < v; x_i^{15} x_j^{16} x_t^3 x_u^5 x_v^6, j < u < v. \end{array}$$

iii) *The following monomials are strictly inadmissible:*

$$\begin{array}{cccccc} x_1 x_2^{14} x_3^{19} x_4^5 x_5^6 & x_1^3 x_2^7 x_3^{11} x_4^4 x_5^{20} & x_1^3 x_2^7 x_3^{11} x_4^{20} x_5^4 & x_1^3 x_2^7 x_3^{24} x_4^5 x_5^6 & x_1^3 x_2^{13} x_3^6 x_4^{16} x_5^7 & \\ x_1^3 x_2^{13} x_3^6 x_4^{17} x_5^6 & x_1^3 x_2^{13} x_3^7 x_4^{16} x_5^6 & x_1^3 x_2^{13} x_3^7 x_4^{18} x_5^4 & x_1^3 x_2^{13} x_3^{16} x_4^6 x_5^7 & x_1^3 x_2^{13} x_3^{16} x_4^7 x_5^6 & \\ x_1^3 x_2^{13} x_3^{17} x_4^6 x_5^6 & x_1^3 x_2^{13} x_3^{18} x_4^4 x_5^7 & x_1^3 x_2^{13} x_3^{18} x_4^5 x_5^6 & x_1^3 x_2^{13} x_3^{18} x_4^7 x_5^4 & x_1^3 x_2^{13} x_3^{19} x_4^4 x_5^6 & \\ x_1^3 x_2^{13} x_3^{19} x_4^6 x_5^4 & x_1^7 x_2^3 x_3^{11} x_4^4 x_5^{20} & x_1^7 x_2^3 x_3^{11} x_4^{20} x_5^4 & x_1^7 x_2^3 x_3^{24} x_4^5 x_5^6 & x_1^7 x_2^9 x_3^2 x_4^5 x_5^{22} & \\ x_1^7 x_2^9 x_3^2 x_4^{21} x_5^6 & x_1^7 x_2^9 x_3^3 x_4^4 x_5^{22} & x_1^7 x_2^9 x_3^3 x_4^{20} x_5^6 & x_1^7 x_2^9 x_3^3 x_4^{22} x_5^4 & x_1^7 x_2^9 x_3^{18} x_4^5 x_5^6 & \\ x_1^7 x_2^9 x_3^{19} x_4^4 x_5^6 & x_1^7 x_2^9 x_3^{19} x_4^6 x_5^4 & x_1^7 x_2^{11} x_3^3 x_4^4 x_5^{20} & x_1^7 x_2^{11} x_3^3 x_4^{20} x_5^4 & x_1^7 x_2^{11} x_3^{16} x_4^5 x_5^6 & \\ x_1^7 x_2^{11} x_3^{19} x_4^4 x_5^4 & x_1^7 x_2^{25} x_3^2 x_4^5 x_5^6 & x_1^7 x_2^{25} x_3^3 x_4^4 x_5^6 & x_1^7 x_2^{25} x_3^3 x_4^6 x_5^4 & & \end{array}$$

*Proof.* Note that all monomials in the lemma are of weight vector  $(3)|^3|(1)|^2$ . Part i) has been prove in [20]. We prove Part ii) for  $x = x_1 x_2^{15} x_3^{19} x_4^4 x_5^6$ ,  $y = x_1^7 x_2^{15} x_3^4 x_4^{16} x_5^6$  and  $z = x_1^{15} x_2^6 x_3^{16} x_4^7 x_5^7$ . We have

$$x = x_1 x_2^8 x_3^{23} x_4^6 x_5^7 + x_1 x_2^9 x_3^{23} x_4^6 x_5^6 + x_1 x_2^{10} x_3^{23} x_4^4 x_5^7 + x_1 x_2^{11} x_3^{23} x_4^4 x_5^6$$

$$\begin{aligned}
& + x_1 x_2^{15} x_3^{16} x_4^6 x_5^7 + x_1 x_2^{15} x_3^{17} x_4^6 x_5^6 + x_1 x_2^{15} x_3^{18} x_4^4 x_5^7 \\
& + Sq^1(x_1 x_2^{15} x_3^{15} x_4^4 x_5^9 + x_1 x_2^{15} x_3^{15} x_4^5 x_5^8 + x_1 x_2^{15} x_3^{16} x_4^5 x_5^7 + x_1 x_2^{15} x_3^{17} x_4^4 x_5^7 \\
& + x_1 x_2^{16} x_3^{15} x_4^5 x_5^7 + x_1 x_2^{17} x_3^{15} x_4^4 x_5^7 + x_1^4 x_2^{15} x_3^{15} x_4^3 x_5^7) + Sq^2(x_1 x_2^{15} x_3^{15} x_4^6 x_5^6 \\
& + x_1^2 x_2^{15} x_3^{15} x_4^3 x_5^8 + x_1^2 x_2^{15} x_3^{16} x_4^3 x_5^7 + x_1^2 x_2^{16} x_3^{15} x_4^3 x_5^7) \\
& + Sq^4(x_1 x_2^{15} x_3^{15} x_4^4 x_5^6) + Sq^8(x_1 x_2^8 x_3^{15} x_4^6 x_5^7 + x_1 x_2^9 x_3^{15} x_4^6 x_5^6 \\
& + x_1 x_2^{10} x_3^{15} x_4^4 x_5^7 + x_1 x_2^{11} x_3^{15} x_4^4 x_5^6) \pmod{(P_5^-((3)|^3|(1)|^2))}, \\
y = & x_1 x_2^{15} x_3^4 x_4^7 x_5^{18} + x_1 x_2^{15} x_3^4 x_4^8 x_5^7 + x_1 x_2^{19} x_3^4 x_4^7 x_5^{14} + x_1 x_2^{19} x_3^4 x_4^{14} x_5^7 \\
& + x_1^4 x_2^{19} x_3^4 x_4^7 x_5^{11} + x_1^4 x_2^{19} x_3^4 x_4^{11} x_5^7 + x_1^5 x_2^{15} x_3 x_4^6 x_5^{18} + x_1^5 x_2^{15} x_3 x_4^{18} x_5^6 \\
& + x_1^5 x_2^{15} x_3^2 x_4^7 x_5^{16} + x_1^5 x_2^{15} x_3^2 x_4^{16} x_5^7 + x_1^5 x_2^{19} x_3 x_4^6 x_5^{14} + x_1^5 x_2^{19} x_3 x_4^{14} x_5^6 \\
& + x_1^5 x_2^{19} x_3^2 x_4^7 x_5^{12} + x_1^5 x_2^{19} x_3^2 x_4^{12} x_5^7 + x_1^7 x_2^8 x_3^4 x_4^7 x_5^{19} + x_1^7 x_2^8 x_3^4 x_4^{19} x_5^7 \\
& + x_1^7 x_2^9 x_3 x_4^6 x_5^{22} + x_1^7 x_2^9 x_3 x_4^{22} x_5^6 + x_1^7 x_2^9 x_3^2 x_4^7 x_5^{20} + x_1^7 x_2^9 x_3^2 x_4^{20} x_5^7 \\
& + x_1^7 x_2^{15} x_3 x_4^6 x_5^{16} + Sq^1(x_1 x_2^{15} x_3^2 x_4^{13} x_5^{13} + x_1 x_2^{15} x_3^8 x_4^7 x_5^{13} + x_1 x_2^{15} x_3^8 x_4^{13} x_5^7 \\
& + x_1^3 x_2^{15} x_3 x_4^{12} x_5^{13} + x_1^3 x_2^{15} x_3 x_4^{13} x_5^{12} + x_1^3 x_2^{15} x_3^4 x_4^{11} x_5^{11} + x_1^7 x_2^{15} x_3 x_4^8 x_5^{13} \\
& + x_1^7 x_2^{15} x_3 x_4^9 x_5^{12} + x_1^7 x_2^{15} x_3 x_4^{12} x_5^9 + x_1^7 x_2^{15} x_3 x_4^{13} x_5^8 + x_1^7 x_2^{15} x_3^4 x_4^7 x_5^{11} \\
& + x_1^7 x_2^{15} x_3^4 x_4^{11} x_5^7) + Sq^2(x_1 x_2^{15} x_3^2 x_4^{11} x_5^{14} + x_1 x_2^{15} x_3^2 x_4^{14} x_5^{11} + x_1^2 x_2^{15} x_3^4 x_4^{11} x_5^{11} \\
& + x_1^2 x_2^{15} x_3^8 x_4^7 x_5^{11} + x_1^2 x_2^{15} x_3^8 x_4^{11} x_5^7 + x_1^3 x_2^{15} x_3 x_4^{10} x_5^{14} + x_1^3 x_2^{15} x_3 x_4^{14} x_5^{10} \\
& + x_1^3 x_2^{15} x_3^2 x_4^{11} x_5^{12} + x_1^3 x_2^{15} x_3^2 x_4^{12} x_5^{11} + x_1^4 x_2^{15} x_3^2 x_4^{11} x_5^{11} + x_1^7 x_2^{15} x_3 x_4^6 x_5^{14} \\
& + x_1^7 x_2^{15} x_3 x_4^{10} x_5^{10} + x_1^7 x_2^{15} x_3 x_4^{14} x_5^6 + x_1^7 x_2^{15} x_3^2 x_4^7 x_5^{12} + x_1^7 x_2^{15} x_3^2 x_4^8 x_5^{11} \\
& + x_1^7 x_2^{15} x_3^2 x_4^{11} x_5^8 + x_1^7 x_2^{15} x_3^2 x_4^{12} x_5^7) + Sq^4(x_1 x_2^{15} x_3^4 x_4^7 x_5^{14} \\
& + x_1 x_2^{15} x_3^4 x_4^{14} x_5^7 + x_1^2 x_2^{15} x_3^2 x_4^{11} x_5^{11} + x_1^4 x_2^{15} x_3^4 x_4^7 x_5^{11} + x_1^4 x_2^{15} x_3^4 x_4^{11} x_5^7 \\
& + x_1^5 x_2^{15} x_3 x_4^6 x_5^{14} + x_1^5 x_2^{15} x_3 x_4^{14} x_5^6 + x_1^5 x_2^{15} x_3^2 x_4^7 x_5^{12} + x_1^5 x_2^{15} x_3^2 x_4^{12} x_5^7) \\
& + Sq^8(x_1^7 x_2^8 x_3^4 x_4^7 x_5^{11} + x_1^7 x_2^8 x_3^4 x_4^{11} x_5^7 + x_1^7 x_2^9 x_3 x_4^6 x_5^{14} + x_1^7 x_2^9 x_3 x_4^{14} x_5^6 \\
& + x_1^7 x_2^9 x_3^2 x_4^7 x_5^{12} + x_1^7 x_2^9 x_3^2 x_4^{12} x_5^7) \pmod{(P_5^-((3)|^3|(1)|^2))},
\end{aligned}$$

Hence, the monomials  $x$  and  $y$  are strictly inadmissible. We have

$$\begin{aligned}
z = & x_1^9 x_2 x_3^6 x_4^7 x_5^{22} + x_1^9 x_2 x_3^6 x_4^{22} x_5^7 + x_1^{11} x_2 x_3^4 x_4^7 x_5^{22} + x_1^{11} x_2 x_3^4 x_4^{22} x_5^7 \\
& + x_1^{15} x_2 x_3^4 x_4^7 x_5^{18} + x_1^{15} x_2 x_3^4 x_4^{18} x_5^7 + x_1^{15} x_2 x_3^6 x_4^7 x_5^{16} + Sq^1(x_1^{15} x_2 x_3^2 x_4^{13} x_5^{13} \\
& + x_1^{15} x_2 x_3^3 x_4^{13} x_5^{12} + x_1^{15} x_2 x_3^5 x_4^{11} x_5^{12} + x_1^{15} x_2^2 x_3^5 x_4^9 x_5^{13} + x_1^{15} x_2^2 x_3^5 x_4^{13} x_5^9 \\
& + x_1^{15} x_2^4 x_3^3 x_4^{11} x_5^{11} + x_1^{19} x_2 x_3^2 x_4^{11} x_5^{11} + x_1^{19} x_2 x_3^3 x_4^{10} x_5^{11}) + Sq^2(x_1^{15} x_2 x_3^2 x_4^{11} x_5^{14} \\
& + x_1^{15} x_2 x_3^2 x_4^{14} x_5^{11} + x_1^{15} x_2 x_3^3 x_4^{14} x_5^{10} + x_1^{15} x_2 x_3^5 x_4^9 x_5^{13} + x_1^{15} x_2 x_3^5 x_4^{13} x_5^9 \\
& + x_1^{15} x_2 x_3^6 x_4^7 x_5^{14} + x_1^{15} x_2 x_3^6 x_4^{11} x_5^{10} + x_1^{15} x_2 x_3^6 x_4^{14} x_5^7 + x_1^{15} x_2^2 x_3^3 x_4^{11} x_5^{12} \\
& + x_1^{15} x_2^2 x_3^3 x_4^{12} x_5^{11} + x_1^{15} x_2^2 x_3^4 x_4^{11} x_5^{11} + x_1^{15} x_2^4 x_3^2 x_4^{11} x_5^{11} + x_1^{15} x_2^4 x_3^3 x_4^{10} x_5^{11} \\
& + x_1^{17} x_2 x_3^3 x_4^9 x_5^{13} + x_1^{17} x_2 x_3^3 x_4^{13} x_5^9) + Sq^4(x_1^{15} x_2 x_3^4 x_4^7 x_5^{14} + x_1^{15} x_2 x_3^4 x_4^{14} x_5^7
\end{aligned}$$

$$+ x_1^{15} x_2^2 x_3^4 x_4^{11} x_5^{11} + x_1^{15} x_2^2 x_3^3 x_4^{10} x_5^{11}) + Sq^8 (x_1^9 x_2 x_3^6 x_4^7 x_5^{14} + x_1^9 x_2 x_3^6 x_4^{14} x_5^7 \\ + x_1^{11} x_2 x_3^4 x_4^7 x_5^{14} + x_1^{11} x_2 x_3^4 x_4^{14} x_5^7) \pmod{(P_5^-((3)|^3|(1)|^2))},$$

Hence, the monomial  $z$  is strictly inadmissible.

We now prove Part iii) for  $u = x_1 x_2^{14} x_3^{19} x_4^5 x_5^6$ . We have

$$u = x_1 x_2^5 x_3^{15} x_4^6 x_5^{18} + x_1 x_2^5 x_3^{19} x_4^6 x_5^{14} + x_1 x_2^6 x_3^{15} x_4^7 x_5^{16} + x_1 x_2^6 x_3^{15} x_4^{16} x_5^7 \\ + x_1 x_2^6 x_3^{17} x_4^7 x_5^{14} + x_1 x_2^6 x_3^{19} x_4^{12} x_5^7 + x_1 x_2^7 x_3^{16} x_4^7 x_5^{14} + x_1 x_2^7 x_3^{17} x_4^6 x_5^{14} \\ + x_1 x_2^7 x_3^{18} x_4^7 x_5^{12} + x_1 x_2^7 x_3^{19} x_4^6 x_5^{12} + x_1 x_2^8 x_3^{23} x_4^6 x_5^7 + x_1 x_2^8 x_3^{23} x_4^7 x_5^6 \\ + x_1 x_2^{10} x_3^{23} x_4^5 x_5^6 + x_1 x_2^{12} x_3^{18} x_4^7 x_5^7 + x_1 x_2^{14} x_3^{16} x_4^7 x_5^7 + x_1 x_2^{14} x_3^{17} x_4^6 x_5^7 \\ + x_1 x_2^{14} x_3^{17} x_4^6 x_5^6 + Sq^1 (x_1 x_2^3 x_3^{15} x_4^{12} x_5^{13} + x_1 x_2^3 x_3^{19} x_4^{10} x_5^{11} + x_1 x_2^5 x_3^{15} x_4^7 x_5^{16} \\ + x_1 x_2^5 x_3^{15} x_4^{10} x_5^{13} + x_1 x_2^5 x_3^{15} x_4^{11} x_5^{12} + x_1 x_2^5 x_3^{15} x_4^{12} x_5^{11} + x_1 x_2^5 x_3^{19} x_4^7 x_5^{12} \\ + x_1 x_2^7 x_3^{15} x_4^8 x_5^{13} + x_1 x_2^7 x_3^{15} x_4^9 x_5^{12} + x_1 x_2^7 x_3^{16} x_4^7 x_5^{13} + x_1 x_2^7 x_3^{17} x_4^7 x_5^{12} \\ + x_1 x_2^8 x_3^{15} x_4^7 x_5^{13} + x_1 x_2^{12} x_3^{15} x_4^7 x_5^9 + x_1 x_2^{12} x_3^{15} x_4^9 x_5^7 + x_1 x_2^{12} x_3^{17} x_4^7 x_5^7 \\ + x_1 x_2^{13} x_3^{15} x_4^7 x_5^8 + x_1 x_2^{13} x_3^{15} x_4^8 x_5^7 + x_1 x_2^{13} x_3^{16} x_4^7 x_5^7 + x_1^2 x_2^5 x_3^{15} x_4^9 x_5^{13} \\ + x_1^2 x_2^{13} x_3^{15} x_4^5 x_5^9 + x_1^4 x_2^7 x_3^{15} x_4^7 x_5^{11} + x_1^4 x_2^{11} x_3^{15} x_4^7 x_5^7) + Sq^2 (x_1 x_2^3 x_3^{15} x_4^{10} x_5^{14} \\ + x_1 x_2^5 x_3^{15} x_4^9 x_5^{13} + x_1 x_2^6 x_3^{15} x_4^7 x_5^{14} + x_1 x_2^7 x_3^{15} x_4^6 x_5^{14} + x_1 x_2^{10} x_3^{15} x_4^7 x_5^{10} \\ + x_1 x_2^{10} x_3^{15} x_4^{10} x_5^7 + x_1 x_2^{11} x_3^{15} x_4^6 x_5^{10} + x_1 x_2^{13} x_3^{15} x_4^5 x_5^9 + x_1 x_2^{14} x_3^{15} x_4^6 x_5^7 \\ + x_1 x_2^{14} x_3^{15} x_4^6 x_5^6 + x_1^2 x_2^7 x_3^{15} x_4^7 x_5^{12} + x_1^2 x_2^7 x_3^{15} x_4^8 x_5^{11} + x_1^2 x_2^7 x_3^{16} x_4^7 x_5^{11} \\ + x_1^2 x_2^8 x_3^{15} x_4^7 x_5^{11} + x_1^2 x_2^{11} x_3^{15} x_4^7 x_5^8 + x_1^2 x_2^{11} x_3^{15} x_4^8 x_5^7 + x_1^2 x_2^{11} x_3^{16} x_4^7 x_5^7 \\ + x_1^2 x_2^{12} x_3^{15} x_4^7 x_5^7 + x_1^4 x_2^3 x_3^{15} x_4^{10} x_5^{11} + x_1 x_2^3 x_3^{17} x_4^9 x_5^{13} + x_1 x_2^{11} x_3^{17} x_4^5 x_5^9) \\ + Sq^4 (x_1 x_2^5 x_3^{15} x_4^6 x_5^{14} + x_1 x_2^6 x_3^{15} x_4^7 x_5^{12} + x_1 x_2^6 x_3^{15} x_4^{12} x_5^7 + x_1 x_2^7 x_3^{15} x_4^6 x_5^{12} \\ + x_1 x_2^{14} x_3^{15} x_4^5 x_5^6 + x_1^2 x_2^5 x_3^{15} x_4^{10} x_5^{11} + x_1^2 x_2^5 x_3^{15} x_4^7 x_5^{12}) + Sq^8 (x_1 x_2^2 x_3^{15} x_4^6 x_5^7 \\ + x_1 x_2^8 x_3^{15} x_4^6 x_5^6 + x_1 x_2^{10} x_3^{15} x_4^5 x_5^6) \pmod{(P_5^-((3)|^3|(1)|^2))},$$

The above equality shows that the monomial  $u$  is strictly inadmissible.  $\square$

**Lemma 3.2.3.** *Let  $(i, j, t, u, v)$  be an arbitrary permutation of  $(1, 2, 3, 4, 5)$ . The following monomials are strictly inadmissible:*

$$x_i^3 x_j^{13} x_t^{14} x_u^{31} x_v^{32}, \quad x_i^3 x_j^{13} x_t^{30} x_u^{15} x_v^{32}, \quad x_i^3 x_j^{29} x_t^{14} x_u^{15} x_v^{32}, \quad i < j < t, u < v$$

and

$$\begin{array}{cccc} x_1^3 x_2^{13} x_3^{30} x_4^{33} x_5^{14} & x_1^3 x_2^{13} x_3^{31} x_4^{34} x_5^{12} & x_1^3 x_2^{15} x_3^{29} x_4^{34} x_5^{12} & x_1^3 x_2^{29} x_3^{14} x_4^{33} x_5^{14} \\ x_1^3 x_2^{29} x_3^{15} x_4^{34} x_5^{12} & x_1^3 x_2^{31} x_3^{13} x_4^{34} x_5^{12} & x_1^3 x_2^{31} x_3^{37} x_4^{10} x_5^{12} & x_1^7 x_2^9 x_3^{10} x_4^{60} x_5^6 \\ x_1^7 x_2^{11} x_3^{13} x_4^{30} x_5^{32} & x_1^7 x_2^{11} x_3^{29} x_4^{14} x_5^{32} & x_1^7 x_2^{11} x_3^{29} x_4^{32} x_5^{14} & x_1^7 x_2^{11} x_3^{29} x_4^{34} x_5^{12} \\ x_1^7 x_2^{27} x_3^{13} x_4^{14} x_5^{32} & x_1^7 x_2^{27} x_3^{13} x_4^{32} x_5^{14} & x_1^7 x_2^{27} x_3^{13} x_4^{34} x_5^{12} & x_1^7 x_2^{27} x_3^{37} x_4^{10} x_5^{12} \\ x_1^{15} x_2^3 x_3^{29} x_4^{34} x_5^{12} & x_1^{15} x_2^3 x_3^{33} x_4^2 x_5^{12} & x_1^{31} x_2^3 x_3^{13} x_4^{34} x_5^{12} & x_1^{31} x_2^3 x_3^{37} x_4^{10} x_5^{12} \\ x_1^{31} x_2^{15} x_3^{33} x_4^2 x_5^{12} & x_1^{31} x_2^{35} x_3^{10} x_4^2 x_5^{12} & & \end{array}$$

*Proof.* The monomials in the lemma are of weight vector  $(3)^4|(1)|^2$ . We prove the lemma for  $v = x_1^3 x_2^{31} x_3^{37} x_4^{10} x_5^{12}$  and  $w = x_1^{15} x_2^3 x_3^{29} x_4^{32} x_5^{14}$ . By a direct computation using the Cartan formula we have

$$\begin{aligned}
 v &= x_1 x_2^{31} x_3^{15} x_4^{32} x_5^{14} + x_1 x_2^{31} x_3^{38} x_4^9 x_5^{14} + x_1 x_2^{39} x_3^{15} x_4^{24} x_5^{14} + x_1 x_2^{39} x_3^{30} x_4^9 x_5^{14} \\
 &\quad + x_1^2 x_2^{35} x_3^{29} x_4^{13} x_5^{14} + x_1^2 x_2^{37} x_3^{27} x_4^{13} x_5^{14} + x_1^3 x_2^{17} x_3^{45} x_4^{14} x_5^{14} + x_1^3 x_2^{19} x_3^{45} x_4^{12} x_5^{14} \\
 &\quad + x_1^3 x_2^{19} x_3^{45} x_4^{14} x_5^{12} + x_1^3 x_2^{20} x_3^{43} x_4^{13} x_5^{14} + x_1^3 x_2^{23} x_3^{15} x_4^{12} x_5^{40} + x_1^3 x_2^{23} x_3^{45} x_4^{10} x_5^{12} \\
 &\quad + x_1^3 x_2^{31} x_3^{15} x_4^{12} x_5^{32} + x_1^3 x_2^{31} x_3^{32} x_4^{13} x_5^{14} + x_1^3 x_2^{31} x_3^{33} x_4^{12} x_5^{14} + x_1^3 x_2^{31} x_3^{33} x_4^{14} x_5^{12} \\
 &\quad + Sq^1(x_1 x_2^{31} x_3^{15} x_4^{21} x_5^{24} + x_1 x_2^{31} x_3^{29} x_4^{11} x_5^{20} + x_1 x_2^{31} x_3^{29} x_4^{17} x_5^{14} \\
 &\quad + x_1 x_2^{35} x_3^{27} x_4^{11} x_5^{18} + x_1^3 x_2^{31} x_3^{15} x_4^{19} x_5^{24} + x_1^3 x_2^{31} x_3^{27} x_4^{13} x_5^{18} + x_1^3 x_2^{31} x_3^{27} x_4^{17} x_5^{14} \\
 &\quad + x_1^3 x_2^{35} x_3^{27} x_4^9 x_5^{18} + x_1^3 x_2^{35} x_3^{27} x_4^{13} x_5^{14}) + Sq^2(x_1 x_2^{31} x_3^{15} x_4^{22} x_5^{22} \\
 &\quad + x_1 x_2^{31} x_3^{27} x_4^{18} x_5^{14} + x_1 x_2^{31} x_3^{30} x_4^7 x_5^{22} + x_1^2 x_2^{31} x_3^{15} x_4^{19} x_5^{24} + x_1^2 x_2^{31} x_3^{27} x_4^{13} x_5^{18} \\
 &\quad + x_1^2 x_2^{31} x_3^{27} x_4^{17} x_5^{14} + x_1^2 x_2^{35} x_3^{27} x_4^{13} x_5^{14} + x_1^4 x_2^{31} x_3^{27} x_4^{11} x_5^{18} + x_1^5 x_2^{31} x_3^{27} x_4^{10} x_5^{18} \\
 &\quad + x_1^5 x_2^{31} x_3^{27} x_4^{14} x_5^{14}) + Sq^4(x_1 x_2^{31} x_3^{23} x_4^{20} x_5^{14} + x_1 x_2^{31} x_3^{30} x_4^7 x_5^{20} \\
 &\quad + x_1^2 x_2^{31} x_3^{27} x_4^{11} x_5^{18} + x_1^3 x_2^{31} x_3^{23} x_4^{12} x_5^{20} + x_1^3 x_2^{31} x_3^{27} x_4^{10} x_5^{18} + x_1^3 x_2^{31} x_3^{27} x_4^{14} x_5^{14} \\
 &\quad + x_1^3 x_2^{31} x_3^{28} x_4^{13} x_5^{14} + x_1^3 x_2^{31} x_3^{29} x_4^{12} x_5^{14} + x_1^3 x_2^{31} x_3^{29} x_4^{14} x_5^{12}) \\
 &\quad + Sq^8(x_1 x_2^{31} x_3^{15} x_4^{24} x_5^{14} + x_1 x_2^{31} x_3^{30} x_4^9 x_5^{14} + x_1^3 x_2^{31} x_3^{15} x_4^{12} x_5^{24} \\
 &\quad + x_1^3 x_2^{31} x_3^{29} x_4^{10} x_5^{12}) + Sq^{16}(x_1^3 x_2^{17} x_3^{29} x_4^{14} x_5^{14} + x_1^3 x_2^{19} x_3^{29} x_4^{12} x_5^{14} \\
 &\quad + x_1^3 x_2^{19} x_3^{29} x_4^{14} x_5^{12} + x_1^3 x_2^{20} x_3^{27} x_4^{13} x_5^{14} + x_1^3 x_2^{23} x_3^{15} x_4^{12} x_5^{24} \\
 &\quad + x_1^3 x_2^{23} x_3^{29} x_4^{10} x_5^{12}) \pmod{(P_5^-(3)^4|(1)|^2)}, \\
 w &= x_1^{11} x_2^3 x_3^{15} x_4^{28} x_5^{36} + x_1^{11} x_2^3 x_3^{15} x_4^{36} x_5^{28} + x_1^{11} x_2^3 x_3^{29} x_4^{14} x_5^{36} + x_1^{11} x_2^3 x_3^{29} x_4^{36} x_5^{14} \\
 &\quad + x_1^{11} x_2^3 x_3^{37} x_4^{14} x_5^{28} + x_1^{11} x_2^3 x_3^{37} x_4^{28} x_5^{14} + x_1^{15} x_2^3 x_3^{15} x_4^{28} x_5^{32} + x_1^{15} x_2^3 x_3^{15} x_4^{32} x_5^{28} \\
 &\quad + x_1^{15} x_2^3 x_3^{17} x_4^{14} x_5^{44} + x_1^{15} x_2^3 x_3^{17} x_4^{44} x_5^{14} + x_1^{15} x_2^3 x_3^{29} x_4^{14} x_5^{32} \\
 &\quad + Sq^1(x_1^{11} x_2^3 x_3^{27} x_4^{25} x_5^{26} + x_1^{19} x_2^3 x_3^{19} x_4^{25} x_5^{26} + x_1^{19} x_2^3 x_3^{23} x_4^{21} x_5^{26} \\
 &\quad + x_1^{19} x_2^3 x_3^{23} x_4^{25} x_5^{22} + x_1^{19} x_2^3 x_3^{27} x_4^{17} x_5^{26} + x_1^{19} x_2^3 x_3^{27} x_4^{21} x_5^{22} + x_1^{19} x_2^3 x_3^{27} x_4^{25} x_5^{18}) \\
 &\quad + Sq^2(x_1^{15} x_2^5 x_3^{27} x_4^{26} x_5^{18} + x_1^{15} x_2^5 x_3^{27} x_4^{22} x_5^{22} + x_1^{15} x_2^5 x_3^{23} x_4^{26} x_5^{22} \\
 &\quad + x_1^{15} x_2^5 x_3^{19} x_4^{26} x_5^{26} + x_1^{15} x_2^5 x_3^{27} x_4^{18} x_5^{26} + x_1^7 x_2^5 x_3^{27} x_4^{26} x_5^{26}) \\
 &\quad + Sq^4(x_1^7 x_2^3 x_3^{23} x_4^{28} x_5^{28} + x_1^7 x_2^3 x_3^{27} x_4^{26} x_5^{26} + x_1^7 x_2^3 x_3^{29} x_4^{22} x_5^{28} + x_1^7 x_2^3 x_3^{29} x_4^{28} x_5^{22} \\
 &\quad + x_1^{15} x_2^3 x_3^{15} x_4^{28} x_5^{28} + x_1^{15} x_2^3 x_3^{19} x_4^{26} x_5^{26} + x_1^{15} x_2^3 x_3^{21} x_4^{22} x_5^{28} + x_1^{15} x_2^3 x_3^{21} x_4^{28} x_5^{22} \\
 &\quad + x_1^{15} x_2^3 x_3^{23} x_4^{20} x_5^{28} + x_1^{15} x_2^3 x_3^{23} x_4^{26} x_5^{22} + x_1^{15} x_2^3 x_3^{23} x_4^{28} x_5^{20} + x_1^{15} x_2^3 x_3^{27} x_4^{18} x_5^{26} \\
 &\quad + x_1^{15} x_2^3 x_3^{27} x_4^{22} x_5^{22} + x_1^{15} x_2^3 x_3^{27} x_4^{26} x_5^{18} + x_1^{15} x_2^3 x_3^{29} x_4^{14} x_5^{28} + x_1^{15} x_2^3 x_3^{29} x_4^{20} x_5^{22} \\
 &\quad + x_1^{15} x_2^3 x_3^{29} x_4^{22} x_5^{20} + x_1^{15} x_2^3 x_3^{29} x_4^{28} x_5^{14}) + Sq^8(x_1^{11} x_2^3 x_3^{15} x_4^{28} x_5^{28} \\
 &\quad + x_1^{11} x_2^3 x_3^{29} x_4^{14} x_5^{28} + x_1^{11} x_2^3 x_3^{29} x_4^{28} x_5^{14}) + Sq^{16}(x_1^{15} x_2^3 x_3^{17} x_4^{14} x_5^{28}
 \end{aligned}$$

$$+ x_1^{15} x_2^3 x_3^{17} x_4^{28} x_5^{14}) \pmod{(P_5^-((3)^4|(1)^2))}.$$

Thus, the monomials  $v$  and  $w$  are strictly inadmissible.  $\square$

**Lemma 3.2.4.** *The following monomials are strictly inadmissible:*

$$\begin{array}{cccc} x_1^3 x_2^{29} x_3^{30} x_4^{31} x_5^{32} & x_1^3 x_2^{29} x_3^{31} x_4^{30} x_5^{32} & x_1^3 x_2^{31} x_3^{29} x_4^{30} x_5^{32} & x_1^7 x_2^{27} x_3^{29} x_4^{30} x_5^{32} \\ x_1^7 x_2^{27} x_3^{29} x_4^{30} x_5^{32} & x_1^7 x_2^{27} x_3^{29} x_4^{32} x_5^{30} & x_1^7 x_2^{27} x_3^{29} x_4^{34} x_5^{28} & x_1^{31} x_2^3 x_3^{29} x_4^{30} x_5^{32} \end{array}$$

*Proof.* The monomials in the lemma are of weight vector  $(3)^5|(1)$ . We prove the lemma for  $t = x_1^3 x_2^{29} x_3^{30} x_4^{31} x_5^{32}$ . A direct computation shows that

$$\begin{aligned} t &= x_1^2 x_2^{27} x_3^{29} x_4^{31} x_5^{36} + x_1^2 x_2^{27} x_3^{29} x_4^{37} x_5^{30} + x_1^2 x_2^{29} x_3^{29} x_4^{31} x_5^{34} + x_1^2 x_2^{29} x_3^{29} x_4^{35} x_5^{30} \\ &\quad + x_1^2 x_2^{29} x_3^{33} x_4^{31} x_5^{30} + x_1^3 x_2^{17} x_3^{30} x_4^{29} x_5^{46} + x_1^3 x_2^{17} x_3^{30} x_4^{45} x_5^{30} + x_1^3 x_2^{17} x_3^{46} x_4^{29} x_5^{30} \\ &\quad + x_1^3 x_2^{21} x_3^{24} x_4^{31} x_5^{46} + x_1^3 x_2^{21} x_3^{24} x_4^{47} x_5^{30} + x_1^3 x_2^{21} x_3^{26} x_4^{29} x_5^{46} + x_1^3 x_2^{21} x_3^{26} x_4^{45} x_5^{30} \\ &\quad + x_1^3 x_2^{21} x_3^{40} x_4^{31} x_5^{30} + x_1^3 x_2^{21} x_3^{42} x_4^{29} x_5^{30} + x_1^3 x_2^{27} x_3^{29} x_4^{36} x_5^{30} + x_1^3 x_2^{28} x_3^{29} x_4^{31} x_5^{34} \\ &\quad + x_1^3 x_2^{28} x_3^{29} x_4^{35} x_5^{30} + x_1^3 x_2^{28} x_3^{33} x_4^{31} x_5^{30} + x_1^3 x_2^{29} x_3^{24} x_4^{31} x_5^{38} + x_1^3 x_2^{29} x_3^{24} x_4^{39} x_5^{30} \\ &\quad + x_1^3 x_2^{29} x_3^{26} x_4^{29} x_5^{38} + x_1^3 x_2^{29} x_3^{26} x_4^{37} x_5^{30} + x_1^3 x_2^{29} x_3^{30} x_4^{29} x_5^{34} \\ &\quad + Sq^1(x_1^3 x_2^{27} x_3^{29} x_4^{31} x_5^{34} + x_1^3 x_2^{27} x_3^{29} x_4^{35} x_5^{30} + x_1^3 x_2^{27} x_3^{33} x_4^{31} x_5^{30}) \\ &\quad + Sq^2(x_1^2 x_2^{27} x_3^{29} x_4^{31} x_5^{34} + x_1^2 x_2^{27} x_3^{29} x_4^{35} x_5^{30} + x_1^2 x_2^{27} x_3^{33} x_4^{31} x_5^{30}) \\ &\quad + x_1^5 x_2^{27} x_3^{30} x_4^{31} x_5^{30}) + Sq^4(x_1^3 x_2^{27} x_3^{30} x_4^{31} x_5^{30} + x_1^3 x_2^{29} x_3^{30} x_4^{29} x_5^{30}) \\ &\quad + Sq^8(x_1^3 x_2^{29} x_3^{24} x_4^{31} x_5^{30} + x_1^3 x_2^{29} x_3^{26} x_4^{29} x_5^{30}) + Sq^{16}(x_1^3 x_2^{17} x_3^{30} x_4^{29} x_5^{30} \\ &\quad + x_1^3 x_2^{21} x_3^{24} x_4^{31} x_5^{30} + x_1^3 x_2^{21} x_3^{26} x_4^{29} x_5^{30}) \pmod{(P_5^-((3)^4|(1)^2))}. \end{aligned}$$

This equality shows that the monomials  $t$  is strictly inadmissible. The others can be proved by a similar computation.  $\square$

We now prove Theorem 1.3.

*Proof of Theorem 1.3.* We observe that for any  $x \in B_5((3)^d)$ , with  $d \geq 5$ , there exists uniquely a sequence  $J_x = (j_1^x, j_2^x, j_3^x)$  such that  $j_u^x > 16$  for  $u = 1, 2, 3$ . We set

$$\begin{aligned} A(d) &= \{x\theta_{J_x}(y) : x \in B_5((3)^d), y \in B_3((1)^2)\}, \\ B(d) &= \left\{x_i^{2^{d+2}-1}\theta_{J_i}(z) : z \in B_4^+((2)^d), 1 \leq i \leq 5\right\}. \end{aligned}$$

Since  $|B_5((3)^d)| = 155$ ,  $|B_3((1)^2)| = 6$  and  $|B_4^+((2)^d)| = 13$ , we get  $|A(d)| = 155 \times 6 = 930$  and  $|B(d)| = 65$ .

By a simple computation we have  $A(d) = B_5^0((3)^d|(1)^2) \cup B(d) \cup C(d)$ , where  $C(d) = \{a_j = a_{d,j} : 1 \leq j \leq 475\}$  with  $a_j$ ,  $1 \leq j \leq 475$ , as listed in the Section 5. We prove Theorem 1.3 by proving  $B_5((3)^d|(1)^2) = A(d)$ .

Let  $X$  be an admissible monomial of weight vector  $(3)^d|(1)^2$ . Then we have  $X = xy^{2^d}$  with  $x$  a monomial of weight vector  $(3)^d$  and  $y$  a monomial of

weight vector  $(1)^2$  in  $P_5$ . Since  $X$  is admissible,  $x$  and  $y$  are also admissible. If  $d \geq 6$ , then  $A(d) = \overline{\mathcal{B}_5}$  as determined in Theorem 2.3.5 with  $k = 5$  and  $\mathcal{B} = \mathcal{B}_5$ . By combining Proposition 2.3.6 and Theorem 2.3.5 we obtain  $B_5((3)^d|(1)^2) \subset A(d)$ .

For  $d < 6$ , let  $x \in B_5((3)^d)$  and  $y \in B_5((1)^2)$ . By a direct computation we see that if  $xy^{2^d} \notin A(d)$ , then there is a monomial  $w$  as given in one of Lemmas from 3.2.1 to 3.2.4 such that  $xy^{2^d} = \bar{x}w^{2^r}z^{2^{r+s}}$ , where  $\bar{x}$  and  $z$  are suitable monomials and  $2 \leq r \leq 6$ ,  $s = 0, 1$ . By Theorem 2.1.6,  $xy^{2^d}$  is inadmissible and we have a contradiction. Hence, we get  $B_5((3)^d|(1)^2) \subset A(d)$ .

We prove  $B_5((3)^d|(1)^2) = A(d)$  by proving the set  $[A(d)]_{\omega_d}$  is linearly independent in  $QP_5(\omega_d)$  with  $\omega_d = (3)^d|(1)^2$ . We prove this for  $d \geq 5$ . The case  $d = 4$  is prove by a similar computation with some minor changes.

Consider the subspaces  $\langle [B(d)]_{\omega_d} \rangle \subset QP_5(\omega_d)$  and  $\langle [C(d)]_{\omega_d} \rangle \subset QP_5(\omega_d)$ . It is easy to see that for any  $x \in B(d)$ , we have  $x = x_i^{2^{d+2}-1}\theta_{J_i}(y)$  with  $y$  an admissible monomial of weight vector  $(2)^d$  in  $P_4$ . By Proposition 2.4.3,  $x$  is admissible. Hence  $\dim \langle [B(d)]_{\omega_d} \rangle = 65$ . Since  $\nu(x) = 2^{d+2} - 1$  for all  $x \in B(d)$  and  $\nu(x) < 2^{d+2} - 1$  for all  $x \in C(d)$ , we obtain  $\langle [B(d)]_{\omega_d} \rangle \cap \langle [C(d)]_{\omega_d} \rangle = \{0\}$ . Hence, we need only to prove the set  $[C(d)]_{\omega_d} = \{[a_{d,t}]_{\omega_d} : 1 \leq t \leq 475\}$  is linearly independent in  $QP_5(\omega_d)$ , where the monomials  $a_t = a_{d,t}$ ,  $1 \leq t \leq 475$ , are determined as in Section 5.

Suppose there is a linear relation

$$\mathcal{U} := \sum_{1 \leq t \leq 475} \gamma_t a_{d,t} \equiv_{\omega_d} 0, \quad (3.1)$$

where  $\gamma_t \in \mathbb{F}_2$ . We denote  $\gamma_{\mathbb{J}} = \sum_{t \in \mathbb{J}} \gamma_t$  for any  $\mathbb{J} \subset \{t \in \mathbb{N} : 1 \leq t \leq 475\}$ .

Let  $w_u = w_{d,u}$ ,  $1 \leq u \leq 66$ , be as in Section 5 and the homomorphism  $p_{(i;I)} : P_5 \rightarrow P_4$  which is defined by (2.2) for  $k = 5$ . From Lemma 2.4.1, we see that  $p_{(i;I)}$  passes to a homomorphism from  $QP_5(\omega_d)$  to  $QP_4(\omega_d)$ . By applying  $p_{(1;2)}$ ,  $p_{(1;3)}$ , and  $p_{(4;5)}$  to (3.1), we obtain

$$\begin{aligned} p_{(1;2)}(\mathcal{U}) &\equiv_{\omega_d} \gamma_{16}w_2 + \gamma_{22}w_5 + \gamma_{23}w_6 + \gamma_{25}w_7 + \gamma_{26}w_8 + \gamma_{28}w_{10} + \gamma_{29}w_{11} \\ &\quad + \gamma_{34}w_{12} + \gamma_{36}w_{13} + \gamma_{37}w_{14} + \gamma_{39}w_{15} + \gamma_{40}w_{16} + \gamma_{17}w_{20} + \gamma_{18}w_{21} \\ &\quad + \gamma_{19}w_{22} + \gamma_{20}w_{23} + \gamma_{21}w_{25} + \gamma_{24}w_{26} + \gamma_{27}w_{27} + \gamma_{30}w_{28} + \gamma_{31}w_{29} \\ &\quad + \gamma_{32}w_{30} + \gamma_{33}w_{31} + \gamma_{35}w_{32} + \gamma_{38}w_{33} + \gamma_{41}w_{34} + \gamma_{42}w_{35} + \gamma_{\{85,227\}}w_{37} \\ &\quad + \gamma_{\{86,228\}}w_{38} + \gamma_{\{87,229\}}w_{39} + \gamma_{\{88,230\}}w_{40} + \gamma_{\{89,231\}}w_{41} + \gamma_{\{90,232\}}w_{42} \\ &\quad + \gamma_{109}w_{44} + \gamma_{110}w_{45} + \gamma_{111}w_{46} + \gamma_{112}w_{47} + \gamma_{113}w_{49} + \gamma_{114}w_{50} + \gamma_{115}w_{51} \\ &\quad + \gamma_{116}w_{52} + \gamma_{117}w_{53} + \gamma_{139}w_{55} + \gamma_{140}w_{56} + \gamma_{141}w_{57} + \gamma_{142}w_{58} \\ &\quad + \gamma_{143}w_{59} + \gamma_{144}w_{60} + \gamma_{145}w_{61} + \gamma_{146}w_{62} + \gamma_{163}w_{63} + \gamma_{164}w_{64} \\ &\quad + \gamma_{165}w_{65} + \gamma_{166}w_{66} \equiv_{\omega_d} 0, \\ p_{(1;3)}(\mathcal{U}) &\equiv_{\omega_d} \gamma_{16}w_2 + \gamma_{\{1,51,167\}}w_5 + \gamma_{\{2,52,168\}}w_6 + \gamma_{\{3,54,169\}}w_7 \end{aligned}$$

$$\begin{aligned}
& + \gamma_{\{4,55,170\}}w_8 + \gamma_{28}w_{10} + \gamma_{29}w_{11} + \gamma_{\{8,69,187\}}w_{12} + \gamma_{\{9,71,188\}}w_{13} \\
& + \gamma_{\{10,72,189\}}w_{14} + \gamma_{\{11,74,190\}}w_{15} + \gamma_{40}w_{16} + \gamma_{\{14,82,203\}}w_{17} \\
& + \gamma_{\{15,84,204\}}w_{18} + \gamma_{43}w_{20} + \gamma_{44}w_{21} + \gamma_{45}w_{22} + \gamma_{46}w_{23} + \gamma_{\{50,209\}}w_{25} \\
& + \gamma_{\{53,210\}}w_{26} + \gamma_{\{56,211\}}w_{27} + \gamma_{63}w_{28} + \gamma_{64}w_{29} + \gamma_{65}w_{30} + \gamma_{66}w_{31} \\
& + \gamma_{\{70,218\}}w_{32} + \gamma_{\{73,219\}}w_{33} + \gamma_{79}w_{34} + \gamma_{80}w_{35} + \gamma_{\{83,224\}}w_{36} + \gamma_{91}w_{37} \\
& + \gamma_{92}w_{38} + \gamma_{93}w_{39} + \gamma_{100}w_{40} + \gamma_{101}w_{41} + \gamma_{106}w_{42} + \gamma_{122}w_{44} + \gamma_{123}w_{45} \\
& + \gamma_{125}w_{46} + \gamma_{126}w_{47} + \gamma_{121}w_{49} + \gamma_{124}w_{50} + \gamma_{127}w_{51} + \gamma_{134}w_{52} + \gamma_{136}w_{53} \\
& + \gamma_{138}w_{54} + \gamma_{149}w_{55} + \gamma_{151}w_{56} + \gamma_{152}w_{57} + \gamma_{154}w_{58} + \gamma_{150}w_{59} + \gamma_{153}w_{60} \\
& + \gamma_{159}w_{61} + \gamma_{161}w_{62} \equiv_{\omega_d} 0, \\
p_{(4;5)}(\mathcal{U}) & \equiv_{\omega_d} \gamma_{116}w_1 + \gamma_{117}w_2 + \gamma_{134}w_4 + \gamma_{135}w_5 + \gamma_{136}w_6 + \gamma_{137}w_7 + \gamma_{138}w_8 \\
& + \gamma_{145}w_{10} + \gamma_{146}w_{11} + \gamma_{159}w_{12} + \gamma_{160}w_{13} + \gamma_{161}w_{14} + \gamma_{162}w_{15} + \gamma_{166}w_{16} \\
& + \gamma_{300}w_{19} + \gamma_{301}w_{20} + \gamma_{302}w_{21} + \gamma_{303}w_{22} + \gamma_{304}w_{23} + \gamma_{314}w_{25} + \gamma_{315}w_{26} \\
& + \gamma_{316}w_{27} + \gamma_{329}w_{28} + \gamma_{330}w_{29} + \gamma_{331}w_{30} + \gamma_{332}w_{31} + \gamma_{339}w_{32} + \gamma_{340}w_{33} \\
& + \gamma_{347}w_{34} + \gamma_{348}w_{35} + \gamma_{397}w_{37} + \gamma_{398}w_{38} + \gamma_{399}w_{39} + \gamma_{406}w_{40} + \gamma_{407}w_{41} \\
& + \gamma_{411}w_{42} + \gamma_{428}w_{43} + \gamma_{429}w_{44} + \gamma_{430}w_{45} + \gamma_{431}w_{46} + \gamma_{432}w_{47} + \gamma_{442}w_{49} \\
& + \gamma_{443}w_{50} + \gamma_{444}w_{51} + \gamma_{445}w_{52} + \gamma_{446}w_{53} + \gamma_{459}w_{55} + \gamma_{460}w_{56} + \gamma_{461}w_{57} \\
& + \gamma_{462}w_{58} + \gamma_{469}w_{59} + \gamma_{470}w_{60} + \gamma_{471}w_{61} + \gamma_{472}w_{62} \equiv_{\omega_d} 0.
\end{aligned}$$

From the above equalities, we get

$$\gamma_j = 0 \text{ for } j \in \mathbb{J}_1, \quad \gamma_i = \gamma_j \text{ for } (i, j) \in \mathbb{K}_1, \quad (3.2)$$

where  $\mathbb{J}_1 = \{16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 63, 64, 65, 66, 79, 80, 91, 92, 93, 100, 101, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 121, 122, 123, 124, 125, 126, 127, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 149, 150, 151, 152, 153, 154, 159, 160, 161, 162, 163, 164, 165, 166, 300, 301, 302, 303, 304, 314, 315, 316, 329, 330, 331, 332, 339, 340, 347, 348, 397, 398, 399, 406, 407, 411, 428, 429, 430, 431, 432, 442, 443, 444, 445, 446, 459, 460, 461, 462, 469, 470, 471, 472\}$  and  $\mathbb{K}_1 = \{(50,209), (53,210), (56,211), (70,218), (73,219), (83,224), (85,227), (86,228), (87,229), (88,230), (89,231), (90,232)\}$ .

Apply the homomorphisms  $p_{(2;3)}$ ,  $p_{(2;4)}$ ,  $p_{(3;4)}$  to (3.1) and using (3.2), to obtain

$$\begin{aligned}
p_{(2;3)}(\mathcal{U}) & \equiv_{\omega_d} \gamma_{\{1,175,234,288\}}w_5 + \gamma_{\{2,176,235,289\}}w_6 + \gamma_{\{3,178,237,291\}}w_7 \\
& + \gamma_{\{4,179,238,292\}}w_8 + \gamma_{\{8,193,249,319\}}w_{12} + \gamma_{\{9,195,251,321\}}w_{13} \\
& + \gamma_{\{10,196,252,322\}}w_{14} + \gamma_{\{11,198,254,324\}}w_{15} + \gamma_{\{14,206,261,342\}}w_{17} \\
& + \gamma_{\{15,208,263,344\}}w_{18} + \gamma_{167}w_{20} + \gamma_{168}w_{21} + \gamma_{169}w_{22} + \gamma_{170}w_{23}
\end{aligned}$$



$$\begin{aligned}
 & + \gamma_{\{50,85,174,233,287\}} w_{25} + \gamma_{\{53,86,177,236,290\}} w_{26} + \gamma_{\{56,87,180,239,293\}} w_{27} \\
 & + \gamma_{187} w_{28} + \gamma_{188} w_{29} + \gamma_{189} w_{30} + \gamma_{190} w_{31} + \gamma_{\{70,88,194,250,320\}} w_{32} \\
 & + \gamma_{\{73,89,197,253,323\}} w_{33} + \gamma_{203} w_{34} + \gamma_{204} w_{35} + \gamma_{\{83,90,207,262,343\}} w_{36} \\
 & + \gamma_{349} w_{37} + \gamma_{350} w_{38} + \gamma_{351} w_{39} + \gamma_{358} w_{40} + \gamma_{359} w_{41} + \gamma_{364} w_{42} \\
 & + \gamma_{416} w_{44} + \gamma_{417} w_{45} + \gamma_{419} w_{46} + \gamma_{420} w_{47} + \gamma_{415} w_{49} + \gamma_{418} w_{50} + \gamma_{421} w_{51} \\
 & + \gamma_{449} w_{55} + \gamma_{451} w_{56} + \gamma_{452} w_{57} + \gamma_{454} w_{58} + \gamma_{450} w_{59} + \gamma_{453} w_{60} \equiv_{\omega_d} 0, \\
 p(2;4)(\mathcal{U}) & \equiv_{\omega_d} \gamma_{\{2,52,168\}} w_2 + \gamma_{\{4,55,170\}} w_3 + \gamma_{\{5,57,182,246\}} w_5 + \gamma_{\{6,59,184,247\}} w_7 \\
 & + \gamma_{\{7,61,186,248\}} w_9 + \gamma_{\{8,69,187\}} w_{10} + \gamma_{\{10,72,189\}} w_{11} + \gamma_{\{12,75,200,259\}} w_{13} \\
 & + \gamma_{\{13,77,202,260\}} w_{15} + \gamma_{\{14,82,203\}} w_{16} + \gamma_{\{171,212,353,367\}} w_{20} + \gamma_{176} w_{21} \\
 & + \gamma_{\{172,214,355,369\}} w_{22} + \gamma_{179} w_{23} + \gamma_{\{173,216,357,371\}} w_{24} + \gamma_{181} w_{25} \\
 & + \gamma_{183} w_{26} + \gamma_{185} w_{27} + \gamma_{193} w_{28} + \gamma_{\{191,220,361,376\}} w_{29} + \gamma_{196} w_{30} \\
 & + \gamma_{\{192,222,363,378\}} w_{31} + \gamma_{199} w_{32} + \gamma_{201} w_{33} + \gamma_{\{205,225,366,382\}} w_{35} \\
 & + \gamma_{206} w_{34} + \gamma_{352} w_{37} + \gamma_{354} w_{38} + \gamma_{356} w_{39} + \gamma_{360} w_{40} + \gamma_{362} w_{41} + \gamma_{365} w_{42} \\
 & + \gamma_{\{412,423,433\}} w_{44} + \gamma_{417} w_{45} + \gamma_{\{413,425,435\}} w_{46} + \gamma_{420} w_{47} \\
 & + \gamma_{\{414,427,437\}} w_{48} + \gamma_{422} w_{49} + \gamma_{424} w_{50} + \gamma_{426} w_{51} + \gamma_{449} w_{55} + \gamma_{452} w_{57} \\
 & + \gamma_{\{447,456,463\}} w_{56} + \gamma_{\{448,458,465\}} w_{58} + \gamma_{455} w_{59} + \gamma_{457} w_{60} \equiv_{\omega_d} 0, \\
 p(3;4)(\mathcal{U}) & \equiv_{\omega_d} \gamma_{\{118,128,306,311\}} w_5 + \gamma_{\{119,130,308,312\}} w_7 + \gamma_{\{120,132,310,313\}} w_9 \\
 & + \gamma_{\{147,155,334,337\}} w_{13} + \gamma_{\{148,157,336,338\}} w_{15} + \gamma_{\{284,287,294,392,473\}} w_{20} \\
 & + \gamma_{289} w_{21} + \gamma_{\{285,290,296,394,474\}} w_{22} + \gamma_{292} w_{23} + \gamma_{\{286,293,298,396,475\}} w_{24} \\
 & + \gamma_{305} w_{25} + \gamma_{307} w_{26} + \gamma_{309} w_{27} + \gamma_{319} w_{28} + \gamma_{\{317,320,325,401,404\}} w_{29} \\
 & + \gamma_{322} w_{30} + \gamma_{\{318,323,327,403,405\}} w_{31} + \gamma_{333} w_{32} + \gamma_{335} w_{33} + \gamma_{342} w_{34} \\
 & + \gamma_{\{341,343,345,409,410\}} w_{35} + \gamma_{391} w_{37} + \gamma_{393} w_{38} + \gamma_{395} w_{39} + \gamma_{400} w_{40} \\
 & + \gamma_{402} w_{41} + \gamma_{408} w_{42} + \gamma_{\{412,415,422,434,439\}} w_{44} + \gamma_{417} w_{45} \\
 & + \gamma_{\{413,418,424,436,440\}} w_{46} + \gamma_{420} w_{47} + \gamma_{\{414,421,426,438,441\}} w_{48} + \gamma_{433} w_{49} \\
 & + \gamma_{435} w_{50} + \gamma_{437} w_{51} + \gamma_{449} w_{55} + \gamma_{\{447,450,455,464,467\}} w_{56} + \gamma_{452} w_{57} \\
 & + \gamma_{\{448,453,457,466,468\}} w_{58} + \gamma_{463} w_{59} + \gamma_{465} w_{60} \equiv_{\omega_d} 0.
 \end{aligned}$$

Computing from these equalities gives

$$\gamma_j = 0 \text{ for } j \in \mathbb{J}_2, \quad \gamma_i = \gamma_j \text{ for } (i, j) \in \mathbb{K}_2, \quad (3.3)$$

where  $\mathbb{J}_2 = \{167, 168, 169, 170, 176, 179, 181, 183, 185, 187, 188, 189, 190, 193, 196, 199, 201, 203, 204, 206, 289, 292, 305, 307, 309, 319, 322, 333, 335, 342, 349, 350, 351, 352, 354, 356, 358, 359, 360, 362, 364, 365, 391, 393, 395, 400, 402, 408, 415, 416, 417, 418, 419, 420, 421, 422, 424, 426, 433, 435, 437, 449, 450, 451, 452, 453, 454, 455, 457, 463, 465\}$  and  $\mathbb{K}_2 = \{(1,51), (2,52), (2,235), (3,54), (4,55), (4,238), (8,69), (8,249), (9,71),$

(10,72), (10,252), (11,74), (14,82), (14,261), (15,84), (412,423), (413,425), (414,427), (447,456), (448,458)}.

Apply the homomorphism  $p_{(1;4)}, p_{(3;5)}$  to (3.1) and using (3.2), (3.3), to get

$$\begin{aligned}
p_{(1;4)}(\mathcal{U}) &\equiv_{\omega_d} \gamma_{\{5,58\}}w_5 + \gamma_{\{6,60\}}w_7 + \gamma_{\{7,62\}}w_9 + \gamma_{\{12,76\}}w_{13} + \gamma_{\{13,78\}}w_{15} \\
&\quad + \gamma_{\{47,95,212,240,266,294,325,345\}}w_{20} + \gamma_2w_{21} + \gamma_{\{48,97,214,242,268,296,327\}}w_{22} \\
&\quad + \gamma_4w_{23} + \gamma_{\{49,99,216,244,270,298\}}w_{24} + \gamma_{57}w_{25} + \gamma_{59}w_{26} + \gamma_{61}w_{27} + \gamma_8w_{28} \\
&\quad + \gamma_{\{67,103,220,255,275,325\}}w_{29} + \gamma_{10}w_{30} + \gamma_{\{68,105,222,257,277,327\}}w_{31} \\
&\quad + \gamma_{75}w_{32} + \gamma_{77}w_{33} + \gamma_{14}w_{34} + \gamma_{\{81,108,225,264,281,345\}}w_{35} + \gamma_{94}w_{37} \\
&\quad + \gamma_{\{96,325\}}w_{38} + \gamma_{\{98,327,345\}}w_{39} + \gamma_{102}w_{40} + \gamma_{104}w_{41} + \gamma_{107}w_{42} \\
&\quad + \gamma_{\{118,129\}}w_{44} + \gamma_{\{119,131\}}w_{46} + \gamma_{\{120,133\}}w_{48} + \gamma_{128}w_{49} + \gamma_{130}w_{50} \\
&\quad + \gamma_{132}w_{51} + \gamma_{\{147,156\}}w_{56} + \gamma_{\{148,158\}}w_{58} + \gamma_{155}w_{59} + \gamma_{157}w_{60} \equiv_{\omega_d} 0, \\
p_{(3;5)}(\mathcal{U}) &\equiv_{\omega_d} \gamma_{\{118,129\}}w_4 + \gamma_{\{119,131\}}w_6 + \gamma_{\{120,133\}}w_8 + \gamma_{\{147,156\}}w_{12} \\
&\quad + \gamma_{\{148,158\}}w_{14} + \gamma_{\{284,295\}}w_{19} + \gamma_{288}w_{20} + \gamma_{\{285,297\}}w_{21} + \gamma_{291}w_{22} \\
&\quad + \gamma_{\{286,299\}}w_{23} + \gamma_{306}w_{25} + \gamma_{308}w_{26} + \gamma_{310}w_{27} + \gamma_{\{317,326\}}w_{28} + \gamma_{321}w_{29} \\
&\quad + \gamma_{\{318,328\}}w_{30} + \gamma_{324}w_{31} + \gamma_{334}w_{32} + \gamma_{336}w_{33} + \gamma_{\{341,346\}}w_{34} + \gamma_{344}w_{35} \\
&\quad + \gamma_{392}w_{37} + \gamma_{394}w_{38} + \gamma_{396}w_{39} + \gamma_{401}w_{40} + \gamma_{403}w_{41} + \gamma_{409}w_{42} + \gamma_{434}w_{49} \\
&\quad + \gamma_{436}w_{50} + \gamma_{438}w_{51} + \gamma_{464}w_{59} + \gamma_{466}w_{60} \equiv_{\omega_d} 0.
\end{aligned}$$

From these equalities it implies

$$\gamma_j = 0 \text{ for } j \in \mathbb{J}_3, \quad \gamma_i = \gamma_j \text{ for } (i, j) \in \mathbb{K}_3, \quad (3.4)$$

where  $\mathbb{J}_3 = \{2, 4, 8, 10, 14, 52, 55, 57, 59, 61, 69, 72, 75, 77, 82, 94, 102, 104, 107, 128, 130, 132, 155, 157, 235, 238, 249, 252, 261, 288, 291, 306, 308, 310, 321, 324, 334, 336, 344, 392, 394, 396, 401, 403, 409, 434, 436, 438, 464, 466\}$  and  $\mathbb{K}_3 = \{(5,58), (6,60), (7,62), (12,76), (13,78), (96,325), (118,129), (118,311), (119,131), (119,312), (120,133), (120,313), (147,156), (147,337), (148,158), (148,338), (284,295), (285,297), (286,299), (317,326)\}$ .

By applying the homomorphisms  $p_{(1;5)}, p_{(2;5)}$  to (3.1) and using (3.2), (3.3), (3.4), we get

$$\begin{aligned}
p_{(1;5)}(\mathcal{U}) &\equiv_{\omega_d} \gamma_{\{1,175,234\}}w_1 + \gamma_{\{3,178,237\}}w_2 + \gamma_{\{5,182,246\}}w_4 + \gamma_{\{6,184,247\}}w_6 \\
&\quad + \gamma_{\{7,186,248\}}w_8 + \gamma_{\{9,195,251\}}w_{10} + \gamma_{\{11,198,254\}}w_{11} + \gamma_{\{12,200,259\}}w_{12} \\
&\quad + \gamma_{\{13,202,260\}}w_{14} + \gamma_{\{15,208,263\}}w_{16} + \gamma_{\{47,50,85,213,241,267,272,284\}}w_{19} \\
&\quad + \gamma_1w_{20} + \gamma_{\{48,53,86,96,215,243,269,273,285,317\}}w_{21} + \gamma_3w_{22} \\
&\quad + \gamma_{\{49,56,87,98,217,245,271,274,286,318,341\}}w_{23} + \gamma_5w_{25} + \gamma_6w_{26} + \gamma_7w_{27} \\
&\quad + \gamma_{\{67,70,88,221,256,276,279,317\}}w_{28} + \gamma_9w_{29} + \gamma_{\{68,73,89,223,258,278,280,318\}}w_{30} \\
&\quad + \gamma_{11}w_{31} + \gamma_{12}w_{32} + \gamma_{13}w_{33} + \gamma_{\{81,83,90,226,265,282,283,341\}}w_{34} + \gamma_{15}w_{35}
\end{aligned}$$

$$\begin{aligned}
 & + \gamma_{\{95,317,341\}}w_{37} + \gamma_{\{97,318\}}w_{38} + \gamma_{99}w_{39} + \gamma_{103}w_{40} + \gamma_{105}w_{41} + \gamma_{108}w_{42} \\
 & + \gamma_{118}w_{49} + \gamma_{119}w_{50} + \gamma_{120}w_{51} + \gamma_{147}w_{59} + \gamma_{148}w_{60} \equiv_{\omega_d} 0, \\
 p_{(2;5)}(\mathcal{U}) & \equiv_{\omega_d} \gamma_{\{171,174,213,368,373\}}w_{19} + \gamma_{175}w_{20} + \gamma_{\{172,177,215,370,374\}}w_{21} \\
 & + \gamma_{178}w_{22} + \gamma_{\{173,180,217,372,375\}}w_{23} + \gamma_{182}w_{25} + \gamma_{184}w_{26} + \gamma_{186}w_{27} \\
 & + \gamma_{\{191,194,221,377,380\}}w_{28} + \gamma_{195}w_{29} + \gamma_{\{192,197,223,379,381\}}w_{30} + \gamma_{198}w_{31} \\
 & + \gamma_{200}w_{32} + \gamma_{202}w_{33} + \gamma_{\{205,207,226,383,384\}}w_{34} + \gamma_{208}w_{35} + \gamma_{353}w_{37} \\
 & + \gamma_{355}w_{38} + \gamma_{357}w_{39} + \gamma_{361}w_{40} + \gamma_{363}w_{41} + \gamma_{366}w_{42} + \gamma_{412}w_{49} + \gamma_{413}w_{50} \\
 & + \gamma_{414}w_{51} + \gamma_{447}w_{59} + \gamma_{448}w_{60} \equiv_{\omega_d} 0.
 \end{aligned}$$

By a direct calculation using the above equalities we have

$$\gamma_j = 0 \text{ for } j \in \mathbb{J}_4, \quad \gamma_{97} = \gamma_{318}, \quad (3.5)$$

where  $\mathbb{J}_4 = \{1, 3, 5, 6, 7, 9, 11, 12, 13, 15, 51, 54, 58, 60, 62, 71, 74, 76, 78, 84, 99, 103, 105, 108, 118, 119, 120, 129, 131, 133, 147, 148, 156, 158, 175, 178, 182, 184, 186, 195, 198, 200, 202, 208, 234, 237, 246, 247, 248, 251, 254, 259, 260, 263, 311, 312, 313, 337, 338, 353, 355, 357, 361, 363, 366, 412, 413, 414, 423, 425, 427, 439, 440, 441, 447, 448, 456, 458, 467, 468\}$ .

Apply the homomorphisms  $p_{(1;(2,3))}, p_{(1;(2,4))}$  to (3.1) and using (3.2), (3.3), (3.4), (3.5) to obtain

$$\begin{aligned}
 p_{(1;(2,3))}(\mathcal{U}) & \equiv_{\omega_d} \gamma_{174}w_{25} + \gamma_{177}w_{26} + \gamma_{180}w_{27} + \gamma_{194}w_{32} + \gamma_{197}w_{33} + \gamma_{207}w_{36} \\
 & + \gamma_{233}w_{37} + \gamma_{236}w_{38} + \gamma_{239}w_{39} + \gamma_{250}w_{40} + \gamma_{253}w_{41} + \gamma_{262}w_{42} + \gamma_{287}w_{49} \\
 & + \gamma_{290}w_{50} + \gamma_{293}w_{51} + \gamma_{320}w_{59} + \gamma_{323}w_{60} + \gamma_{343}w_{65} \equiv_{\omega_d} 0, \\
 p_{(1;(2,4))}(\mathcal{U}) & \equiv_{\omega_d} \gamma_{\{47,85,95,96,171,212,240,241,266,294,345,367\}}w_{20} \\
 & + \gamma_{\{48,86,97,172,214,242,243,268,296,327,369\}}w_{22} \\
 & + \gamma_{\{49,87,173,216,244,245,270,298,371\}}w_{24} + \gamma_{\{67,88,96,191,220,255,256,275,376\}}w_{29} \\
 & + \gamma_{\{68,89,192,222,257,258,277,327,378\}}w_{31} + \gamma_{\{81,90,205,225,264,265,281,345,382\}}w_{35} \\
 & + \gamma_{240}w_{37} + \gamma_{242}w_{38} + \gamma_{\{98,244,327,345\}}w_{39} + \gamma_{255}w_{40} + \gamma_{257}w_{41} + \gamma_{264}w_{42} \\
 & + \gamma_{294}w_{49} + \gamma_{296}w_{50} + \gamma_{298}w_{51} + \gamma_{96}w_{59} + \gamma_{327}w_{60} + \gamma_{345}w_{65} \equiv_{\omega_d} 0.
 \end{aligned}$$

From the above relations we obtain

$$\gamma_j = 0 \text{ for } j \in \mathbb{J}_5, \quad \gamma_i = \gamma_j \text{ for } (i, j) \in \mathbb{K}_5, \quad (3.6)$$

where  $\mathbb{J}_5 = \{96, 98, 174, 177, 180, 194, 197, 207, 233, 236, 239, 240, 242, 244, 250, 253, 255, 257, 262, 264, 287, 290, 293, 294, 296, 298, 320, 323, 325, 327, 343, 345\}$  and  $\mathbb{K}_5 = \{(50,85), (53,86), (56,87), (70,88), (73,89), (83,90), (97,405), (284,473), (285,474), (286,475), (317,404), (341,410)\}$ .

Applying the homomorphisms  $p_{(1;(3,4))}$ ,  $p_{(1;(2,5))}$  to (3.1) and using (3.2), (3.3), (3.4), (3.5), (3.6) gives

$$\begin{aligned}
p_{(1;(3,4))}(\mathcal{U}) &\equiv_{\omega_d} \gamma_{\{171,213,368,373\}}w_5 + \gamma_{\{172,215,370,374\}}w_7 + \gamma_{\{173,217,372,375\}}w_9 \\
&+ \gamma_{\{191,221,377,380\}}w_{13} + \gamma_{\{192,223,379,381\}}w_{15} + \gamma_{\{205,226,383,384\}}w_{18} \\
&+ \gamma_{\{47,50,95,212,266,267,367,385\}}w_{20} + \gamma_{\{48,53,97,214,268,269,369,386\}}w_{22} \\
&+ \gamma_{\{49,56,216,270,271,371,387\}}w_{24} + \gamma_{\{212,272,367\}}w_{25} + \gamma_{\{214,273,369\}}w_{26} \\
&+ \gamma_{\{216,274,371\}}w_{27} + \gamma_{\{67,70,220,275,276,376,388\}}w_{29} \\
&+ \gamma_{\{68,73,222,277,278,378,389\}}w_{31} + \gamma_{\{220,279,376\}}w_{32} + \gamma_{\{222,280,378\}}w_{33} \\
&+ \gamma_{\{81,83,225,281,282,382,390\}}w_{35} + \gamma_{\{225,283,382\}}w_{36} + \gamma_{266}w_{37} + \gamma_{268}w_{38} \\
&+ \gamma_{270}w_{39} + \gamma_{275}w_{40} + \gamma_{277}w_{41} + \gamma_{281}w_{42} \equiv_{\omega_d} 0, \\
p_{(1;(2,5))}(\mathcal{U}) &\equiv_{\omega_d} \gamma_{\{47,50,171,213,241,267,272,284,368,373,385\}}w_{19} \\
&+ \gamma_{\{48,53,172,215,243,269,273,285,317,370,374,386\}}w_{21} \\
&+ \gamma_{\{49,56,97,173,217,245,271,274,286,341,372,375,387\}}w_{23} \\
&+ \gamma_{\{67,70,191,221,256,276,279,317,377,380,388\}}w_{28} \\
&+ \gamma_{\{68,73,97,192,223,258,278,280,379,381,389\}}w_{30} \\
&+ \gamma_{\{81,83,205,226,265,282,283,341,383,384,390\}}w_{34} + \gamma_{\{95,241,317,341\}}w_{37} \\
&+ \gamma_{243}w_{38} + \gamma_{245}w_{39} + \gamma_{256}w_{40} + \gamma_{258}w_{41} + \gamma_{265}w_{42} + \gamma_{284}w_{49} \\
&+ \gamma_{285}w_{50} + \gamma_{286}w_{51} + \gamma_{317}w_{59} + \gamma_{97}w_{60} + \gamma_{341}w_{65} \equiv_{\omega_d} 0.
\end{aligned}$$

By computing from these relations we obtain

$$\gamma_j = 0 \text{ for } j \in \mathbb{J}_6, \quad \gamma_i = \gamma_j \text{ for } (i, j) \in \mathbb{K}_6, \quad (3.7)$$

where  $\mathbb{J}_6 = \{95, 97, 241, 243, 245, 256, 258, 265, 266, 268, 270, 275, 277, 281, 284, 285, 286, 295, 297, 299, 317, 318, 326, 328, 341, 346, 404, 405, 410, 473, 474, 475\}$  and  $\mathbb{K}_6 = \{(47,212), (48,214), (49,216), (67,220), (68,222), (81,225)\}$ .

Apply the homomorphisms  $p_{(1;(3,5))}$ ,  $p_{(1;(4,5))}$  to (3.1) and using (3.2), (3.3), (3.4), (3.5), (3.6), (3.6) to get

$$\begin{aligned}
p_{(1;(3,5))}(\mathcal{U}) &\equiv_{\omega_d} \gamma_{\{47,171,367\}}w_4 + \gamma_{\{48,172,369\}}w_6 + \gamma_{\{49,173,371\}}w_8 \\
&+ \gamma_{\{67,191,376\}}w_{12} + \gamma_{\{68,192,378\}}w_{14} + \gamma_{\{81,205,382\}}w_{17} \\
&+ \gamma_{\{47,50,213,267,272,368\}}w_{19} + \gamma_{\{48,53,215,269,273,370\}}w_{21} \\
&+ \gamma_{\{49,56,217,271,274,372\}}w_{23} + \gamma_{\{213,272,368\}}w_{25} + \gamma_{\{215,273,370\}}w_{26} \\
&+ \gamma_{\{217,274,372\}}w_{27} + \gamma_{\{67,70,221,276,279,377\}}w_{28} + \gamma_{\{68,73,223,278,280,379\}}w_{30} \\
&+ \gamma_{\{221,279,377\}}w_{32} + \gamma_{\{223,280,379\}}w_{33} + \gamma_{\{81,83,226,282,283,383\}}w_{34} \\
&+ \gamma_{\{226,283,383\}}w_{36} + \gamma_{267}w_{37} + \gamma_{269}w_{38} + \gamma_{271}w_{39} + \gamma_{276}w_{40} \\
&+ \gamma_{278}w_{41} + \gamma_{282}w_{42} \equiv_{\omega_d} 0,
\end{aligned}$$

$$\begin{aligned}
 P_{(1;(4,5))}(\mathcal{U}) &\equiv_{\omega_d} \gamma_{\{213,267,272,373,385\}} w_{19} + \gamma_{\{213,267,373,385\}} w_{20} \\
 &+ \gamma_{\{215,269,273,374,386\}} w_{21} + \gamma_{\{215,269,374,386\}} w_{22} + \gamma_{\{217,271,274,375,387\}} w_{23} \\
 &+ \gamma_{\{217,271,375,387\}} w_{24} + \gamma_{\{221,276,279,380,388\}} w_{28} + \gamma_{\{221,276,380,388\}} w_{29} \\
 &+ \gamma_{\{223,278,280,381,389\}} w_{30} + \gamma_{\{223,278,381,389\}} w_{31} + \gamma_{\{226,282,283,384,390\}} w_{34} \\
 &+ \gamma_{\{226,282,384,390\}} w_{35} + \gamma_{272} w_{37} + \gamma_{273} w_{38} + \gamma_{274} w_{39} + \gamma_{279} w_{40} \\
 &+ \gamma_{280} w_{41} + \gamma_{283} w_{42} \equiv_{\omega_d} \mathbf{0}.
 \end{aligned}$$

By a direct computation from the above equalities we obtain  $\gamma_j = 0$  for  $1 \leq j \leq 475$ . The theorem is proved.  $\square$

### 3.3 On a structure of of the space $\text{Ker}((\widetilde{S}q_*^0)_{(5,20)})$

In this section we present some results on a structure of  $\text{Ker}((\widetilde{S}q_*^0)_{(5,20)})$ . Consider the homomorphism  $p_{(i;I)} : P_k \rightarrow P_{k-1}$  as defined by (2.2). We set

$$\begin{aligned}
 \widetilde{S}F_k(\omega) &= \bigcap_{(i;I) \in \mathcal{N}_k} \text{Ker}(p_{(i;I)}^{(\omega)}) \text{ and } \widetilde{Q}P_k(\omega) = QP_k(\omega) / \widetilde{S}F_k(\omega), \\
 (\widetilde{S}F_k)_m &= \bigcap_{(i;I) \in \mathcal{N}_k} \text{Ker}(p_{(i;I)}^{(m)}) \text{ and } (\widetilde{Q}P_k)_m = (QP_k)_m / (\widetilde{S}F_k)_m,
 \end{aligned}$$

where  $m = \deg \omega$ . Note that  $\widetilde{S}F_k(\omega) \subset QP_k^+(\omega)$  and  $(\widetilde{S}F_k)_m \subset (QP_k)_m$ .

We see that  $\widetilde{S}F_k(\omega)$  and  $(\widetilde{S}F_k)_m$  are respectively  $\mathbb{F}_2$ -subspaces of  $QP_k(\omega)$  and  $(\widetilde{Q}P_k)_m$ . Then we have

$$QP_k(\omega) \cong \widetilde{Q}P_k(\omega) \bigoplus \widetilde{S}F_k(\omega), \quad (QP_k)_m \cong (\widetilde{Q}P_k)_m \bigoplus (\widetilde{S}F_k)_m.$$

We denote by  $\widetilde{B}_k^+(\omega)$  the set of all elements in  $B_k^+(\omega)$  such that the set of all classes in  $\widetilde{Q}P_k(\omega)$  represented by  $x \in \widetilde{B}_k^+(\omega)$  is a basis of  $\widetilde{Q}P_k^+(\omega) = QP_k^+(\omega) / \widetilde{S}F_k(\omega)$ .

By an argument analogous to the one in Section 3.2 and using Lemmas 3.2.1 and 3.2.2, we have determined a set of generators for  $\text{Ker}((\widetilde{S}q_*^0)_{(5,20)})$  consisting of 939 monomials (see Subsection 5.2). In [22], we have prove the following.

**Proposition 3.3.1.** *We have*

$$\dim \widetilde{Q}P_5((3)^4) = 154, \quad \dim \widetilde{S}F_5((3)^4) \leq 5.$$

By a direct computation using this generating set we have

$$\widetilde{B}_5^+((3)^3|(1)^2) = B(3) \cup \{a_j : 1 \leq j \leq 450\}$$

$$\widetilde{SF}_5((3)|^3|(1)^2) = \langle \{[h_t] : 1 \leq t \leq 34\} \rangle,$$

where  $h_t$  are the polynomials with the leading monomials  $Y_t$  which are determined as follows:

$$\begin{array}{lll} Y_1 = x_1^3 x_2^3 x_3^4 x_4^7 x_5^{28} & Y_2 = x_1^3 x_2^3 x_3^7 x_4^4 x_5^{28} & Y_3 = x_1^3 x_2^3 x_3^7 x_4^{12} x_5^{20} \\ Y_4 = x_1^3 x_2^3 x_3^7 x_4^{28} x_5^4 & Y_5 = x_1^3 x_2^5 x_3^{14} x_4^7 x_5^{16} & Y_6 = x_1^3 x_2^5 x_3^{14} x_4^{16} x_5^7 \\ Y_7 = x_1^3 x_2^5 x_3^{14} x_4^{17} x_5^6 & Y_8 = x_1^3 x_2^5 x_3^{15} x_4^{16} x_5^6 & Y_9 = x_1^3 x_2^7 x_3^3 x_4^4 x_5^{28} \\ Y_{10} = x_1^3 x_2^7 x_3^3 x_4^{12} x_5^{20} & Y_{11} = x_1^3 x_2^7 x_3^3 x_4^{28} x_5^4 & Y_{12} = x_1^3 x_2^7 x_3^8 x_4^5 x_5^{22} \\ Y_{13} = x_1^3 x_2^7 x_3^8 x_4^{21} x_5^6 & Y_{14} = x_1^3 x_2^7 x_3^{13} x_4^{16} x_5^6 & Y_{15} = x_1^3 x_2^7 x_3^{27} x_4^4 x_5^4 \\ Y_{16} = x_1^3 x_2^{13} x_3^6 x_4^7 x_5^{16} & Y_{17} = x_1^3 x_2^{13} x_3^7 x_4^6 x_5^{16} & Y_{18} = x_1^3 x_2^{15} x_3^5 x_4^{16} x_5^6 \\ Y_{19} = x_1^7 x_2^3 x_3^3 x_4^4 x_5^{28} & Y_{20} = x_1^7 x_2^3 x_3^3 x_4^{12} x_5^{20} & Y_{21} = x_1^7 x_2^3 x_3^3 x_4^{28} x_5^4 \\ Y_{22} = x_1^7 x_2^3 x_3^8 x_4^5 x_5^{22} & Y_{23} = x_1^7 x_2^3 x_3^8 x_4^{21} x_5^6 & Y_{24} = x_1^7 x_2^3 x_3^{13} x_4^{16} x_5^6 \\ Y_{25} = x_1^7 x_2^3 x_3^{27} x_4^4 x_5^4 & Y_{26} = x_1^7 x_2^7 x_3^8 x_4^4 x_5^{22} & Y_{27} = x_1^7 x_2^7 x_3^8 x_4^{17} x_5^6 \\ Y_{28} = x_1^7 x_2^7 x_3^9 x_4^{16} x_5^6 & Y_{29} = x_1^7 x_2^7 x_3^{24} x_4^6 x_5^6 & Y_{30} = x_1^7 x_2^{11} x_3^5 x_4^{16} x_5^6 \\ Y_{31} = x_1^7 x_2^{11} x_3^{17} x_4^4 x_5^6 & Y_{32} = x_1^7 x_2^{11} x_3^{17} x_4^6 x_5^4 & Y_{33} = x_1^7 x_2^{27} x_3^3 x_4^4 x_5^4 \\ Y_{34} = x_1^{15} x_2^3 x_3^5 x_4^{16} x_5^6 & & \end{array}$$

By combining the above results and the ones in Subsection 4.1 we get the following.

**Proposition 3.3.2.** *We have*

$$\begin{aligned} \dim \widetilde{QP}_5((3)|^3|(1)^2) &= 905, \quad \dim \widetilde{SF}_5((3)|^3|(1)^2) \leq 34, \\ 1690 &\leq \dim(QP)_{45} \leq 1739. \end{aligned}$$

**Remark 3.3.3.** Dr. Nguyen Khac Tin informs us that by using a computer program based on the algorithm of MAGMA, he obtains  $\dim(QP)_{45} = 1731$ . However, we have been unable to verify this result by hand computation.

## 4 Structures of $(QP_5)_{2^{d+1}+2^d-4}$ for $3 \leq d \leq 5$

In this section we present a sketch of structure of the space  $(QP_5)_{2^{d+1}+2^d-4}$  with  $3 \leq d \leq 5$ . The detailed computations will be published elsewhere.

Let  $x$  be an admissible monomial of degree  $q = 2^{d+1} + 2^d - 4$  in  $P_5$  with  $d \geq 3$ . By Lemma 2.14 in [19],  $\omega_i(x) = 4$  for  $1 \leq i \leq d - 2$ . Hence,  $x = \bar{x}y^{2^{d-2}}$  with  $\bar{x}$  a monomial of weight vector  $(4)^{|d-2}$  and  $y$  a monomial of degree 8 in  $P_5$ . By combining this and a result in [25] we easily obtain

$$\omega(x) = (4)^{|d-2}|(2)|(1)^2 \text{ or } \omega(x) = (4)^{|d-2}|(2)|(3) \text{ or } \omega(x) = (4)^{|d-1}|(2). \quad (4.1)$$

This result is also presented in [8].

#### 4.1 On a structure of the space $(QP_5)_{20}$

By (4.1), we have

$$(QP_5)_{20} \cong QP_5(4, 2, 1, 1) \bigoplus QP_5(4, 2, 3) \bigoplus QP_5(4, 4, 2).$$

In [26] and [8], the authors say that

$$\dim QP_5(4, 2, 1, 1) = 450, \quad \dim QP_5(4, 2, 3) = 70, \quad \dim QP_5(4, 4, 2) = 121.$$

However, the detailed computation is only presented for the space  $QP_5(4, 2, 3)$ . The computation of  $QP_5(4, 2, 3)$  and  $QP_5(4, 4, 2)$  is easy and the above dimensional results are true. However, the computation of  $QP_5(4, 2, 1, 1)$  is rather complicated. To determine a generating set for  $QP_5(4, 2, 1, 1)$  we need the following which is easy to checked the accuracy.

**Lemma 4.1.1.** *If  $(i, j, t, u, v)$  is a permutation of  $(1, 2, 3, 4, 5)$  such that  $i < j < t < u$ , then the monomials  $x_i^2 x_j x_t x_u x_v^3$  is strictly inadmissible.*

By a direct computation analogous to the one in Subsection 3.2 and using Lemma 4.1.1, we have determined a set of generators for  $QP_5(4, 2, 1, 1)$  consisting of 450 monomials (see Subsection 6.1). By using this generating set we get

$$\widetilde{SF}_5(4, 2, 1, 1) = \langle \{g_t : 1 \leq t \leq 10\} \rangle,$$

where  $g_t$  are the polynomials with the leading monomials  $U_t$  which are determined as follows:

$$\begin{aligned} U_1 &= x_1^3 x_2^4 x_3 x_4^9 x_5^3 & U_2 &= x_1^3 x_2^4 x_3^9 x_4 x_5^3 & U_3 &= x_1^3 x_2^4 x_3^9 x_4^3 x_5 & U_4 &= x_1^3 x_2^{12} x_3 x_4 x_5^3 \\ U_5 &= x_1^3 x_2^{12} x_3 x_4^3 x_5 & U_6 &= x_1^3 x_2^{12} x_3^3 x_4 x_5 & U_7 &= x_1^7 x_2^8 x_3 x_4 x_5^3 & U_8 &= x_1^7 x_2^8 x_3 x_4^3 x_5 \\ U_9 &= x_1^7 x_2^8 x_3^3 x_4 x_5 & U_{10} &= x_1^7 x_2^9 x_3^2 x_4 x_5 \end{aligned}$$

If  $\dim QP_5(4, 2, 1, 1) = 450$ , then  $\dim \widetilde{SF}_5(4, 2, 1, 1) = 10$ . However, we have been unable to prove this by hand computation. Perhaps, this result be can only be checked by using a computer calculation.

From [20] we see that the monomials  $x_1^3 x_2^{12} x_3^3 x_4$ ,  $x_1^3 x_2^{12} x_3 x_4^3$ ,  $x_1^7 x_2^8 x_3^3 x_4$ ,  $x_1^7 x_2^8 x_3 x_4^3$ ,  $x_1^7 x_2^9 x_3^2 x_4$  are admissible. So, from Proposition 2.4.3,  $U_t$ ,  $4 \leq t \leq 10$ , are admissible. Hence, from the above equalities and by a simple calculation, we obtain the following.

**Proposition 4.1.2.** *We have*

- i)  $\dim \widetilde{QP}_k(4, 2, 1, 1) = 440$ .
- ii)  $\widetilde{SF}_5(4, 2, 1, 1)$  is an  $\mathbb{F}_2$ -subspace of  $QP_5(4, 2, 1, 1)$  and

$$7 \leq \dim \widetilde{SF}_5((4)|(2)^3) \leq 10.$$

**Remark 4.1.3.** In [8], the author presents the detailed computation for the easiest case,  $QP_5(4, 2, 3)$ , and he says that “the remaining ones can be acquired through analogous computational techniques”. However, Proposition 4.1.2 shows that his calculation techniques are not applicable to the case of  $QP_5(4, 2, 1, 1)$ .

## 4.2 On a structure of the space $(QP_5)_{44}$

By (4.1), we have

$$(QP_5)_{44} \cong QP_5((4)^2|(2)|^2) \oplus QP_5((4)^2|(2)|(3)) \oplus QP_5((4)^3|(2)).$$

The authors of [26] and [8] say that by using computer calculation, they obtain the dimensional result that  $\dim(QP_5)_{44} = 1426$ . However, at the time of writing, we do not find any detailed results for this space. We need the following technical lemmas.

**Lemma 4.2.1** (See [10]). *Let  $(i, j, t, u, v)$  is an arbitrary permutation of  $(1, 2, 3, 4, 5)$ . The following monomials are strictly inadmissible:*

$$x_i^2 x_j x_t^3 x_u^3 x_v^3, x_i^3 x_j^4 x_t^7 x_u^7 x_v^7, i < j; x_i^7 x_j^8 x_t^7 x_u^3 x_v^3, i = 1, j = 2; x_1^7 x_2^9 x_3^6 x_4^3 x_5^3.$$

**Lemma 4.2.2** (See [10]). *Let  $(i, j, t, u, v)$  is an arbitrary permutation of  $(1, 2, 3, 4, 5)$ . The following monomials are strictly inadmissible:*

$$\begin{aligned} & x_i^3 x_j^4 x_t^{11} x_u^{11} x_v^{15}, x_i^3 x_j^8 x_t^7 x_u^{11} x_v^{15}, x_i^3 x_j^{12} x_t^7 x_u^{11} x_v^{11}, i < j \\ & x_i^7 x_j^8 x_t^7 x_u^{11} x_v^{11}, x_i^7 x_j^9 x_t^6 x_u^{11} x_v^{11}, x_i^7 x_j^9 x_t^7 x_u^{11} x_v^{10}, x_i^7 x_j^{11} x_t^4 x_u^{11} x_v^{11}, i < j < t, \\ & x_i^7 x_j^7 x_t^8 x_u^7 x_v^{15}, i < j < t < u. \end{aligned}$$

**Lemma 4.2.3.** *Let  $(i, j, t, u, v)$  is an arbitrary permutation of  $(1, 2, 3, 4, 5)$ . The following monomials are strictly inadmissible:*

$$\begin{aligned} & x_i^3 x_j^4 x_t^3 x_u^7 x_v^{27}, x_i^3 x_j^4 x_t^3 x_u^{23} x_v^{11}, x_i^3 x_j^4 x_t^3 x_u^{15} x_v^{19}, x_i^3 x_j^{12} x_t^3 x_u^7 x_v^{19}, 1 < j < t, j < v; \\ & x_i^3 x_j^4 x_t^3 x_u^3 x_v^{31}, x_i^3 x_j^{28} x_t^3 x_u^3 x_v^7, x_i^3 x_j^{12} x_t^3 x_u^3 x_v^{23}, x_i^3 x_j^{15} x_t^{20} x_u^3 x_v^3, i < j < t < u, \\ & x_1^7 x_2^{11} x_3^4 x_4^3 x_5^{19}, x_1^7 x_2^{11} x_3^4 x_4^{19} x_5^3, x_1^7 x_2^{11} x_3^{20} x_4^3 x_5^3, x_1^7 x_2^{27} x_3^4 x_4^3 x_5^3, x_1^{15} x_2^{16} x_3^3 x_4^3 x_5^7, \\ & x_1^{15} x_2^{16} x_3^3 x_4^3 x_5^3, x_1^{15} x_2^{16} x_3^7 x_4^3 x_5^3, x_1^{15} x_2^{17} x_3^6 x_4^3 x_5^3, x_1^{15} x_2^{19} x_3^4 x_4^3 x_5^3. \end{aligned}$$

By a similar computation as in Subsection 3.2 and using Lemmas 4.2.1-4.2.3, we have determined a generating set for  $(QP_5)_{44}$  consisting of 1426 monomials and proved that  $\dim QP_5((4)^3|(2)) = 176$ ,  $\dim QP_5((4)^2|(2)|(3)) = 110$ , and  $1095 \leq \dim QP_5((4)^2|(2)|(1)^2) \leq 1140$ . The monomials of degree 44 in this



generating set are given in Subsection 6.2 and they are admissible. By a direct computation using this generating set we obtain

$$\begin{aligned} \widetilde{SF}_5((4)^3|(2)) &= 0; \quad \widetilde{SF}_5((4)^2|(2)|(3)) = 0 \\ \widetilde{SF}_5((4)^2|(2)|(1)^2) &= \langle \{\bar{g}_s : 1 \leq s \leq 45\} \rangle, \end{aligned}$$

where  $\bar{g}_s$  are the polynomials with the leading monomials  $V_s$  which are determined as follows:

$$\begin{array}{lll} V_1 = x_1^3 x_2^7 x_3^8 x_4^{19} x_5^7 & V_2 = x_1^3 x_2^7 x_3^{24} x_4^3 x_5^7 & V_3 = x_1^3 x_2^7 x_3^{24} x_4^7 x_5^3 \\ V_4 = x_1^3 x_2^{15} x_3^{16} x_4^3 x_5^7 & V_5 = x_1^3 x_2^{15} x_3^{16} x_4^7 x_5^3 & V_6 = x_1^3 x_2^{15} x_3^{19} x_4^4 x_5^3 \\ V_7 = x_1^7 x_2^3 x_3^8 x_4^{19} x_5^7 & V_8 = x_1^7 x_2^3 x_3^{24} x_4^3 x_5^7 & V_9 = x_1^7 x_2^3 x_3^{24} x_4^7 x_5^3 \\ V_{10} = x_1^7 x_2^9 x_3^2 x_4^{19} x_5^7 & V_{11} = x_1^7 x_2^9 x_3^3 x_4^{18} x_5^7 & V_{12} = x_1^7 x_2^9 x_3^3 x_4^{19} x_5^6 \\ V_{13} = x_1^7 x_2^9 x_3^{18} x_4^3 x_5^7 & V_{14} = x_1^7 x_2^9 x_3^{18} x_4^7 x_5^3 & V_{15} = x_1^7 x_2^9 x_3^{19} x_4^2 x_5^7 \\ V_{16} = x_1^7 x_2^9 x_3^{19} x_4^3 x_5^6 & V_{17} = x_1^7 x_2^9 x_3^{19} x_4^6 x_5^3 & V_{18} = x_1^7 x_2^9 x_3^{19} x_4^7 x_5^2 \\ V_{19} = x_1^7 x_2^{11} x_3^{16} x_4^3 x_5^7 & V_{20} = x_1^7 x_2^{11} x_3^{16} x_4^7 x_5^3 & V_{21} = x_1^7 x_2^{11} x_3^{19} x_4^4 x_5^3 \\ V_{22} = x_1^7 x_2^{15} x_3^{16} x_4^3 x_5^3 & V_{23} = x_1^7 x_2^{25} x_3^2 x_4^3 x_5^7 & V_{24} = x_1^7 x_2^{25} x_3^2 x_4^7 x_5^3 \\ V_{25} = x_1^7 x_2^{25} x_3^3 x_4^2 x_5^7 & V_{26} = x_1^7 x_2^{25} x_3^3 x_4^6 x_5^3 & V_{27} = x_1^7 x_2^{25} x_3^3 x_4^6 x_5^3 \\ V_{28} = x_1^7 x_2^{25} x_3^3 x_4^2 x_5^2 & V_{29} = x_1^7 x_2^{25} x_3^7 x_4^2 x_5^3 & V_{30} = x_1^7 x_2^{25} x_3^7 x_4^2 x_5^2 \\ V_{31} = x_1^{15} x_2^3 x_3^{16} x_4^3 x_5^7 & V_{32} = x_1^{15} x_2^3 x_3^{16} x_4^7 x_5^3 & V_{33} = x_1^{15} x_2^3 x_3^{19} x_4^4 x_5^3 \\ V_{34} = x_1^{15} x_2^7 x_3^{16} x_4^3 x_5^3 & V_{35} = x_1^{15} x_2^{17} x_3^2 x_4^3 x_5^7 & V_{36} = x_1^{15} x_2^{17} x_3^2 x_4^7 x_5^3 \\ V_{37} = x_1^{15} x_2^{17} x_3^3 x_4^2 x_5^7 & V_{38} = x_1^{15} x_2^{17} x_3^3 x_4^6 x_5^3 & V_{39} = x_1^{15} x_2^{17} x_3^3 x_4^6 x_5^3 \\ V_{40} = x_1^{15} x_2^{17} x_3^3 x_4^2 x_5^2 & V_{41} = x_1^{15} x_2^{17} x_3^7 x_4^2 x_5^3 & V_{42} = x_1^{15} x_2^{17} x_3^7 x_4^2 x_5^2 \\ V_{43} = x_1^{15} x_2^{19} x_3^3 x_4^3 x_5^3 & V_{44} = x_1^{15} x_2^{19} x_3^5 x_4^2 x_5^3 & V_{45} = x_1^{15} x_2^{19} x_3^5 x_4^2 x_5^2 \end{array}$$

If  $\dim(QP_5)_{44} = 1426$ , then  $\dim \widetilde{SF}_5((4)^2|(2)|(1)^2) = 45$ . However, this result can not be proved by hand computation. From the above result, we obtain the following.

**Proposition 4.2.4.** *We have*

- i)  $\dim \widetilde{QP}_k((4)^2|(2)|(1)^2) = 1095$ .
- ii)  $\widetilde{SF}_5((4)^2|(2)|(1)^2)$  is an  $\mathbb{F}_2$ -subspace of  $QP_5((4)^2|(2)|(1)^2)$  and  $\dim \widetilde{SF}_5((4)^2|(2)|(1)^2) \leq 45$ .

### 4.3 On a structure of the space $(QP_5)_{92}$

By (4.1), we have

$$(QP_5)_{92} \cong QP_5((4)^3|(2)|(1)^2) \oplus QP_5((4)^3|(2)|(3)) \oplus QP_5((4)^4|(2)).$$

The authors of [26] and [8] say that by using computer calculation, they obtains the dimensional result  $\dim(QP_5)_{92} = 1706$ . However, at the time of writing, the detailed computations were not available in their works. Moreover, we also do not find the detailed results for this space in any documents. To determined  $(QP_5)_{92}$  we need some technical lemmas.

**Lemma 4.3.1** (See [10]). *Let  $(i, j, t, u, v)$  is an arbitrary permutation of  $(1, 2, 3, 4, 5)$ . The following monomials are strictly inadmissible:*

$$x_i^7 x_j^7 x_t^{15} x_u^{16} x_v^{15}, t < u < v; x_i^7 x_j^{15} x_t^{17} x_u^6 x_v^{15}, i, j < t = 3; x_1^{15} x_2^{19} x_3^5 x_4^{14} x_5^7;$$

$$x_i^7 x_j^{15} x_t^{17} x_u^{14} x_v^7, (u, v) = (4, 5), x_i^{15} x_j^{19} x_t^5 x_u^6 x_v^{15}, (i, j) = (1, 2).$$

**Lemma 4.3.2.** *Let  $(i, j, t, u, v)$  is an arbitrary permutation of  $(1, 2, 3, 4, 5)$ . The following monomials are strictly inadmissible:*

$$x_i^7 x_j^7 x_t^{15} x_u^{31} x_v^{32}; x_i^7 x_j^{15} x_t^{23} x_u^{39} x_v^8, x_i^7 x_j^{15} x_t^{31} x_u^{33} x_v^6; x_i^7 x_j^{31} x_t^{39} x_u^9 x_v^6; x_i^7 x_j^7 x_t^{31} x_u^{39} x_v^8;$$

$$x_i^7 x_j^{15} x_t^{23} x_u^{15} x_v^{32}, j < t < u; x_i^7 x_j^{31} x_t^{33} x_u^7 x_v^{14}, i, j < t = 3;$$

$$x_i^7 x_j^{15} x_t^{17} x_u^{39} x_v^{14}, (u, v) = (4, 5); x_i^{15} x_j^{31} x_t^{35} x_u^5 x_v^6, j < j < u < v;$$

$$x_i^{15} x_j^{19} x_t^5 x_u^{14} x_v^{39}, x_i^{15} x_j^{19} x_t^7 x_u^{37} x_v^{14}, x_i^{15} x_j^{51} x_t^5 x_u^7 x_v^{14}, x_i^{31} x_j^{35} x_t^5 x_u^7 x_v^{14}, (i, j) = (1, 2);$$

$$x_1^7 x_2^{15} x_3^{49} x_4^7 x_5^{14}, x_1^{15} x_2^7 x_3^{49} x_4^7 x_5^{14}, x_1^{15} x_2^{19} x_3^5 x_4^{46} x_5^7, x_1^{15} x_2^{19} x_3^{39} x_4^{13} x_5^6,$$

$$x_1^{15} x_2^{23} x_3^{39} x_4^9 x_5^6, x_1^{15} x_2^{51} x_3^7 x_4^{13} x_5^6, x_1^{31} x_2^{35} x_3^7 x_4^{13} x_5^6, x_1^{31} x_2^{39} x_3^{11} x_4^5 x_5^6,$$

**Lemma 4.3.3.** *Let  $(i, j, t, u, v)$  is an arbitrary permutation of  $(1, 2, 3, 4, 5)$ . The following monomials are strictly inadmissible:*

$$x_i^{15} x_j^{15} x_t^{15} x_u^{16} x_v^{31}, x_i^{15} x_j^{15} x_t^{16} x_u^{23} x_v^{23}, x_1^{15} x_2^{15} x_3^{23} x_4^{17} x_5^{22}.$$

By using the above lemma and an analogous argument as in Subsection 3.2, we have determined a generating set for  $(QP_5)_{92}$  consisting of 1711 monomials and proved that  $\dim QP_5((4)|^4|(2)) = 186$ ,  $\dim QP_5((4)|^2|(2)|(3)) = 124$  and  $1395 \leq \dim QP_5((4)|^2|(2)|(1)|^2) \leq 1401$ . The monomials in this generating set are explicitly given in Subsection 6.3. By a direct computation using this generating set, we get

$$\widetilde{SF}_5((4)|^4|(2)) = 0; \quad \widetilde{SF}_5((4)|^3|(2)|(3)) = 0$$

$$\widetilde{SF}_5((4)|^3|(2)|(1)|^2) = \langle \{[\tilde{g}_u] : 1 \leq u \leq 6\} \rangle,$$

where  $\tilde{g}_u$  are the polynomials with the leading monomials  $W_u$  which are determined as follows:

$$W_1 = x_1^{15} x_2^{15} x_3^{23} x_4^{32} x_5^7 \quad W_2 = x_1^7 x_2^{15} x_3^{17} x_4^7 x_5^{46} \quad W_3 = x_1^{15} x_2^7 x_3^{17} x_4^7 x_5^{46}$$

By computing from the above result, we obtain the following.

**Proposition 4.3.4.** *We have*

- i)  $\dim \widetilde{QP}_k((4)|^3|(2)|(1)|^2) = 1395$ .
- ii)  $\widetilde{SF}_5((4)|^3|(2)|(1)|^2)$  is an  $\mathbb{F}_2$ -subspace of  $QP_5((4)|^3|(2)|(1)|^2)$  and

$$\dim \widetilde{SF}_5((4)|^3|(2)|(1)|^2) \leq 6.$$

From our computation of  $\widetilde{SF}_5((4)^3(2)|(1)^2)$ , we obtain the following.

**Proposition 4.3.5.** *If  $\dim(QP_5)_{92} = 1706$ , then  $[\tilde{g}_1] \neq 0$  and*

$$\widetilde{SF}_5((4)^3(2)|(1)^2) = \langle [\tilde{g}_1] \rangle.$$

*Proof.* For  $1 \leq s < 5$ , denote by  $\psi_s : P_5 \rightarrow P_5$  the homomorphism of  $\mathcal{A}$ -algebras given by

$$\psi_s(x_t) = \begin{cases} x_{s+1}, & \text{if } t = s, \\ x_s, & \text{if } t = s + 1, \\ x_t, & \text{if } t \neq s. \end{cases}$$

Then,  $\{\psi_1, \psi_2, \psi_3, \psi_4\}$  is a set of generators for the symmetric group  $\Sigma_5 \subset GL_5$ .

We have

$$\begin{aligned} \tilde{g}_2 = & x_1 x_2^7 x_3^{15} x_4^7 x_5^{62} + x_1 x_2^{15} x_3^{15} x_4^7 x_5^{54} + x_1 x_2^{15} x_3^{15} x_4^{22} x_5^{39} + x_1 x_2^{15} x_3^{15} x_4^{23} x_5^{38} \\ & + x_1^3 x_2^5 x_3^{15} x_4^{23} x_5^{46} + x_1^3 x_2^5 x_3^{15} x_4^{30} x_5^{39} + x_1^3 x_2^7 x_3^7 x_4^{13} x_5^{62} + x_1^3 x_2^7 x_3^7 x_4^{29} x_5^{46} \\ & + x_1^3 x_2^7 x_3^{13} x_4^{23} x_5^{46} + x_1^3 x_2^7 x_3^{13} x_4^{30} x_5^{39} + x_1^3 x_2^7 x_3^{15} x_4^{13} x_5^{54} + x_1^3 x_2^7 x_3^{29} x_4^7 x_5^{46} \\ & + x_1^3 x_2^{15} x_3^5 x_4^{23} x_5^{46} + x_1^3 x_2^{15} x_3^5 x_4^{30} x_5^{39} + x_1^3 x_2^{15} x_3^7 x_4^{13} x_5^{54} + x_1^3 x_2^{15} x_3^7 x_4^{21} x_5^{46} \\ & + x_1^3 x_2^{15} x_3^{15} x_4^{21} x_5^{38} + x_1^3 x_2^{15} x_3^{15} x_4^{21} x_5^{46} + x_1^7 x_2 x_3^{15} x_4^{23} x_5^{46} + x_1^7 x_2 x_3^{15} x_4^{30} x_5^{39} \\ & + x_1^7 x_2 x_3^7 x_4^9 x_5^{62} + x_1^7 x_2 x_3^7 x_4^{25} x_5^{46} + x_1^7 x_2 x_3^9 x_4^{23} x_5^{46} + x_1^7 x_2 x_3^9 x_4^{30} x_5^{39} \\ & + x_1^7 x_2 x_3^{15} x_4^7 x_5^{56} + x_1^7 x_2 x_3^{15} x_4^9 x_5^{54} + x_1^7 x_2 x_3^{25} x_4^7 x_5^{46} + x_1^7 x_2^{15} x_3 x_4^{23} x_5^{46} \\ & + x_1^7 x_2^{15} x_3 x_4^{30} x_5^{39} + x_1^7 x_2^{15} x_3^9 x_4^9 x_5^{54} + x_1^7 x_2^{15} x_3^7 x_4^{17} x_5^{46} + x_1^7 x_2^{15} x_3^{15} x_4^7 x_5^{48} \\ & + x_1^7 x_2^{15} x_3^{15} x_4^{16} x_5^{39} + x_1^7 x_2^{15} x_3^{15} x_4^{17} x_5^{38} + x_1^7 x_2^{15} x_3^{15} x_4^{23} x_5^{32} + x_1^7 x_2^{15} x_3^{17} x_4^7 x_5^{46}, \end{aligned}$$

and  $\tilde{g}_3 = \psi_1(\tilde{g}_2)$ . Hence, if  $[\tilde{g}_2] \neq 0$ , then  $[\tilde{g}_3] \neq 0$  and  $\dim(QP_5)_{92} \geq 1707$ . This is a contradiction. Therefore,  $[\tilde{g}_2] = [\tilde{g}_3] = 0$ .

We have

$$\begin{aligned} \tilde{g}_5 = & x_1 x_2^{15} x_3^7 x_4^{30} x_5^{39} + x_1 x_2^{15} x_3^{23} x_4^7 x_5^{46} + x_1^3 x_2^7 x_3^7 x_4^{29} x_5^{46} + x_1^3 x_2^7 x_3^{13} x_4^{30} x_5^{39} \\ & + x_1^3 x_2^7 x_3^{15} x_4^{13} x_5^{54} + x_1^3 x_2^7 x_3^{15} x_4^{21} x_5^{46} + x_1^3 x_2^7 x_3^{29} x_4^7 x_5^{46} + x_1^3 x_2^{13} x_3^7 x_4^{30} x_5^{39} \\ & + x_1^3 x_2^{13} x_3^{23} x_4^7 x_5^{46} + x_1^3 x_2^{15} x_3^7 x_4^{21} x_5^{46} + x_1^3 x_2^{15} x_3^{13} x_4^{22} x_5^{39} + x_1^3 x_2^{15} x_3^{21} x_4^7 x_5^{46} \\ & + x_1^3 x_2^{15} x_3^{21} x_4^{14} x_5^{39} + x_1^3 x_2^{15} x_3^{23} x_4^5 x_5^{46} + x_1^3 x_2^{15} x_3^{29} x_4^6 x_5^{39} + x_1^7 x_2 x_3^7 x_4^9 x_5^{62} \\ & + x_1^7 x_2 x_3^9 x_4^{30} x_5^{39} + x_1^7 x_2 x_3^{11} x_4^5 x_5^{62} + x_1^7 x_2 x_3^{11} x_4^{13} x_5^{54} + x_1^7 x_2 x_3^{15} x_4^9 x_5^{54} \\ & + x_1^7 x_2 x_3^{15} x_4^{17} x_5^{46} + x_1^7 x_2 x_3^{25} x_4^7 x_5^{46} + x_1^7 x_2 x_3^{27} x_4^5 x_5^{46} + x_1^7 x_2^{11} x_3^7 x_4^{21} x_5^{46} \\ & + x_1^7 x_2^{11} x_3^{13} x_4^{22} x_5^{39} + x_1^7 x_2^{11} x_3^{21} x_4^7 x_5^{46} + x_1^7 x_2^{11} x_3^{21} x_4^{14} x_5^{39} + x_1^7 x_2^{11} x_3^{23} x_4^5 x_5^{46} \\ & + x_1^7 x_2^{11} x_3^{29} x_4^6 x_5^{39} + x_1^7 x_2^{15} x_3^7 x_4^6 x_5^{62} + x_1^7 x_2^{15} x_3^9 x_4^5 x_5^{54} + x_1^7 x_2^{15} x_3^{11} x_4^5 x_5^{54} \\ & + x_1^7 x_2^{27} x_3^7 x_4^5 x_5^{46} + x_1^{15} x_2 x_3^7 x_4^{30} x_5^{39} + x_1^{15} x_2 x_3^{23} x_4^7 x_5^{46} + x_1^{15} x_2 x_3^3 x_4^{21} x_5^{46} \\ & + x_1^{15} x_2^3 x_3^{13} x_4^{22} x_5^{39} + x_1^{15} x_2^3 x_3^{21} x_4^7 x_5^{46} + x_1^{15} x_2^3 x_3^{21} x_4^{14} x_5^{39} + x_1^{15} x_2^3 x_3^{23} x_4^5 x_5^{46} \end{aligned}$$

$$\begin{aligned}
& + x_1^{15} x_2^3 x_3^{29} x_4^6 x_5^{39} + x_1^{15} x_2^7 x_3^7 x_4^9 x_5^{54} + x_1^{15} x_2^7 x_3^7 x_4^{17} x_5^{46} + x_1^{15} x_2^7 x_3^{11} x_4^5 x_5^{54} \\
& + x_1^{15} x_2^7 x_3^{19} x_4^5 x_5^{46} + x_1^{15} x_2^{15} x_3^7 x_4 x_5^{54} + x_1^{15} x_2^{15} x_3^7 x_4^{16} x_5^{39} + x_1^{15} x_2^{15} x_3^{17} x_4^6 x_5^{39} \\
& + x_1^{15} x_2^{15} x_3^{17} x_4^7 x_5^{38} + x_1^{15} x_2^{15} x_3^{19} x_4^5 x_5^{38} + x_1^{15} x_2^{15} x_3^{23} x_4 x_5^{38} + x_1^{15} x_2^{15} x_3^{23} x_4^7 x_5^{32} \\
& + x_1^{15} x_2^{19} x_3^7 x_4^5 x_5^{46},
\end{aligned}$$

and  $\tilde{g}_4 \equiv \psi_3(\tilde{g}_5)$ ,  $\tilde{g}_6 \equiv \psi_4(\tilde{g}_5)$ . So, if  $[\tilde{g}_5] \neq 0$ , then  $\dim(QP_5)_{92} \geq 1708$  and we have a contradiction. This implies  $[\tilde{g}_4] = [\tilde{g}_5] = [\tilde{g}_6] = 0$ . Combining these facts and the hypothesis that  $\dim(QP_5)_{92} = 1706$ , we must have  $[\tilde{g}_1] \neq 0$  and the proposition is proved.  $\square$

**Remark 4.3.6.** For any  $d \geq 6$ , by combining Theorem 1.3 and 1.4 in [20] we easily obtain

$$\dim(QP_5)_{2^{d+1}+2^d-4} = (2^5 - 1) \dim(QP_4)_8 = 31 \times 55 = 1705.$$

The author of [8] says that by using a computer program, he has checked the accuracy of the above result for any  $d \geq 6$ , but we do not believe that there is a computer program that can check this result for infinitely many values of  $d$ .

## 5 The admissible monomials of degree $2^{d+2} + 2^{d+1} - 3$

In this section, we list all elements in the set

$$B_5((3)|^d|(1)|^2) = B_5^0((3)|^d|(1)|^2) \cup B_5^+((3)|^d|(1)|^2)$$

for all  $d \geq 3$ .

By Kameko [4], we have  $B_3^+((3)|^d|(1)|^2) = \{v_j = v_{d,j} : 1 \leq j \leq 6\}$ , where

$$\begin{aligned}
v_1 &= x_1^{2^d-1} x_2^{2^d-1} x_3^{2^{d+2}-1} & v_2 &= x_1^{2^d-1} x_2^{2^{d+1}-1} x_3^{2^{d+1}+2^d-1} \\
v_3 &= x_1^{2^d-1} x_2^{2^{d+2}-1} x_3^{2^d-1} & v_4 &= x_1^{2^{d+1}-1} x_2^{2^d-1} x_3^{2^{d+1}+2^d-1} \\
v_5 &= x_1^{2^{d+1}-1} x_2^{2^{d+1}+2^d-1} x_3^{2^d-1} & v_6 &= x_1^{2^{d+2}-1} x_2^{2^d-1} x_3^{2^d-1}
\end{aligned}$$

From [20], for,  $d \geq 4$ , we have  $B_4^+((3)|^d|(1)|^2) = \{w_j = w_{d,j} : 1 \leq j \leq 66\}$ , where

$$\begin{aligned}
 w_1 &= x_1 x_2^{2^d-2} x_3^{2^d-1} x_4^{2^{d+2}-1} & w_2 &= x_1 x_2^{2^d-2} x_3^{2^{d+1}-1} x_4^{2^{d+1}+2^d-1} \\
 w_3 &= x_1 x_2^{2^d-2} x_3^{2^{d+2}-1} x_4^{2^d-1} & w_4 &= x_1 x_2^{2^d-1} x_3^{2^d-2} x_4^{2^{d+2}-1} \\
 w_5 &= x_1 x_2^{2^d-1} x_3^{2^d-1} x_4^{2^{d+2}-2} & w_6 &= x_1 x_2^{2^d-1} x_3^{2^{d+1}-2} x_4^{2^{d+1}+2^d-1} \\
 w_7 &= x_1 x_2^{2^d-1} x_3^{2^{d+1}-1} x_4^{2^{d+1}+2^d-2} & w_8 &= x_1 x_2^{2^d-1} x_3^{2^{d+2}-2} x_4^{2^d-1} \\
 w_9 &= x_1 x_2^{2^d-1} x_3^{2^{d+2}-1} x_4^{2^d-2} & w_{10} &= x_1 x_2^{2^{d+1}-2} x_3^{2^d-1} x_4^{2^{d+1}+2^d-1} \\
 w_{11} &= x_1 x_2^{2^{d+1}-2} x_3^{2^{d+1}+2^d-1} x_4^{2^d-1} & w_{12} &= x_1 x_2^{2^{d+1}-1} x_3^{2^d-2} x_4^{2^{d+1}+2^d-1} \\
 w_{13} &= x_1 x_2^{2^{d+1}-1} x_3^{2^d-1} x_4^{2^{d+1}+2^d-2} & w_{14} &= x_1 x_2^{2^{d+1}-1} x_3^{2^{d+1}+2^d-2} x_4^{2^d-1} \\
 w_{15} &= x_1 x_2^{2^{d+1}-1} x_3^{2^{d+1}+2^d-1} x_4^{2^d-2} & w_{16} &= x_1 x_2^{2^{d+2}-2} x_3^{2^d-1} x_4^{2^d-1} \\
 w_{17} &= x_1 x_2^{2^{d+2}-1} x_3^{2^d-2} x_4^{2^d-1} & w_{18} &= x_1 x_2^{2^{d+2}-1} x_3^{2^d-1} x_4^{2^d-2} \\
 w_{19} &= x_1^3 x_2^{2^d-3} x_3^{2^d-2} x_4^{2^{d+2}-1} & w_{20} &= x_1^3 x_2^{2^d-3} x_3^{2^d-1} x_4^{2^{d+2}-2} \\
 w_{21} &= x_1^3 x_2^{2^d-3} x_3^{2^{d+1}-2} x_4^{2^{d+1}+2^d-1} & w_{22} &= x_1^3 x_2^{2^d-3} x_3^{2^{d+1}-1} x_4^{2^{d+1}+2^d-2} \\
 w_{23} &= x_1^3 x_2^{2^d-3} x_3^{2^{d+2}-2} x_4^{2^d-1} & w_{24} &= x_1^3 x_2^{2^d-3} x_3^{2^{d+2}-1} x_4^{2^d-2} \\
 w_{25} &= x_1^3 x_2^{2^d-1} x_3^{2^d-3} x_4^{2^{d+2}-2} & w_{26} &= x_1^3 x_2^{2^d-1} x_3^{2^{d+1}-3} x_4^{2^{d+1}+2^d-2} \\
 w_{27} &= x_1^3 x_2^{2^d-1} x_3^{2^{d+2}-3} x_4^{2^d-2} & w_{28} &= x_1^3 x_2^{2^{d+1}-3} x_3^{2^d-2} x_4^{2^{d+1}+2^d-1} \\
 w_{29} &= x_1^3 x_2^{2^{d+1}-3} x_3^{2^d-1} x_4^{2^{d+1}+2^d-2} & w_{30} &= x_1^3 x_2^{2^{d+1}-3} x_3^{2^{d+1}+2^d-2} x_4^{2^d-1} \\
 w_{31} &= x_1^3 x_2^{2^{d+1}-3} x_3^{2^{d+1}+2^d-1} x_4^{2^d-2} & w_{32} &= x_1^3 x_2^{2^{d+1}-1} x_3^{2^d-3} x_4^{2^{d+1}+2^d-2} \\
 w_{33} &= x_1^3 x_2^{2^{d+1}-1} x_3^{2^{d+1}+2^d-3} x_4^{2^d-2} & w_{34} &= x_1^3 x_2^{2^{d+2}-3} x_3^{2^d-2} x_4^{2^d-1} \\
 w_{35} &= x_1^3 x_2^{2^{d+2}-3} x_3^{2^d-1} x_4^{2^d-2} & w_{36} &= x_1^3 x_2^{2^{d+2}-1} x_3^{2^d-3} x_4^{2^d-2} \\
 w_{37} &= x_1^7 x_2^{2^d-5} x_3^{2^d-3} x_4^{2^{d+2}-2} & w_{38} &= x_1^7 x_2^{2^d-5} x_3^{2^{d+1}-3} x_4^{2^{d+1}+2^d-2} \\
 w_{39} &= x_1^7 x_2^{2^d-5} x_3^{2^{d+2}-3} x_4^{2^d-2} & w_{40} &= x_1^7 x_2^{2^{d+1}-5} x_3^{2^d-3} x_4^{2^{d+1}+2^d-2} \\
 w_{41} &= x_1^7 x_2^{2^{d+1}-5} x_3^{2^{d+1}+2^d-3} x_4^{2^d-2} & w_{42} &= x_1^7 x_2^{2^{d+2}-5} x_3^{2^d-3} x_4^{2^d-2} \\
 w_{43} &= x_1^{2^d-1} x_2 x_3^{2^d-2} x_4^{2^{d+2}-1} & w_{44} &= x_1^{2^d-1} x_2 x_3^{2^d-1} x_4^{2^{d+2}-2} \\
 w_{45} &= x_1^{2^d-1} x_2 x_3^{2^{d+1}-2} x_4^{2^{d+1}+2^d-1} & w_{46} &= x_1^{2^d-1} x_2 x_3^{2^{d+1}-1} x_4^{2^{d+1}+2^d-2} \\
 w_{47} &= x_1^{2^d-1} x_2 x_3^{2^{d+2}-2} x_4^{2^d-1} & w_{48} &= x_1^{2^d-1} x_2 x_3^{2^{d+2}-1} x_4^{2^d-2} \\
 w_{49} &= x_1^{2^d-1} x_2^3 x_3^{2^d-3} x_4^{2^{d+2}-2} & w_{50} &= x_1^{2^d-1} x_2^3 x_3^{2^{d+1}-3} x_4^{2^{d+1}+2^d-2} \\
 w_{51} &= x_1^{2^d-1} x_2^3 x_3^{2^{d+2}-3} x_4^{2^d-2} & w_{52} &= x_1^{2^d-1} x_2^2 x_3^{2^d-1} x_4^{2^{d+2}-2} \\
 w_{53} &= x_1^{2^d-1} x_2^{2^{d+1}-1} x_3 x_4^{2^{d+1}+2^d-2} & w_{54} &= x_1^{2^d-1} x_2^{2^{d+2}-1} x_3 x_4^{2^d-2} \\
 w_{55} &= x_1^{2^{d+1}-1} x_2 x_3^{2^d-2} x_4^{2^{d+1}+2^d-1} & w_{56} &= x_1^{2^{d+1}-1} x_2 x_3^{2^d-1} x_4^{2^{d+1}+2^d-2} \\
 w_{57} &= x_1^{2^{d+1}-1} x_2 x_3^{2^{d+1}+2^d-2} x_4^{2^d-1} & w_{58} &= x_1^{2^{d+1}-1} x_2 x_3^{2^{d+1}+2^d-1} x_4^{2^d-2} \\
 w_{59} &= x_1^{2^{d+1}-1} x_2^3 x_3^{2^d-3} x_4^{2^{d+1}+2^d-2} & w_{60} &= x_1^{2^{d+1}-1} x_2^3 x_3^{2^{d+1}+2^d-3} x_4^{2^d-2} \\
 w_{61} &= x_1^{2^{d+1}-1} x_2^{2^d-1} x_3 x_4^{2^{d+1}+2^d-2} & w_{62} &= x_1^{2^{d+1}-1} x_2^{2^{d+1}+2^d-1} x_3 x_4^{2^d-2} \\
 w_{63} &= x_1^{2^{d+2}-1} x_2 x_3^{2^d-2} x_4^{2^d-1} & w_{64} &= x_1^{2^{d+2}-1} x_2 x_3^{2^d-1} x_4^{2^d-2} \\
 w_{65} &= x_1^{2^{d+2}-1} x_2^3 x_3^{2^d-3} x_4^{2^d-2} & w_{66} &= x_1^{2^{d+2}-1} x_2^{2^d-1} x_3 x_4^{2^d-2}
 \end{aligned}$$

For  $d = 4$ ,  $B_5^+ = \{w_{4,j} : j \neq 37, 38, 39\} \cup \{w_{37}, w_{38}, w_{39}\}$  where

$$w_{37} = x_1^7 x_2^7 x_3^7 x_4^{24}, \quad w_{38} = x_1^7 x_2^7 x_3^9 x_4^{22}, \quad w_{39} = x_1^7 x_2^7 x_3^{25} x_4^6.$$

By Proposition 2.4.2, we have  $|B_5^0((3)^d|(1)^2)| = \binom{5}{3} \times 6 + \binom{5}{4} \times 66 = 390$ .

### 5.1 The cases $d \geq 4$

For  $d \geq 4$ ,  $B_5^+((3)^d|(1)^2) = B(d) \cup C(d)$ , where  $C(d)$  is the set of the monomials  $a_j = a_{d,j}$ ,  $1 \leq j \leq 475$ , which are determined as follows:

For  $d \geq 4$ ,

$$\begin{aligned}
a_1 &= x_1 x_2 x_3^{2^d-2} x_4^{2^d-1} x_5^{2^{d+2}-2} & a_2 &= x_1 x_2 x_3^{2^d-2} x_4^{2^{d+1}-2} x_5^{2^{d+1}+2^d-1} \\
a_3 &= x_1 x_2 x_3^{2^d-2} x_4^{2^{d+1}-1} x_5^{2^{d+1}+2^d-2} & a_4 &= x_1 x_2 x_3^{2^d-2} x_4^{2^{d+2}-2} x_5^{2^d-1} \\
a_5 &= x_1 x_2 x_3^{2^d-1} x_4^{2^d-2} x_5^{2^{d+2}-2} & a_6 &= x_1 x_2 x_3^{2^d-1} x_4^{2^{d+1}-2} x_5^{2^{d+1}+2^d-2} \\
a_7 &= x_1 x_2 x_3^{2^d-1} x_4^{2^{d+2}-2} x_5^{2^d-2} & a_8 &= x_1 x_2 x_3^{2^{d+1}-2} x_4^{2^d-2} x_5^{2^{d+1}+2^d-1} \\
a_9 &= x_1 x_2 x_3^{2^{d+1}-2} x_4^{2^d-1} x_5^{2^{d+1}+2^d-2} & a_{10} &= x_1 x_2 x_3^{2^{d+1}-2} x_4^{2^{d+1}+2^d-2} x_5^{2^d-1} \\
a_{11} &= x_1 x_2 x_3^{2^{d+1}-2} x_4^{2^{d+1}+2^d-1} x_5^{2^d-2} & a_{12} &= x_1 x_2 x_3^{2^{d+1}-1} x_4^{2^d-2} x_5^{2^{d+1}+2^d-2} \\
a_{13} &= x_1 x_2 x_3^{2^{d+1}-1} x_4^{2^{d+1}+2^d-2} x_5^{2^d-2} & a_{14} &= x_1 x_2 x_3^{2^{d+2}-2} x_4^{2^d-2} x_5^{2^d-1} \\
a_{15} &= x_1 x_2 x_3^{2^{d+2}-2} x_4^{2^d-1} x_5^{2^d-2} & a_{16} &= x_1 x_2 x_3^{2^d-4} x_4^{2^{d+1}-1} x_5^{2^{d+1}+2^d-1} \\
a_{17} &= x_1 x_2 x_3^{2^d-3} x_4^{2^d-1} x_5^{2^{d+2}-2} & a_{18} &= x_1 x_2 x_3^{2^d-3} x_4^{2^{d+1}-2} x_5^{2^{d+1}+2^d-1} \\
a_{19} &= x_1 x_2 x_3^{2^d-3} x_4^{2^{d+1}-1} x_5^{2^{d+1}+2^d-2} & a_{20} &= x_1 x_2 x_3^{2^d-3} x_4^{2^{d+2}-2} x_5^{2^d-1} \\
a_{21} &= x_1 x_2 x_3^{2^d-1} x_4^{2^d-3} x_5^{2^{d+2}-2} & a_{22} &= x_1 x_2 x_3^{2^d-1} x_4^{2^d-1} x_5^{2^{d+2}-4} \\
a_{23} &= x_1 x_2 x_3^{2^d-1} x_4^{2^{d+1}-4} x_5^{2^{d+1}+2^d-1} & a_{24} &= x_1 x_2 x_3^{2^d-1} x_4^{2^{d+1}-3} x_5^{2^{d+1}+2^d-2} \\
a_{25} &= x_1 x_2 x_3^{2^d-1} x_4^{2^{d+1}-1} x_5^{2^{d+1}+2^d-4} & a_{26} &= x_1 x_2 x_3^{2^d-1} x_4^{2^{d+2}-4} x_5^{2^d-1} \\
a_{27} &= x_1 x_2 x_3^{2^d-1} x_4^{2^{d+2}-3} x_5^{2^d-2} & a_{28} &= x_1 x_2 x_3^{2^{d+1}-4} x_4^{2^d-1} x_5^{2^{d+1}+2^d-1} \\
a_{29} &= x_1 x_2 x_3^{2^{d+1}-4} x_4^{2^{d+1}+2^d-1} x_5^{2^d-1} & a_{30} &= x_1 x_2 x_3^{2^{d+1}-3} x_4^{2^d-2} x_5^{2^{d+1}+2^d-1} \\
a_{31} &= x_1 x_2 x_3^{2^{d+1}-3} x_4^{2^d-1} x_5^{2^{d+1}+2^d-2} & a_{32} &= x_1 x_2 x_3^{2^{d+1}-3} x_4^{2^{d+1}+2^d-2} x_5^{2^d-1} \\
a_{33} &= x_1 x_2 x_3^{2^{d+1}-3} x_4^{2^{d+1}+2^d-1} x_5^{2^d-2} & a_{34} &= x_1 x_2 x_3^{2^{d+1}-1} x_4^{2^d-4} x_5^{2^{d+1}+2^d-1} \\
a_{35} &= x_1 x_2 x_3^{2^{d+1}-1} x_4^{2^d-3} x_5^{2^{d+1}+2^d-2} & a_{36} &= x_1 x_2 x_3^{2^{d+1}-1} x_4^{2^d-1} x_5^{2^{d+1}+2^d-4} \\
a_{37} &= x_1 x_2 x_3^{2^{d+1}-1} x_4^{2^{d+1}+2^d-4} x_5^{2^d-1} & a_{38} &= x_1 x_2 x_3^{2^{d+1}-1} x_4^{2^{d+1}+2^d-3} x_5^{2^d-2} \\
a_{39} &= x_1 x_2 x_3^{2^{d+1}-1} x_4^{2^{d+1}+2^d-1} x_5^{2^d-4} & a_{40} &= x_1 x_2 x_3^{2^{d+2}-4} x_4^{2^d-1} x_5^{2^d-1} \\
a_{41} &= x_1 x_2 x_3^{2^{d+2}-3} x_4^{2^d-2} x_5^{2^d-1} & a_{42} &= x_1 x_2 x_3^{2^{d+2}-3} x_4^{2^d-1} x_5^{2^d-2} \\
a_{43} &= x_1 x_2 x_3^{2^d-4} x_4^{2^d-1} x_5^{2^{d+2}-2} & a_{44} &= x_1 x_2 x_3^{2^d-4} x_4^{2^{d+1}-2} x_5^{2^{d+1}+2^d-1} \\
a_{45} &= x_1 x_2 x_3^{2^d-4} x_4^{2^{d+1}-1} x_5^{2^{d+1}+2^d-2} & a_{46} &= x_1 x_2 x_3^{2^d-4} x_4^{2^{d+2}-2} x_5^{2^d-1} \\
a_{47} &= x_1 x_2 x_3^{2^d-3} x_4^{2^d-2} x_5^{2^{d+2}-2} & a_{48} &= x_1 x_2 x_3^{2^d-3} x_4^{2^{d+1}-2} x_5^{2^{d+1}+2^d-2} \\
a_{49} &= x_1 x_2 x_3^{2^d-3} x_4^{2^{d+2}-2} x_5^{2^d-2} & a_{50} &= x_1 x_2 x_3^{2^d-2} x_4^{2^d-3} x_5^{2^{d+2}-2} \\
a_{51} &= x_1 x_2 x_3^{2^d-2} x_4^{2^d-1} x_5^{2^{d+2}-4} & a_{52} &= x_1 x_2 x_3^{2^d-2} x_4^{2^{d+1}-4} x_5^{2^{d+1}+2^d-1} \\
a_{53} &= x_1 x_2 x_3^{2^d-2} x_4^{2^{d+1}-3} x_5^{2^{d+1}+2^d-2} & a_{54} &= x_1 x_2 x_3^{2^d-2} x_4^{2^{d+1}-1} x_5^{2^{d+1}+2^d-4} \\
a_{55} &= x_1 x_2 x_3^{2^d-2} x_4^{2^{d+2}-4} x_5^{2^d-1} & a_{56} &= x_1 x_2 x_3^{2^d-2} x_4^{2^{d+2}-3} x_5^{2^d-2}
\end{aligned}$$

$$\begin{aligned}
 a_{57} &= x_1 x_2^3 x_3^{2^d-1} x_4^{2^d-4} x_5^{2^{d+2}-2} & a_{58} &= x_1 x_2^3 x_3^{2^d-1} x_4^{2^d-2} x_5^{2^{d+2}-4} \\
 a_{59} &= x_1 x_2^3 x_3^{2^d-1} x_4^{2^{d+1}-4} x_5^{2^{d+1}+2^d-2} & a_{60} &= x_1 x_2^3 x_3^{2^d-1} x_4^{2^{d+1}-2} x_5^{2^{d+1}+2^d-4} \\
 a_{61} &= x_1 x_2^3 x_3^{2^d-1} x_4^{2^{d+2}-4} x_5^{2^d-2} & a_{62} &= x_1 x_2^3 x_3^{2^d-1} x_4^{2^{d+2}-2} x_5^{2^d-4} \\
 a_{63} &= x_1 x_2^3 x_3^{2^{d+1}-4} x_4^{2^d-2} x_5^{2^{d+1}+2^d-1} & a_{64} &= x_1 x_2^3 x_3^{2^{d+1}-4} x_4^{2^d-1} x_5^{2^{d+1}+2^d-2} \\
 a_{65} &= x_1 x_2^3 x_3^{2^{d+1}-4} x_4^{2^{d+1}+2^d-2} x_5^{2^d-1} & a_{66} &= x_1 x_2^3 x_3^{2^{d+1}-4} x_4^{2^{d+1}+2^d-1} x_5^{2^d-2} \\
 a_{67} &= x_1 x_2^3 x_3^{2^{d+1}-3} x_4^{2^d-2} x_5^{2^{d+1}+2^d-2} & a_{68} &= x_1 x_2^3 x_3^{2^{d+1}-3} x_4^{2^{d+1}+2^d-2} x_5^{2^d-2} \\
 a_{69} &= x_1 x_2^3 x_3^{2^{d+1}-2} x_4^{2^d-4} x_5^{2^{d+1}+2^d-1} & a_{70} &= x_1 x_2^3 x_3^{2^{d+1}-2} x_4^{2^d-3} x_5^{2^{d+1}+2^d-2} \\
 a_{71} &= x_1 x_2^3 x_3^{2^{d+1}-2} x_4^{2^d-1} x_5^{2^{d+1}+2^d-4} & a_{72} &= x_1 x_2^3 x_3^{2^{d+1}-2} x_4^{2^{d+1}+2^d-4} x_5^{2^d-1} \\
 a_{73} &= x_1 x_2^3 x_3^{2^{d+1}-2} x_4^{2^{d+1}+2^d-3} x_5^{2^d-2} & a_{74} &= x_1 x_2^3 x_3^{2^{d+1}-2} x_4^{2^{d+1}+2^d-1} x_5^{2^d-4} \\
 a_{75} &= x_1 x_2^3 x_3^{2^{d+1}-1} x_4^{2^d-4} x_5^{2^{d+1}+2^d-2} & a_{76} &= x_1 x_2^3 x_3^{2^{d+1}-1} x_4^{2^d-2} x_5^{2^{d+1}+2^d-4} \\
 a_{77} &= x_1 x_2^3 x_3^{2^{d+1}-1} x_4^{2^{d+1}+2^d-4} x_5^{2^d-2} & a_{78} &= x_1 x_2^3 x_3^{2^{d+1}-1} x_4^{2^{d+1}+2^d-2} x_5^{2^d-4} \\
 a_{79} &= x_1 x_2^3 x_3^{2^{d+2}-4} x_4^{2^d-2} x_5^{2^d-1} & a_{80} &= x_1 x_2^3 x_3^{2^{d+2}-4} x_4^{2^d-1} x_5^{2^d-2} \\
 a_{81} &= x_1 x_2^3 x_3^{2^{d+2}-3} x_4^{2^d-2} x_5^{2^d-2} & a_{82} &= x_1 x_2^3 x_3^{2^{d+2}-2} x_4^{2^d-4} x_5^{2^d-1} \\
 a_{83} &= x_1 x_2^3 x_3^{2^{d+2}-2} x_4^{2^d-3} x_5^{2^d-2} & a_{84} &= x_1 x_2^3 x_3^{2^{d+2}-2} x_4^{2^d-1} x_5^{2^d-4} \\
 a_{85} &= x_1 x_2^6 x_3^{2^d-5} x_4^{2^d-3} x_5^{2^{d+2}-2} & a_{86} &= x_1 x_2^6 x_3^{2^d-5} x_4^{2^{d+1}-3} x_5^{2^{d+1}+2^d-2} \\
 a_{87} &= x_1 x_2^6 x_3^{2^d-5} x_4^{2^{d+2}-3} x_5^{2^d-2} & a_{88} &= x_1 x_2^6 x_3^{2^{d+1}-5} x_4^{2^d-3} x_5^{2^{d+1}+2^d-2} \\
 a_{89} &= x_1 x_2^6 x_3^{2^{d+1}-5} x_4^{2^{d+1}+2^d-3} x_5^{2^d-2} & a_{90} &= x_1 x_2^6 x_3^{2^{d+2}-5} x_4^{2^d-3} x_5^{2^d-2} \\
 a_{91} &= x_1 x_2^7 x_3^{2^d-6} x_4^{2^d-3} x_5^{2^{d+2}-2} & a_{92} &= x_1 x_2^7 x_3^{2^d-6} x_4^{2^{d+1}-3} x_5^{2^{d+1}+2^d-2} \\
 a_{93} &= x_1 x_2^7 x_3^{2^d-6} x_4^{2^{d+2}-3} x_5^{2^d-2} & a_{94} &= x_1 x_2^7 x_3^{2^d-5} x_4^{2^d-4} x_5^{2^{d+2}-2} \\
 a_{95} &= x_1 x_2^7 x_3^{2^d-5} x_4^{2^d-2} x_5^{2^{d+2}-4} & a_{96} &= x_1 x_2^7 x_3^{2^d-5} x_4^{2^{d+1}-4} x_5^{2^{d+1}+2^d-2} \\
 a_{97} &= x_1 x_2^7 x_3^{2^d-5} x_4^{2^{d+1}-2} x_5^{2^{d+1}+2^d-4} & a_{98} &= x_1 x_2^7 x_3^{2^d-5} x_4^{2^{d+2}-4} x_5^{2^d-2} \\
 a_{99} &= x_1 x_2^7 x_3^{2^d-5} x_4^{2^{d+2}-2} x_5^{2^d-4} & a_{100} &= x_1 x_2^7 x_3^{2^{d+1}-6} x_4^{2^d-3} x_5^{2^{d+1}+2^d-2} \\
 a_{101} &= x_1 x_2^7 x_3^{2^{d+1}-6} x_4^{2^{d+1}+2^d-3} x_5^{2^d-2} & a_{102} &= x_1 x_2^7 x_3^{2^{d+1}-5} x_4^{2^d-4} x_5^{2^{d+1}+2^d-2} \\
 a_{103} &= x_1 x_2^7 x_3^{2^{d+1}-5} x_4^{2^d-2} x_5^{2^{d+1}+2^d-4} & a_{104} &= x_1 x_2^7 x_3^{2^{d+1}-5} x_4^{2^{d+1}+2^d-4} x_5^{2^d-2} \\
 a_{105} &= x_1 x_2^7 x_3^{2^{d+1}-5} x_4^{2^{d+1}+2^d-2} x_5^{2^d-4} & a_{106} &= x_1 x_2^7 x_3^{2^{d+2}-6} x_4^{2^d-3} x_5^{2^d-2} \\
 a_{107} &= x_1 x_2^7 x_3^{2^{d+2}-5} x_4^{2^d-4} x_5^{2^d-2} & a_{108} &= x_1 x_2^7 x_3^{2^{d+2}-5} x_4^{2^d-2} x_5^{2^d-4} \\
 a_{109} &= x_1 x_2^{2^d-2} x_3 x_4^{2^d-1} x_5^{2^{d+2}-2} & a_{110} &= x_1 x_2^{2^d-2} x_3 x_4^{2^{d+1}-2} x_5^{2^{d+1}+2^d-1} \\
 a_{111} &= x_1 x_2^{2^d-2} x_3 x_4^{2^{d+1}-1} x_5^{2^{d+1}+2^d-2} & a_{112} &= x_1 x_2^{2^d-2} x_3 x_4^{2^{d+2}-2} x_5^{2^d-1} \\
 a_{113} &= x_1 x_2^{2^d-2} x_3^2 x_4^{2^d-3} x_5^{2^{d+2}-2} & a_{114} &= x_1 x_2^{2^d-2} x_3^2 x_4^{2^{d+1}-3} x_5^{2^{d+1}+2^d-2} \\
 a_{115} &= x_1 x_2^{2^d-2} x_3^2 x_4^{2^{d+2}-3} x_5^{2^d-2} & a_{116} &= x_1 x_2^{2^d-2} x_3^2 x_4^{2^d-1} x_5^{2^{d+2}-2} \\
 a_{117} &= x_1 x_2^{2^d-2} x_3^{2^{d+1}-1} x_4 x_5^{2^{d+1}+2^d-2} & a_{118} &= x_1 x_2^{2^d-1} x_3 x_4^{2^d-2} x_5^{2^{d+2}-2} \\
 a_{119} &= x_1 x_2^{2^d-1} x_3 x_4^{2^{d+1}-2} x_5^{2^{d+1}+2^d-2} & a_{120} &= x_1 x_2^{2^d-1} x_3 x_4^{2^{d+2}-2} x_5^{2^d-2} \\
 a_{121} &= x_1 x_2^{2^d-1} x_3^2 x_4^{2^d-3} x_5^{2^{d+2}-2} & a_{122} &= x_1 x_2^{2^d-1} x_3^2 x_4^{2^d-1} x_5^{2^{d+2}-4} \\
 a_{123} &= x_1 x_2^{2^d-1} x_3^2 x_4^{2^{d+1}-4} x_5^{2^{d+1}+2^d-1} & a_{124} &= x_1 x_2^{2^d-1} x_3^2 x_4^{2^{d+1}-3} x_5^{2^{d+1}+2^d-2} \\
 a_{125} &= x_1 x_2^{2^d-1} x_3^2 x_4^{2^{d+1}-1} x_5^{2^{d+1}+2^d-4} & a_{126} &= x_1 x_2^{2^d-1} x_3^2 x_4^{2^{d+2}-4} x_5^{2^d-1} \\
 a_{127} &= x_1 x_2^{2^d-1} x_3^2 x_4^{2^{d+2}-3} x_5^{2^d-2} & a_{128} &= x_1 x_2^{2^d-1} x_3^3 x_4^{2^d-4} x_5^{2^{d+2}-2}
 \end{aligned}$$

$$\begin{aligned}
a_{129} &= x_1 x_2^{2^d-1} x_3^3 x_4^{2^d-2} x_5^{2^{d+2}-4} & a_{130} &= x_1 x_2^{2^d-1} x_3^3 x_4^{2^{d+1}-4} x_5^{2^{d+1}+2^d-2} \\
a_{131} &= x_1 x_2^{2^d-1} x_3^3 x_4^{2^{d+1}-2} x_5^{2^{d+1}+2^d-4} & a_{132} &= x_1 x_2^{2^d-1} x_3^3 x_4^{2^{d+2}-4} x_5^{2^d-2} \\
a_{133} &= x_1 x_2^{2^d-1} x_3^3 x_4^{2^{d+2}-2} x_5^{2^d-4} & a_{134} &= x_1 x_2^{2^d-1} x_3^2 x_4^2 x_5^{2^{d+2}-2} \\
a_{135} &= x_1 x_2^{2^d-1} x_3^2 x_4^2 x_5^{2^{d+2}-4} & a_{136} &= x_1 x_2^{2^d-1} x_3^{2^{d+1}-2} x_4 x_5^{2^{d+1}+2^d-2} \\
a_{137} &= x_1 x_2^{2^d-1} x_3^{2^{d+1}-1} x_4^2 x_5^{2^{d+1}+2^d-4} & a_{138} &= x_1 x_2^{2^d-1} x_3^{2^{d+2}-2} x_4 x_5^{2^d-2} \\
a_{139} &= x_1 x_2^{2^{d+1}-2} x_3 x_4^{2^d-2} x_5^{2^{d+1}+2^d-1} & a_{140} &= x_1 x_2^{2^{d+1}-2} x_3 x_4^{2^d-1} x_5^{2^{d+1}+2^d-2} \\
a_{141} &= x_1 x_2^{2^{d+1}-2} x_3 x_4^{2^{d+1}+2^d-2} x_5^{2^d-1} & a_{142} &= x_1 x_2^{2^{d+1}-2} x_3 x_4^{2^{d+1}+2^d-1} x_5^{2^d-2} \\
a_{143} &= x_1 x_2^{2^{d+1}-2} x_3^3 x_4^{2^d-3} x_5^{2^{d+1}+2^d-2} & a_{144} &= x_1 x_2^{2^{d+1}-2} x_3^3 x_4^{2^{d+1}+2^d-3} x_5^{2^d-2} \\
a_{145} &= x_1 x_2^{2^{d+1}-2} x_3^2 x_4^2 x_5^{2^{d+1}+2^d-2} & a_{146} &= x_1 x_2^{2^{d+1}-2} x_3^{2^{d+1}+2^d-1} x_4 x_5^{2^d-2} \\
a_{147} &= x_1 x_2^{2^{d+1}-1} x_3 x_4^{2^d-2} x_5^{2^{d+1}+2^d-2} & a_{148} &= x_1 x_2^{2^{d+1}-1} x_3 x_4^{2^{d+1}+2^d-2} x_5^{2^d-2} \\
a_{149} &= x_1 x_2^{2^{d+1}-1} x_3^2 x_4^{2^d-4} x_5^{2^{d+1}+2^d-1} & a_{150} &= x_1 x_2^{2^{d+1}-1} x_3^2 x_4^{2^d-3} x_5^{2^{d+1}+2^d-2} \\
a_{151} &= x_1 x_2^{2^{d+1}-1} x_3^2 x_4^{2^d-1} x_5^{2^{d+1}+2^d-4} & a_{152} &= x_1 x_2^{2^{d+1}-1} x_3^2 x_4^{2^{d+1}+2^d-4} x_5^{2^d-1} \\
a_{153} &= x_1 x_2^{2^{d+1}-1} x_3^2 x_4^{2^{d+1}+2^d-3} x_5^{2^d-2} & a_{154} &= x_1 x_2^{2^{d+1}-1} x_3^2 x_4^{2^{d+1}+2^d-1} x_5^{2^d-4} \\
a_{155} &= x_1 x_2^{2^{d+1}-1} x_3^3 x_4^{2^d-4} x_5^{2^{d+1}+2^d-2} & a_{156} &= x_1 x_2^{2^{d+1}-1} x_3^3 x_4^{2^d-2} x_5^{2^{d+1}+2^d-4} \\
a_{157} &= x_1 x_2^{2^{d+1}-1} x_3^3 x_4^{2^{d+1}+2^d-4} x_5^{2^d-2} & a_{158} &= x_1 x_2^{2^{d+1}-1} x_3^3 x_4^{2^{d+1}+2^d-2} x_5^{2^d-4} \\
a_{159} &= x_1 x_2^{2^{d+1}-1} x_3^2 x_4^2 x_5^{2^{d+1}+2^d-2} & a_{160} &= x_1 x_2^{2^{d+1}-1} x_3^2 x_4^2 x_5^{2^{d+1}+2^d-4} \\
a_{161} &= x_1 x_2^{2^{d+1}-1} x_3^{2^{d+1}+2^d-2} x_4 x_5^{2^d-2} & a_{162} &= x_1 x_2^{2^{d+1}-1} x_3^{2^{d+1}+2^d-1} x_4 x_5^{2^d-4} \\
a_{163} &= x_1 x_2^{2^{d+2}-2} x_3 x_4^{2^d-2} x_5^{2^d-1} & a_{164} &= x_1 x_2^{2^{d+2}-2} x_3 x_4^{2^d-1} x_5^{2^d-2} \\
a_{165} &= x_1 x_2^{2^{d+2}-2} x_3^3 x_4^{2^d-3} x_5^{2^d-2} & a_{166} &= x_1 x_2^{2^{d+2}-2} x_3^{2^d-1} x_4 x_5^{2^d-2} \\
a_{167} &= x_1^3 x_2 x_3^{2^d-4} x_4^{2^d-1} x_5^{2^{d+2}-2} & a_{168} &= x_1^3 x_2 x_3^{2^d-4} x_4^{2^{d+1}-2} x_5^{2^{d+1}+2^d-1} \\
a_{169} &= x_1^3 x_2 x_3^{2^d-4} x_4^{2^{d+1}-1} x_5^{2^{d+1}+2^d-2} & a_{170} &= x_1^3 x_2 x_3^{2^d-4} x_4^{2^{d+2}-2} x_5^{2^d-1} \\
a_{171} &= x_1^3 x_2 x_3^{2^d-3} x_4^{2^d-2} x_5^{2^{d+2}-2} & a_{172} &= x_1^3 x_2 x_3^{2^d-3} x_4^{2^{d+1}-2} x_5^{2^{d+1}+2^d-2} \\
a_{173} &= x_1^3 x_2 x_3^{2^d-3} x_4^{2^{d+2}-2} x_5^{2^d-2} & a_{174} &= x_1^3 x_2 x_3^{2^d-2} x_4^{2^d-3} x_5^{2^{d+2}-2} \\
a_{175} &= x_1^3 x_2 x_3^{2^d-2} x_4^{2^d-1} x_5^{2^{d+2}-4} & a_{176} &= x_1^3 x_2 x_3^{2^d-2} x_4^{2^{d+1}-4} x_5^{2^{d+1}+2^d-1} \\
a_{177} &= x_1^3 x_2 x_3^{2^d-2} x_4^{2^{d+1}-3} x_5^{2^{d+1}+2^d-2} & a_{178} &= x_1^3 x_2 x_3^{2^d-2} x_4^{2^{d+1}-1} x_5^{2^{d+1}+2^d-4} \\
a_{179} &= x_1^3 x_2 x_3^{2^d-2} x_4^{2^{d+2}-4} x_5^{2^d-1} & a_{180} &= x_1^3 x_2 x_3^{2^d-2} x_4^{2^{d+2}-3} x_5^{2^d-2} \\
a_{181} &= x_1^3 x_2 x_3^{2^d-1} x_4^{2^d-4} x_5^{2^{d+2}-2} & a_{182} &= x_1^3 x_2 x_3^{2^d-1} x_4^{2^d-2} x_5^{2^{d+2}-4} \\
a_{183} &= x_1^3 x_2 x_3^{2^d-1} x_4^{2^{d+1}-4} x_5^{2^{d+1}+2^d-2} & a_{184} &= x_1^3 x_2 x_3^{2^d-1} x_4^{2^{d+1}-2} x_5^{2^{d+1}+2^d-4} \\
a_{185} &= x_1^3 x_2 x_3^{2^d-1} x_4^{2^{d+2}-4} x_5^{2^d-2} & a_{186} &= x_1^3 x_2 x_3^{2^d-1} x_4^{2^{d+2}-2} x_5^{2^d-4} \\
a_{187} &= x_1^3 x_2 x_3^{2^{d+1}-4} x_4^{2^d-2} x_5^{2^{d+1}+2^d-1} & a_{188} &= x_1^3 x_2 x_3^{2^{d+1}-4} x_4^{2^d-1} x_5^{2^{d+1}+2^d-2} \\
a_{189} &= x_1^3 x_2 x_3^{2^{d+1}-4} x_4^{2^{d+1}+2^d-2} x_5^{2^d-1} & a_{190} &= x_1^3 x_2 x_3^{2^{d+1}-4} x_4^{2^{d+1}+2^d-1} x_5^{2^d-2} \\
a_{191} &= x_1^3 x_2 x_3^{2^{d+1}-3} x_4^{2^d-2} x_5^{2^{d+1}+2^d-2} & a_{192} &= x_1^3 x_2 x_3^{2^{d+1}-3} x_4^{2^{d+1}+2^d-2} x_5^{2^d-2} \\
a_{193} &= x_1^3 x_2 x_3^{2^{d+1}-2} x_4^{2^d-4} x_5^{2^{d+1}+2^d-1} & a_{194} &= x_1^3 x_2 x_3^{2^{d+1}-2} x_4^{2^d-3} x_5^{2^{d+1}+2^d-2} \\
a_{195} &= x_1^3 x_2 x_3^{2^{d+1}-2} x_4^{2^d-1} x_5^{2^{d+1}+2^d-4} & a_{196} &= x_1^3 x_2 x_3^{2^{d+1}-2} x_4^{2^{d+1}+2^d-4} x_5^{2^d-1} \\
a_{197} &= x_1^3 x_2 x_3^{2^{d+1}-2} x_4^{2^{d+1}+2^d-3} x_5^{2^d-2} & a_{198} &= x_1^3 x_2 x_3^{2^{d+1}-2} x_4^{2^{d+1}+2^d-1} x_5^{2^d-4}
\end{aligned}$$



$$\begin{aligned}
 a_{199} &= x_1^3 x_2 x_3^{2^{d+1}-1} x_4^{2^d-4} x_5^{2^{d+1}+2^d-2} & a_{200} &= x_1^3 x_2 x_3^{2^{d+1}-1} x_4^{2^d-2} x_5^{2^{d+1}+2^d-4} \\
 a_{201} &= x_1^3 x_2 x_3^{2^{d+1}-1} x_4^{2^{d+1}+2^d-4} x_5^{2^d-2} & a_{202} &= x_1^3 x_2 x_3^{2^{d+1}-1} x_4^{2^{d+1}+2^d-2} x_5^{2^d-4} \\
 a_{203} &= x_1^3 x_2 x_3^{2^{d+2}-4} x_4^{2^d-2} x_5^{2^d-1} & a_{204} &= x_1^3 x_2 x_3^{2^{d+2}-4} x_4^{2^d-1} x_5^{2^d-2} \\
 a_{205} &= x_1^3 x_2 x_3^{2^{d+2}-3} x_4^{2^d-2} x_5^{2^d-2} & a_{206} &= x_1^3 x_2 x_3^{2^{d+2}-2} x_4^{2^d-4} x_5^{2^d-1} \\
 a_{207} &= x_1^3 x_2 x_3^{2^{d+2}-2} x_4^{2^d-3} x_5^{2^d-2} & a_{208} &= x_1^3 x_2 x_3^{2^{d+2}-2} x_4^{2^d-1} x_5^{2^d-4} \\
 a_{209} &= x_1^3 x_2^3 x_3^{2^d-4} x_4^{2^d-3} x_5^{2^{d+2}-2} & a_{210} &= x_1^3 x_2^3 x_3^{2^d-4} x_4^{2^{d+1}-3} x_5^{2^{d+1}+2^d-2} \\
 a_{211} &= x_1^3 x_2^3 x_3^{2^d-4} x_4^{2^{d+2}-3} x_5^{2^d-2} & a_{212} &= x_1^3 x_2^3 x_3^{2^d-3} x_4^{2^d-4} x_5^{2^{d+2}-2} \\
 a_{213} &= x_1^3 x_2^3 x_3^{2^d-3} x_4^{2^d-2} x_5^{2^{d+2}-4} & a_{214} &= x_1^3 x_2^3 x_3^{2^d-3} x_4^{2^{d+1}-4} x_5^{2^{d+1}+2^d-2} \\
 a_{215} &= x_1^3 x_2^3 x_3^{2^d-3} x_4^{2^{d+1}-2} x_5^{2^{d+1}+2^d-4} & a_{216} &= x_1^3 x_2^3 x_3^{2^d-3} x_4^{2^{d+2}-4} x_5^{2^d-2} \\
 a_{217} &= x_1^3 x_2^3 x_3^{2^d-3} x_4^{2^{d+2}-2} x_5^{2^d-4} & a_{218} &= x_1^3 x_2^3 x_3^{2^{d+1}-4} x_4^{2^d-3} x_5^{2^{d+1}+2^d-2} \\
 a_{219} &= x_1^3 x_2^3 x_3^{2^{d+1}-4} x_4^{2^{d+1}+2^d-3} x_5^{2^d-2} & a_{220} &= x_1^3 x_2^3 x_3^{2^{d+1}-3} x_4^{2^d-4} x_5^{2^{d+1}+2^d-2} \\
 a_{221} &= x_1^3 x_2^3 x_3^{2^{d+1}-3} x_4^{2^d-2} x_5^{2^{d+1}+2^d-4} & a_{222} &= x_1^3 x_2^3 x_3^{2^{d+1}-3} x_4^{2^{d+1}+2^d-4} x_5^{2^d-2} \\
 a_{223} &= x_1^3 x_2^3 x_3^{2^{d+1}-3} x_4^{2^{d+1}+2^d-2} x_5^{2^d-4} & a_{224} &= x_1^3 x_2^3 x_3^{2^{d+2}-4} x_4^{2^d-3} x_5^{2^d-2} \\
 a_{225} &= x_1^3 x_2^3 x_3^{2^{d+2}-3} x_4^{2^d-4} x_5^{2^d-2} & a_{226} &= x_1^3 x_2^3 x_3^{2^{d+2}-3} x_4^{2^d-2} x_5^{2^d-4} \\
 a_{227} &= x_1^3 x_2^4 x_3^{2^d-5} x_4^{2^d-3} x_5^{2^{d+2}-2} & a_{228} &= x_1^3 x_2^4 x_3^{2^d-5} x_4^{2^{d+1}-3} x_5^{2^{d+1}+2^d-2} \\
 a_{229} &= x_1^3 x_2^4 x_3^{2^d-5} x_4^{2^{d+2}-3} x_5^{2^d-2} & a_{230} &= x_1^3 x_2^4 x_3^{2^{d+1}-5} x_4^{2^d-3} x_5^{2^{d+1}+2^d-2} \\
 a_{231} &= x_1^3 x_2^4 x_3^{2^{d+1}-5} x_4^{2^{d+1}+2^d-3} x_5^{2^d-2} & a_{232} &= x_1^3 x_2^4 x_3^{2^{d+2}-5} x_4^{2^d-3} x_5^{2^d-2} \\
 a_{233} &= x_1^3 x_2^5 x_3^{2^d-6} x_4^{2^d-3} x_5^{2^{d+2}-2} & a_{234} &= x_1^3 x_2^5 x_3^{2^d-6} x_4^{2^d-1} x_5^{2^{d+2}-4} \\
 a_{235} &= x_1^3 x_2^5 x_3^{2^d-6} x_4^{2^{d+1}-4} x_5^{2^{d+1}+2^d-1} & a_{236} &= x_1^3 x_2^5 x_3^{2^d-6} x_4^{2^{d+1}-3} x_5^{2^{d+1}+2^d-2} \\
 a_{237} &= x_1^3 x_2^5 x_3^{2^d-6} x_4^{2^{d+1}-1} x_5^{2^{d+1}+2^d-4} & a_{238} &= x_1^3 x_2^5 x_3^{2^d-6} x_4^{2^{d+2}-4} x_5^{2^d-1} \\
 a_{239} &= x_1^3 x_2^5 x_3^{2^d-6} x_4^{2^{d+2}-3} x_5^{2^d-2} & a_{240} &= x_1^3 x_2^5 x_3^{2^d-5} x_4^{2^d-4} x_5^{2^{d+2}-2} \\
 a_{241} &= x_1^3 x_2^5 x_3^{2^d-5} x_4^{2^d-2} x_5^{2^{d+2}-4} & a_{242} &= x_1^3 x_2^5 x_3^{2^d-5} x_4^{2^{d+1}-4} x_5^{2^{d+1}+2^d-2} \\
 a_{243} &= x_1^3 x_2^5 x_3^{2^d-5} x_4^{2^{d+1}-2} x_5^{2^{d+1}+2^d-4} & a_{244} &= x_1^3 x_2^5 x_3^{2^d-5} x_4^{2^{d+2}-4} x_5^{2^d-2} \\
 a_{245} &= x_1^3 x_2^5 x_3^{2^d-5} x_4^{2^{d+2}-2} x_5^{2^d-4} & a_{246} &= x_1^3 x_2^5 x_3^{2^d-1} x_4^{2^d-6} x_5^{2^{d+2}-4} \\
 a_{247} &= x_1^3 x_2^5 x_3^{2^d-1} x_4^{2^{d+1}-6} x_5^{2^{d+1}+2^d-4} & a_{248} &= x_1^3 x_2^5 x_3^{2^d-1} x_4^{2^{d+2}-6} x_5^{2^d-4} \\
 a_{249} &= x_1^3 x_2^5 x_3^{2^{d+1}-6} x_4^{2^d-4} x_5^{2^{d+1}+2^d-1} & a_{250} &= x_1^3 x_2^5 x_3^{2^{d+1}-6} x_4^{2^d-3} x_5^{2^{d+1}+2^d-2} \\
 a_{251} &= x_1^3 x_2^5 x_3^{2^{d+1}-6} x_4^{2^d-1} x_5^{2^{d+1}+2^d-4} & a_{252} &= x_1^3 x_2^5 x_3^{2^{d+1}-6} x_4^{2^{d+1}+2^d-4} x_5^{2^d-1} \\
 a_{253} &= x_1^3 x_2^5 x_3^{2^{d+1}-6} x_4^{2^{d+1}+2^d-3} x_5^{2^d-2} & a_{254} &= x_1^3 x_2^5 x_3^{2^{d+1}-6} x_4^{2^{d+1}+2^d-1} x_5^{2^d-4} \\
 a_{255} &= x_1^3 x_2^5 x_3^{2^{d+1}-5} x_4^{2^d-4} x_5^{2^{d+1}+2^d-2} & a_{256} &= x_1^3 x_2^5 x_3^{2^{d+1}-5} x_4^{2^d-2} x_5^{2^{d+1}+2^d-4} \\
 a_{257} &= x_1^3 x_2^5 x_3^{2^{d+1}-5} x_4^{2^{d+1}+2^d-4} x_5^{2^d-2} & a_{258} &= x_1^3 x_2^5 x_3^{2^{d+1}-5} x_4^{2^{d+1}+2^d-2} x_5^{2^d-4} \\
 a_{259} &= x_1^3 x_2^5 x_3^{2^{d+1}-1} x_4^{2^d-6} x_5^{2^{d+1}+2^d-4} & a_{260} &= x_1^3 x_2^5 x_3^{2^{d+1}-1} x_4^{2^{d+1}+2^d-6} x_5^{2^d-4} \\
 a_{261} &= x_1^3 x_2^5 x_3^{2^{d+2}-6} x_4^{2^d-4} x_5^{2^d-1} & a_{262} &= x_1^3 x_2^5 x_3^{2^{d+2}-6} x_4^{2^d-3} x_5^{2^d-2} \\
 a_{263} &= x_1^3 x_2^5 x_3^{2^{d+2}-6} x_4^{2^d-1} x_5^{2^d-4} & a_{264} &= x_1^3 x_2^5 x_3^{2^{d+2}-5} x_4^{2^d-4} x_5^{2^d-2} \\
 a_{265} &= x_1^3 x_2^5 x_3^{2^{d+2}-5} x_4^{2^d-2} x_5^{2^d-4} & a_{266} &= x_1^3 x_2^7 x_3^{2^d-7} x_4^{2^d-4} x_5^{2^{d+2}-2} \\
 a_{267} &= x_1^3 x_2^7 x_3^{2^d-7} x_4^{2^d-2} x_5^{2^{d+2}-4} & a_{268} &= x_1^3 x_2^7 x_3^{2^d-7} x_4^{2^{d+1}-4} x_5^{2^{d+1}+2^d-2}
 \end{aligned}$$

$$\begin{aligned}
a_{269} &= x_1^3 x_2^7 x_3^{2^d-7} x_4^{2^{d+1}-2} x_5^{2^{d+1}+2^d-4} & a_{270} &= x_1^3 x_2^7 x_3^{2^d-7} x_4^{2^{d+2}-4} x_5^{2^d-2} \\
a_{271} &= x_1^3 x_2^7 x_3^{2^d-7} x_4^{2^{d+2}-2} x_5^{2^d-4} & a_{272} &= x_1^3 x_2^7 x_3^{2^d-3} x_4^{2^d-6} x_5^{2^{d+2}-4} \\
a_{273} &= x_1^3 x_2^7 x_3^{2^d-3} x_4^{2^{d+1}-6} x_5^{2^{d+1}+2^d-4} & a_{274} &= x_1^3 x_2^7 x_3^{2^d-3} x_4^{2^{d+2}-6} x_5^{2^d-4} \\
a_{275} &= x_1^3 x_2^7 x_3^{2^{d+1}-7} x_4^{2^d-4} x_5^{2^{d+1}+2^d-2} & a_{276} &= x_1^3 x_2^7 x_3^{2^{d+1}-7} x_4^{2^d-2} x_5^{2^{d+1}+2^d-4} \\
a_{277} &= x_1^3 x_2^7 x_3^{2^{d+1}-7} x_4^{2^{d+1}+2^d-4} x_5^{2^d-2} & a_{278} &= x_1^3 x_2^7 x_3^{2^{d+1}-7} x_4^{2^{d+1}+2^d-2} x_5^{2^d-4} \\
a_{279} &= x_1^3 x_2^7 x_3^{2^{d+1}-3} x_4^{2^d-6} x_5^{2^{d+1}+2^d-4} & a_{280} &= x_1^3 x_2^7 x_3^{2^{d+1}-3} x_4^{2^{d+1}+2^d-6} x_5^{2^d-4} \\
a_{281} &= x_1^3 x_2^7 x_3^{2^{d+2}-7} x_4^{2^d-4} x_5^{2^d-2} & a_{282} &= x_1^3 x_2^7 x_3^{2^{d+2}-7} x_4^{2^d-2} x_5^{2^d-4} \\
a_{283} &= x_1^3 x_2^7 x_3^{2^{d+2}-3} x_4^{2^d-6} x_5^{2^d-4} & a_{284} &= x_1^3 x_2^{2^d-3} x_3 x_4^{2^d-2} x_5^{2^{d+2}-2} \\
a_{285} &= x_1^3 x_2^{2^d-3} x_3 x_4^{2^{d+1}-2} x_5^{2^{d+1}+2^d-2} & a_{286} &= x_1^3 x_2^{2^d-3} x_3 x_4^{2^{d+2}-2} x_5^{2^d-2} \\
a_{287} &= x_1^3 x_2^{2^d-3} x_3 x_4^{2^d-3} x_5^{2^{d+2}-2} & a_{288} &= x_1^3 x_2^{2^d-3} x_3 x_4^{2^d-1} x_5^{2^{d+2}-4} \\
a_{289} &= x_1^3 x_2^{2^d-3} x_3 x_4^{2^{d+1}-4} x_5^{2^{d+1}+2^d-1} & a_{290} &= x_1^3 x_2^{2^d-3} x_3 x_4^{2^{d+1}-3} x_5^{2^{d+1}+2^d-2} \\
a_{291} &= x_1^3 x_2^{2^d-3} x_3 x_4^{2^{d+1}-1} x_5^{2^{d+1}+2^d-4} & a_{292} &= x_1^3 x_2^{2^d-3} x_3 x_4^{2^{d+2}-4} x_5^{2^d-1} \\
a_{293} &= x_1^3 x_2^{2^d-3} x_3 x_4^{2^{d+2}-3} x_5^{2^d-2} & a_{294} &= x_1^3 x_2^{2^d-3} x_3 x_4^{2^d-4} x_5^{2^{d+2}-2} \\
a_{295} &= x_1^3 x_2^{2^d-3} x_3 x_4^{2^d-2} x_5^{2^{d+2}-4} & a_{296} &= x_1^3 x_2^{2^d-3} x_3 x_4^{2^{d+1}-4} x_5^{2^{d+1}+2^d-2} \\
a_{297} &= x_1^3 x_2^{2^d-3} x_3 x_4^{2^{d+1}-2} x_5^{2^{d+1}+2^d-4} & a_{298} &= x_1^3 x_2^{2^d-3} x_3 x_4^{2^{d+2}-4} x_5^{2^d-2} \\
a_{299} &= x_1^3 x_2^{2^d-3} x_3 x_4^{2^{d+2}-2} x_5^{2^d-4} & a_{300} &= x_1^3 x_2^{2^d-3} x_3^{2^d-2} x_4 x_5^{2^{d+2}-2} \\
a_{301} &= x_1^3 x_2^{2^d-3} x_3^{2^d-1} x_4 x_5^{2^{d+2}-4} & a_{302} &= x_1^3 x_2^{2^d-3} x_3^{2^{d+1}-2} x_4 x_5^{2^{d+1}+2^d-2} \\
a_{303} &= x_1^3 x_2^{2^d-3} x_3^{2^{d+1}-1} x_4 x_5^{2^{d+1}+2^d-4} & a_{304} &= x_1^3 x_2^{2^d-3} x_3^{2^{d+2}-2} x_4 x_5^{2^d-2} \\
a_{305} &= x_1^3 x_2^{2^d-1} x_3 x_4^{2^d-4} x_5^{2^{d+2}-2} & a_{306} &= x_1^3 x_2^{2^d-1} x_3 x_4^{2^d-2} x_5^{2^{d+2}-4} \\
a_{307} &= x_1^3 x_2^{2^d-1} x_3 x_4^{2^{d+1}-4} x_5^{2^{d+1}+2^d-2} & a_{308} &= x_1^3 x_2^{2^d-1} x_3 x_4^{2^{d+1}-2} x_5^{2^{d+1}+2^d-4} \\
a_{309} &= x_1^3 x_2^{2^d-1} x_3 x_4^{2^{d+2}-4} x_5^{2^d-2} & a_{310} &= x_1^3 x_2^{2^d-1} x_3 x_4^{2^{d+2}-2} x_5^{2^d-4} \\
a_{311} &= x_1^3 x_2^{2^d-1} x_3 x_4^{2^d-6} x_5^{2^{d+2}-4} & a_{312} &= x_1^3 x_2^{2^d-1} x_3 x_4^{2^{d+1}-6} x_5^{2^{d+1}+2^d-4} \\
a_{313} &= x_1^3 x_2^{2^d-1} x_3 x_4^{2^{d+2}-6} x_5^{2^d-4} & a_{314} &= x_1^3 x_2^{2^d-1} x_3^{2^d-3} x_4 x_5^{2^{d+2}-4} \\
a_{315} &= x_1^3 x_2^{2^d-1} x_3^{2^{d+1}-3} x_4 x_5^{2^{d+1}+2^d-4} & a_{316} &= x_1^3 x_2^{2^d-1} x_3^{2^{d+2}-3} x_4 x_5^{2^d-4} \\
a_{317} &= x_1^3 x_2^{2^{d+1}-3} x_3 x_4^{2^d-2} x_5^{2^{d+1}+2^d-2} & a_{318} &= x_1^3 x_2^{2^{d+1}-3} x_3 x_4^{2^{d+1}+2^d-2} x_5^{2^d-2} \\
a_{319} &= x_1^3 x_2^{2^{d+1}-3} x_3 x_4^{2^d-4} x_5^{2^{d+1}+2^d-1} & a_{320} &= x_1^3 x_2^{2^{d+1}-3} x_3 x_4^{2^d-3} x_5^{2^{d+1}+2^d-2} \\
a_{321} &= x_1^3 x_2^{2^{d+1}-3} x_3 x_4^{2^d-1} x_5^{2^{d+1}+2^d-4} & a_{322} &= x_1^3 x_2^{2^{d+1}-3} x_3 x_4^{2^{d+1}+2^d-4} x_5^{2^d-1} \\
a_{323} &= x_1^3 x_2^{2^{d+1}-3} x_3 x_4^{2^{d+1}+2^d-3} x_5^{2^d-2} & a_{324} &= x_1^3 x_2^{2^{d+1}-3} x_3 x_4^{2^{d+1}+2^d-1} x_5^{2^d-4} \\
a_{325} &= x_1^3 x_2^{2^{d+1}-3} x_3 x_4^{2^d-4} x_5^{2^{d+1}+2^d-2} & a_{326} &= x_1^3 x_2^{2^{d+1}-3} x_3 x_4^{2^d-2} x_5^{2^{d+1}+2^d-4} \\
a_{327} &= x_1^3 x_2^{2^{d+1}-3} x_3 x_4^{2^{d+1}+2^d-4} x_5^{2^d-2} & a_{328} &= x_1^3 x_2^{2^{d+1}-3} x_3 x_4^{2^{d+1}+2^d-2} x_5^{2^d-4} \\
a_{329} &= x_1^3 x_2^{2^{d+1}-3} x_3^{2^d-2} x_4 x_5^{2^{d+1}+2^d-2} & a_{330} &= x_1^3 x_2^{2^{d+1}-3} x_3^{2^d-1} x_4 x_5^{2^{d+1}+2^d-4} \\
a_{331} &= x_1^3 x_2^{2^{d+1}-3} x_3^{2^{d+1}+2^d-2} x_4 x_5^{2^d-2} & a_{332} &= x_1^3 x_2^{2^{d+1}-3} x_3^{2^{d+1}+2^d-1} x_4 x_5^{2^d-4} \\
a_{333} &= x_1^3 x_2^{2^{d+1}-1} x_3 x_4^{2^d-4} x_5^{2^{d+1}+2^d-2} & a_{334} &= x_1^3 x_2^{2^{d+1}-1} x_3 x_4^{2^d-2} x_5^{2^{d+1}+2^d-4} \\
a_{335} &= x_1^3 x_2^{2^{d+1}-1} x_3 x_4^{2^{d+1}+2^d-4} x_5^{2^d-2} & a_{336} &= x_1^3 x_2^{2^{d+1}-1} x_3 x_4^{2^{d+1}+2^d-2} x_5^{2^d-4} \\
a_{337} &= x_1^3 x_2^{2^{d+1}-1} x_3 x_4^{2^d-6} x_5^{2^{d+1}+2^d-4} & a_{338} &= x_1^3 x_2^{2^{d+1}-1} x_3 x_4^{2^{d+1}+2^d-6} x_5^{2^d-4}
\end{aligned}$$

$$\begin{aligned}
 a_{339} &= x_1^3 x_2^{2^{d+1}-1} x_3^{2^d-3} x_4^2 x_5^{2^{d+1}+2^d-4} \\
 a_{341} &= x_1^3 x_2^{2^{d+2}-3} x_3 x_4^{2^d-2} x_5^{2^d-2} \\
 a_{343} &= x_1^3 x_2^{2^{d+2}-3} x_3^2 x_4^{2^d-3} x_5^{2^d-2} \\
 a_{345} &= x_1^3 x_2^{2^{d+2}-3} x_3^3 x_4^{2^d-4} x_5^{2^d-2} \\
 a_{347} &= x_1^3 x_2^{2^{d+2}-3} x_3^{2^d-2} x_4 x_5^{2^d-2} \\
 a_{349} &= x_1^7 x_2 x_3^{2^d-6} x_4^{2^d-3} x_5^{2^{d+2}-2} \\
 a_{351} &= x_1^7 x_2 x_3^{2^d-6} x_4^{2^{d+2}-3} x_5^{2^d-2} \\
 a_{353} &= x_1^7 x_2 x_3^{2^d-5} x_4^{2^d-2} x_5^{2^{d+2}-4} \\
 a_{355} &= x_1^7 x_2 x_3^{2^d-5} x_4^{2^{d+1}-2} x_5^{2^{d+1}+2^d-4} \\
 a_{357} &= x_1^7 x_2 x_3^{2^d-5} x_4^{2^{d+2}-2} x_5^{2^d-4} \\
 a_{359} &= x_1^7 x_2 x_3^{2^{d+1}-6} x_4^{2^{d+1}+2^d-3} x_5^{2^d-2} \\
 a_{361} &= x_1^7 x_2 x_3^{2^{d+1}-5} x_4^{2^d-2} x_5^{2^{d+1}+2^d-4} \\
 a_{363} &= x_1^7 x_2 x_3^{2^{d+1}-5} x_4^{2^{d+1}+2^d-2} x_5^{2^d-4} \\
 a_{365} &= x_1^7 x_2 x_3^{2^{d+2}-5} x_4^{2^d-4} x_5^{2^d-2} \\
 a_{367} &= x_1^7 x_2^3 x_3^{2^d-7} x_4^{2^d-4} x_5^{2^{d+2}-2} \\
 a_{369} &= x_1^7 x_2^3 x_3^{2^d-7} x_4^{2^{d+1}-4} x_5^{2^{d+1}+2^d-2} \\
 a_{371} &= x_1^7 x_2^3 x_3^{2^d-7} x_4^{2^{d+2}-4} x_5^{2^d-2} \\
 a_{373} &= x_1^7 x_2^3 x_3^{2^d-3} x_4^{2^d-6} x_5^{2^{d+2}-4} \\
 a_{375} &= x_1^7 x_2^3 x_3^{2^d-3} x_4^{2^{d+2}-6} x_5^{2^d-4} \\
 a_{377} &= x_1^7 x_2^3 x_3^{2^{d+1}-7} x_4^{2^d-2} x_5^{2^{d+1}+2^d-4} \\
 a_{379} &= x_1^7 x_2^3 x_3^{2^{d+1}-7} x_4^{2^{d+1}+2^d-2} x_5^{2^d-4} \\
 a_{381} &= x_1^7 x_2^3 x_3^{2^{d+1}-3} x_4^{2^{d+1}+2^d-6} x_5^{2^d-4} \\
 a_{383} &= x_1^7 x_2^3 x_3^{2^{d+2}-7} x_4^{2^d-2} x_5^{2^d-4} \\
 a_{385} &= x_1^7 x_2^{11} x_3^{2^d-11} x_4^{2^d-6} x_5^{2^{d+2}-4} \\
 a_{387} &= x_1^7 x_2^{11} x_3^{2^d-11} x_4^{2^{d+2}-6} x_5^{2^d-4} \\
 a_{389} &= x_1^7 x_2^{11} x_3^{2^{d+1}-11} x_4^{2^{d+1}+2^d-6} x_5^{2^d-4} \\
 a_{391} &= x_1^7 x_2^{2^d-5} x_3 x_4^{2^d-4} x_5^{2^{d+2}-2} \\
 a_{393} &= x_1^7 x_2^{2^d-5} x_3 x_4^{2^{d+1}-4} x_5^{2^{d+1}+2^d-2} \\
 a_{395} &= x_1^7 x_2^{2^d-5} x_3 x_4^{2^{d+2}-4} x_5^{2^d-2} \\
 a_{397} &= x_1^7 x_2^{2^d-5} x_3^2 x_4^2 x_5^{2^{d+2}-4} \\
 a_{399} &= x_1^7 x_2^{2^d-5} x_3^{2^{d+2}-3} x_4^2 x_5^{2^d-4} \\
 a_{401} &= x_1^7 x_2^{2^{d+1}-5} x_3 x_4^{2^d-2} x_5^{2^{d+1}+2^d-4} \\
 a_{403} &= x_1^7 x_2^{2^{d+1}-5} x_3 x_4^{2^{d+1}+2^d-2} x_5^{2^d-4} \\
 a_{405} &= x_1^7 x_2^{2^{d+1}-5} x_3^5 x_4^{2^{d+1}+2^d-6} x_5^{2^d-4} \\
 a_{407} &= x_1^7 x_2^{2^{d+1}-5} x_3^{2^{d+1}+2^d-3} x_4^2 x_5^{2^d-4} \\
 a_{409} &= x_1^7 x_2^{2^{d+2}-5} x_3 x_4^{2^d-2} x_5^{2^d-4} \\
 a_{340} &= x_1^3 x_2^{2^{d+1}-1} x_3^{2^{d+1}+2^d-3} x_4^2 x_5^{2^d-4} \\
 a_{342} &= x_1^3 x_2^{2^{d+2}-3} x_3^2 x_4^{2^d-4} x_5^{2^d-1} \\
 a_{344} &= x_1^3 x_2^{2^{d+2}-3} x_3^2 x_4^{2^d-1} x_5^{2^d-4} \\
 a_{346} &= x_1^3 x_2^{2^{d+2}-3} x_3^3 x_4^{2^d-2} x_5^{2^d-4} \\
 a_{348} &= x_1^3 x_2^{2^{d+2}-3} x_3^{2^d-1} x_4^2 x_5^{2^d-4} \\
 a_{350} &= x_1^7 x_2 x_3^{2^d-6} x_4^{2^{d+1}-3} x_5^{2^{d+1}+2^d-2} \\
 a_{352} &= x_1^7 x_2 x_3^{2^d-5} x_4^{2^d-4} x_5^{2^{d+2}-2} \\
 a_{354} &= x_1^7 x_2 x_3^{2^d-5} x_4^{2^{d+1}-4} x_5^{2^{d+1}+2^d-2} \\
 a_{356} &= x_1^7 x_2 x_3^{2^d-5} x_4^{2^{d+2}-4} x_5^{2^d-2} \\
 a_{358} &= x_1^7 x_2 x_3^{2^{d+1}-6} x_4^{2^d-3} x_5^{2^{d+1}+2^d-2} \\
 a_{360} &= x_1^7 x_2 x_3^{2^{d+1}-5} x_4^{2^d-4} x_5^{2^{d+1}+2^d-2} \\
 a_{362} &= x_1^7 x_2 x_3^{2^{d+1}-5} x_4^{2^{d+1}+2^d-4} x_5^{2^d-2} \\
 a_{364} &= x_1^7 x_2 x_3^{2^{d+2}-6} x_4^{2^d-3} x_5^{2^d-2} \\
 a_{366} &= x_1^7 x_2 x_3^{2^{d+2}-5} x_4^{2^d-2} x_5^{2^d-4} \\
 a_{368} &= x_1^7 x_2^3 x_3^{2^d-7} x_4^{2^d-2} x_5^{2^{d+2}-4} \\
 a_{370} &= x_1^7 x_2^3 x_3^{2^d-7} x_4^{2^{d+1}-2} x_5^{2^{d+1}+2^d-4} \\
 a_{372} &= x_1^7 x_2^3 x_3^{2^d-7} x_4^{2^{d+2}-2} x_5^{2^d-4} \\
 a_{374} &= x_1^7 x_2^3 x_3^{2^d-3} x_4^{2^{d+1}-6} x_5^{2^{d+1}+2^d-4} \\
 a_{376} &= x_1^7 x_2^3 x_3^{2^{d+1}-7} x_4^{2^d-4} x_5^{2^{d+1}+2^d-2} \\
 a_{378} &= x_1^7 x_2^3 x_3^{2^{d+1}-7} x_4^{2^{d+1}+2^d-4} x_5^{2^d-2} \\
 a_{380} &= x_1^7 x_2^3 x_3^{2^{d+1}-3} x_4^{2^d-6} x_5^{2^{d+1}+2^d-4} \\
 a_{382} &= x_1^7 x_2^3 x_3^{2^{d+2}-7} x_4^{2^d-4} x_5^{2^d-2} \\
 a_{384} &= x_1^7 x_2^3 x_3^{2^{d+2}-3} x_4^{2^d-6} x_5^{2^d-4} \\
 a_{386} &= x_1^7 x_2^{11} x_3^{2^d-11} x_4^{2^{d+1}-6} x_5^{2^{d+1}+2^d-4} \\
 a_{388} &= x_1^7 x_2^{11} x_3^{2^{d+1}-11} x_4^{2^d-6} x_5^{2^{d+1}+2^d-4} \\
 a_{390} &= x_1^7 x_2^{11} x_3^{2^{d+2}-11} x_4^{2^d-6} x_5^{2^d-4} \\
 a_{392} &= x_1^7 x_2^{2^d-5} x_3 x_4^{2^d-2} x_5^{2^{d+2}-4} \\
 a_{394} &= x_1^7 x_2^{2^d-5} x_3 x_4^{2^{d+1}-2} x_5^{2^{d+1}+2^d-4} \\
 a_{396} &= x_1^7 x_2^{2^d-5} x_3 x_4^{2^{d+2}-2} x_5^{2^d-4} \\
 a_{398} &= x_1^7 x_2^{2^d-5} x_3^{2^{d+1}-3} x_4^2 x_5^{2^{d+1}+2^d-4} \\
 a_{400} &= x_1^7 x_2^{2^{d+1}-5} x_3 x_4^{2^d-4} x_5^{2^{d+1}+2^d-2} \\
 a_{402} &= x_1^7 x_2^{2^{d+1}-5} x_3 x_4^{2^{d+1}+2^d-4} x_5^{2^d-2} \\
 a_{404} &= x_1^7 x_2^{2^{d+1}-5} x_3^5 x_4^{2^d-6} x_5^{2^{d+1}+2^d-4} \\
 a_{406} &= x_1^7 x_2^{2^{d+1}-5} x_3^2 x_4^2 x_5^{2^{d+1}+2^d-4} \\
 a_{408} &= x_1^7 x_2^{2^{d+2}-5} x_3 x_4^{2^d-4} x_5^{2^d-2} \\
 a_{410} &= x_1^7 x_2^{2^{d+2}-5} x_3^5 x_4^{2^d-6} x_5^{2^d-4}
 \end{aligned}$$

$$\begin{aligned}
a_{411} &= x_1^7 x_2^{2^{d+2}-5} x_3^{2^d-3} x_4^2 x_5^{2^d-4} & a_{412} &= x_1^{2^d-1} x_2 x_3 x_4^{2^d-2} x_5^{2^{d+2}-2} \\
a_{413} &= x_1^{2^d-1} x_2 x_3 x_4^{2^{d+1}-2} x_5^{2^{d+1}+2^d-2} & a_{414} &= x_1^{2^d-1} x_2 x_3 x_4^{2^{d+2}-2} x_5^{2^d-2} \\
a_{415} &= x_1^{2^d-1} x_2 x_3 x_4^{2^d-3} x_5^{2^{d+2}-2} & a_{416} &= x_1^{2^d-1} x_2 x_3 x_4^{2^d-1} x_5^{2^{d+2}-4} \\
a_{417} &= x_1^{2^d-1} x_2 x_3 x_4^{2^{d+1}-4} x_5^{2^{d+1}+2^d-1} & a_{418} &= x_1^{2^d-1} x_2 x_3 x_4^{2^{d+1}-3} x_5^{2^{d+1}+2^d-2} \\
a_{419} &= x_1^{2^d-1} x_2 x_3 x_4^{2^{d+1}-1} x_5^{2^{d+1}+2^d-4} & a_{420} &= x_1^{2^d-1} x_2 x_3 x_4^{2^{d+2}-4} x_5^{2^d-1} \\
a_{421} &= x_1^{2^d-1} x_2 x_3 x_4^{2^{d+2}-3} x_5^{2^d-2} & a_{422} &= x_1^{2^d-1} x_2 x_3 x_4^{2^d-4} x_5^{2^{d+2}-2} \\
a_{423} &= x_1^{2^d-1} x_2 x_3 x_4^{2^d-2} x_5^{2^{d+2}-4} & a_{424} &= x_1^{2^d-1} x_2 x_3 x_4^{2^{d+1}-4} x_5^{2^{d+1}+2^d-2} \\
a_{425} &= x_1^{2^d-1} x_2 x_3 x_4^{2^{d+1}-2} x_5^{2^{d+1}+2^d-4} & a_{426} &= x_1^{2^d-1} x_2 x_3 x_4^{2^{d+2}-4} x_5^{2^d-2} \\
a_{427} &= x_1^{2^d-1} x_2 x_3 x_4^{2^{d+2}-2} x_5^{2^d-4} & a_{428} &= x_1^{2^d-1} x_2 x_3 x_4^{2^d-2} x_4 x_5^{2^{d+2}-2} \\
a_{429} &= x_1^{2^d-1} x_2 x_3 x_4^{2^d-1} x_4 x_5^{2^{d+2}-4} & a_{430} &= x_1^{2^d-1} x_2 x_3^{2^{d+1}-2} x_4 x_5^{2^{d+1}+2^d-2} \\
a_{431} &= x_1^{2^d-1} x_2 x_3^{2^{d+1}-1} x_4 x_5^{2^{d+1}+2^d-4} & a_{432} &= x_1^{2^d-1} x_2 x_3^{2^{d+2}-2} x_4 x_5^{2^d-2} \\
a_{433} &= x_1^{2^d-1} x_2^3 x_3 x_4^{2^d-4} x_5^{2^{d+2}-2} & a_{434} &= x_1^{2^d-1} x_2^3 x_3 x_4^{2^d-2} x_5^{2^{d+2}-4} \\
a_{435} &= x_1^{2^d-1} x_2^3 x_3 x_4^{2^{d+1}-4} x_5^{2^{d+1}+2^d-2} & a_{436} &= x_1^{2^d-1} x_2^3 x_3 x_4^{2^{d+1}-2} x_5^{2^{d+1}+2^d-4} \\
a_{437} &= x_1^{2^d-1} x_2^3 x_3 x_4^{2^{d+2}-4} x_5^{2^d-2} & a_{438} &= x_1^{2^d-1} x_2^3 x_3 x_4^{2^{d+2}-2} x_5^{2^d-4} \\
a_{439} &= x_1^{2^d-1} x_2^3 x_3 x_4^{2^d-6} x_5^{2^{d+2}-4} & a_{440} &= x_1^{2^d-1} x_2^3 x_3 x_4^{2^{d+1}-6} x_5^{2^{d+1}+2^d-4} \\
a_{441} &= x_1^{2^d-1} x_2^3 x_3 x_4^{2^{d+2}-6} x_5^{2^d-4} & a_{442} &= x_1^{2^d-1} x_2^3 x_3^{2^d-3} x_4 x_5^{2^{d+2}-4} \\
a_{443} &= x_1^{2^d-1} x_2^3 x_3^{2^{d+1}-3} x_4 x_5^{2^{d+1}+2^d-4} & a_{444} &= x_1^{2^d-1} x_2^3 x_3^{2^{d+2}-3} x_4 x_5^{2^d-4} \\
a_{445} &= x_1^{2^d-1} x_2^{2^d-1} x_3 x_4 x_5^{2^{d+2}-4} & a_{446} &= x_1^{2^d-1} x_2^{2^{d+1}-1} x_3 x_4 x_5^{2^{d+1}+2^d-4} \\
a_{447} &= x_1^{2^{d+1}-1} x_2 x_3 x_4^{2^d-2} x_5^{2^{d+1}+2^d-2} & a_{448} &= x_1^{2^{d+1}-1} x_2 x_3 x_4^{2^{d+1}+2^d-2} x_5^{2^d-2} \\
a_{449} &= x_1^{2^{d+1}-1} x_2 x_3 x_4^{2^d-4} x_5^{2^{d+1}+2^d-1} & a_{450} &= x_1^{2^{d+1}-1} x_2 x_3 x_4^{2^d-3} x_5^{2^{d+1}+2^d-2} \\
a_{451} &= x_1^{2^{d+1}-1} x_2 x_3 x_4^{2^d-1} x_5^{2^{d+1}+2^d-4} & a_{452} &= x_1^{2^{d+1}-1} x_2 x_3 x_4^{2^{d+1}+2^d-4} x_5^{2^d-1} \\
a_{453} &= x_1^{2^{d+1}-1} x_2 x_3 x_4^{2^{d+1}+2^d-3} x_5^{2^d-2} & a_{454} &= x_1^{2^{d+1}-1} x_2 x_3 x_4^{2^{d+1}+2^d-1} x_5^{2^d-4} \\
a_{455} &= x_1^{2^{d+1}-1} x_2 x_3 x_4^{2^d-4} x_5^{2^{d+1}+2^d-2} & a_{456} &= x_1^{2^{d+1}-1} x_2 x_3 x_4^{2^d-2} x_5^{2^{d+1}+2^d-4} \\
a_{457} &= x_1^{2^{d+1}-1} x_2 x_3 x_4^{2^{d+1}+2^d-4} x_5^{2^d-2} & a_{458} &= x_1^{2^{d+1}-1} x_2 x_3 x_4^{2^{d+1}+2^d-2} x_5^{2^d-4} \\
a_{459} &= x_1^{2^{d+1}-1} x_2 x_3^{2^d-2} x_4 x_5^{2^{d+1}+2^d-2} & a_{460} &= x_1^{2^{d+1}-1} x_2 x_3^{2^d-1} x_4 x_5^{2^{d+1}+2^d-4} \\
a_{461} &= x_1^{2^{d+1}-1} x_2 x_3^{2^{d+1}+2^d-2} x_4 x_5^{2^d-2} & a_{462} &= x_1^{2^{d+1}-1} x_2 x_3^{2^{d+1}+2^d-1} x_4 x_5^{2^d-4} \\
a_{463} &= x_1^{2^{d+1}-1} x_2^3 x_3 x_4^{2^d-4} x_5^{2^{d+1}+2^d-2} & a_{464} &= x_1^{2^{d+1}-1} x_2^3 x_3 x_4^{2^d-2} x_5^{2^{d+1}+2^d-4} \\
a_{465} &= x_1^{2^{d+1}-1} x_2^3 x_3 x_4^{2^{d+1}+2^d-4} x_5^{2^d-2} & a_{466} &= x_1^{2^{d+1}-1} x_2^3 x_3 x_4^{2^{d+1}+2^d-2} x_5^{2^d-4} \\
a_{467} &= x_1^{2^{d+1}-1} x_2^3 x_3 x_4^{2^d-6} x_5^{2^{d+1}+2^d-4} & a_{468} &= x_1^{2^{d+1}-1} x_2^3 x_3 x_4^{2^{d+1}+2^d-6} x_5^{2^d-4} \\
a_{469} &= x_1^{2^{d+1}-1} x_2^3 x_3^{2^d-3} x_4 x_5^{2^{d+1}+2^d-4} & a_{470} &= x_1^{2^{d+1}-1} x_2^3 x_3^{2^{d+1}+2^d-3} x_4 x_5^{2^d-4} \\
a_{471} &= x_1^{2^{d+1}-1} x_2^{2^d-1} x_3 x_4 x_5^{2^{d+1}+2^d-4} & a_{472} &= x_1^{2^{d+1}-1} x_2^{2^{d+1}+2^d-1} x_3 x_4 x_5^{2^d-4}
\end{aligned}$$

For  $d = 4$ ,

$$a_{473} = x_1^7 x_2^{11} x_3^{13} x_4^{14} x_5^{48}, \quad a_{474} = x_1^7 x_2^{11} x_3^{13} x_4^{18} x_5^{44}, \quad a_{475} = x_1^7 x_2^{11} x_3^{13} x_4^{50} x_5^{12}.$$

For  $d \geq 5$ ,

$$a_{473} = x_1^7 x_2^{2^d-5} x_3^5 x_4^{2^d-6} x_5^{2^{d+2}-4} \quad a_{474} = x_1^7 x_2^{2^d-5} x_3^5 x_4^{2^{d+1}-6} x_5^{2^{d+1}+2^d-4}$$

$$a_{475} = x_1^7 x_2^{2^d-5} x_3^5 x_4^{2^{d+2}-6} x_5^{2^d-4}$$

## 5.2 The cases $d = 3$

For  $d = 3$ , consider the monomials  $a_t = a_{3,t}$  as in Subsection 5.1 with  $d = 3$ ,  $1 \leq t \leq 84$  and  $B(3)$  as in the proof of Theorem 1.3. Then, we have

$$\tilde{B}_5^+((3)^3|(1)^2) = B(3) \cup \{a_{3,j} : 1 \leq j \leq 84\} \cup C(3),$$

where  $C(3)$  is the set of the monomials  $a_j$ ,  $85 \leq j \leq 450$ , which are determined as follows:

- |  |  |  |  |
|--|--|--|--|
| 85. $x_1 x_2^6 x_3 x_4^7 x_5^{30}$       | 86. $x_1 x_2^6 x_3 x_4^{14} x_5^{23}$    | 87. $x_1 x_2^6 x_3 x_4^{15} x_5^{22}$    | 88. $x_1 x_2^6 x_3 x_4^{30} x_5^7$       |
| 89. $x_1 x_2^6 x_3^3 x_4^5 x_5^{30}$     | 90. $x_1 x_2^6 x_3^3 x_4^{13} x_5^{22}$  | 91. $x_1 x_2^6 x_3^3 x_4^{29} x_5^6$     | 92. $x_1 x_2^6 x_3^7 x_4 x_5^{30}$       |
| 93. $x_1 x_2^6 x_3^7 x_4^7 x_5^{24}$     | 94. $x_1 x_2^6 x_3^9 x_4^5 x_5^{22}$     | 95. $x_1 x_2^6 x_3^7 x_4^{25} x_5^6$     | 96. $x_1 x_2^6 x_3^{11} x_4^5 x_5^{22}$  |
| 97. $x_1 x_2^6 x_3^{11} x_4^{21} x_5^6$  | 98. $x_1 x_2^6 x_3^{15} x_4 x_5^{22}$    | 99. $x_1 x_2^6 x_3^{27} x_4^5 x_5^6$     | 100. $x_1 x_2^7 x_3 x_4^6 x_5^{30}$      |
| 101. $x_1 x_2^7 x_3 x_4^{14} x_5^{22}$   | 102. $x_1 x_2^7 x_3 x_4^{30} x_5^6$      | 103. $x_1 x_2^7 x_3^2 x_4^5 x_5^{30}$    | 104. $x_1 x_2^7 x_3^2 x_4^7 x_5^{28}$    |
| 105. $x_1 x_2^7 x_3^2 x_4^{12} x_5^{23}$ | 106. $x_1 x_2^7 x_3^2 x_4^{13} x_5^{22}$ | 107. $x_1 x_2^7 x_3^2 x_4^{15} x_5^{20}$ | 108. $x_1 x_2^7 x_3^2 x_4^{28} x_5^7$    |
| 109. $x_1 x_2^7 x_3^2 x_4^{29} x_5^6$    | 110. $x_1 x_2^7 x_3^3 x_4^4 x_5^{30}$    | 111. $x_1 x_2^7 x_3^3 x_4^6 x_5^{28}$    | 112. $x_1 x_2^7 x_3^3 x_4^{12} x_5^{22}$ |
| 113. $x_1 x_2^7 x_3^3 x_4^{14} x_5^{20}$ | 114. $x_1 x_2^7 x_3^3 x_4^{28} x_5^6$    | 115. $x_1 x_2^7 x_3^3 x_4^{30} x_5^4$    | 116. $x_1 x_2^7 x_3^6 x_4 x_5^{30}$      |
| 117. $x_1 x_2^7 x_3^6 x_4^7 x_5^{24}$    | 118. $x_1 x_2^7 x_3^6 x_4^9 x_5^{22}$    | 119. $x_1 x_2^7 x_3^6 x_4^{25} x_5^6$    | 120. $x_1 x_2^7 x_3^7 x_4^2 x_5^{28}$    |
| 121. $x_1 x_2^7 x_3^7 x_4^6 x_5^{24}$    | 122. $x_1 x_2^7 x_3^7 x_4^8 x_5^{22}$    | 123. $x_1 x_2^7 x_3^7 x_4^{10} x_5^{20}$ | 124. $x_1 x_2^7 x_3^7 x_4^{14} x_5^{16}$ |
| 125. $x_1 x_2^7 x_3^7 x_4^{24} x_5^6$    | 126. $x_1 x_2^7 x_3^7 x_4^{26} x_5^4$    | 127. $x_1 x_2^7 x_3^{10} x_4^5 x_5^{22}$ | 128. $x_1 x_2^7 x_3^{10} x_4^{21} x_5^6$ |
| 129. $x_1 x_2^7 x_3^{11} x_4^4 x_5^{22}$ | 130. $x_1 x_2^7 x_3^{11} x_4^6 x_5^{20}$ | 131. $x_1 x_2^7 x_3^{11} x_4^{20} x_5^6$ | 132. $x_1 x_2^7 x_3^{11} x_4^{22} x_5^4$ |
| 133. $x_1 x_2^7 x_3^{14} x_4 x_5^{22}$   | 134. $x_1 x_2^7 x_3^{15} x_4^2 x_5^{20}$ | 135. $x_1 x_2^7 x_3^{26} x_4^5 x_5^6$    | 136. $x_1 x_2^7 x_3^{27} x_4^4 x_5^6$    |
| 137. $x_1 x_2^7 x_3^{27} x_4^6 x_5^4$    | 138. $x_1 x_2^7 x_3^{30} x_4 x_5^6$      | 139. $x_1 x_2^{14} x_3 x_4^6 x_5^{23}$   | 140. $x_1 x_2^{14} x_3 x_4^7 x_5^{22}$   |
| 141. $x_1 x_2^{14} x_3 x_4^{22} x_5^7$   | 142. $x_1 x_2^{14} x_3 x_4^{23} x_5^6$   | 143. $x_1 x_2^{14} x_3^3 x_4^5 x_5^{22}$ | 144. $x_1 x_2^{14} x_3^3 x_4^{21} x_5^6$ |
| 145. $x_1 x_2^{14} x_3^7 x_4 x_5^{22}$   | 146. $x_1 x_2^{14} x_3^{23} x_4 x_5^6$   | 147. $x_1 x_2^{15} x_3 x_4^6 x_5^{22}$   | 148. $x_1 x_2^{15} x_3 x_4^{22} x_5^6$   |
| 149. $x_1 x_2^{15} x_3^2 x_4^4 x_5^{23}$ | 150. $x_1 x_2^{15} x_3^2 x_4^5 x_5^{22}$ | 151. $x_1 x_2^{15} x_3^2 x_4^7 x_5^{20}$ | 152. $x_1 x_2^{15} x_3^2 x_4^{20} x_5^7$ |
| 153. $x_1 x_2^{15} x_3^2 x_4^{21} x_5^6$ | 154. $x_1 x_2^{15} x_3^2 x_4^{23} x_5^4$ | 155. $x_1 x_2^{15} x_3^3 x_4^4 x_5^{22}$ | 156. $x_1 x_2^{15} x_3^3 x_4^6 x_5^{20}$ |
| 157. $x_1 x_2^{15} x_3^3 x_4^{20} x_5^6$ | 158. $x_1 x_2^{15} x_3^3 x_4^{22} x_5^4$ | 159. $x_1 x_2^{15} x_3^6 x_4 x_5^{22}$   | 160. $x_1 x_2^{15} x_3^7 x_4^2 x_5^{20}$ |
| 161. $x_1 x_2^{15} x_3^{22} x_4 x_5^6$   | 162. $x_1 x_2^{15} x_3^{23} x_4^2 x_5^4$ | 163. $x_1 x_2^{30} x_3 x_4^6 x_5^7$      | 164. $x_1 x_2^{30} x_3 x_4^7 x_5^6$      |
| 165. $x_1 x_2^{30} x_3^3 x_4^5 x_5^6$    | 166. $x_1 x_2^{30} x_3^7 x_4 x_5^6$      | 167. $x_1^3 x_2 x_3^4 x_4^7 x_5^{30}$    | 168. $x_1^3 x_2 x_3^4 x_4^{14} x_5^{23}$ |
| 169. $x_1^3 x_2 x_3^4 x_4^{15} x_5^{22}$ | 170. $x_1^3 x_2 x_3^4 x_4^{30} x_5^7$    | 171. $x_1^3 x_2 x_3^5 x_4^6 x_5^{30}$    | 172. $x_1^3 x_2 x_3^5 x_4^{14} x_5^{22}$ |
| 173. $x_1^3 x_2 x_3^5 x_4^{30} x_5^6$    | 174. $x_1^3 x_2 x_3^6 x_4^5 x_5^{30}$    | 175. $x_1^3 x_2 x_3^6 x_4^7 x_5^{28}$    | 176. $x_1^3 x_2 x_3^6 x_4^{12} x_5^{23}$ |
| 177. $x_1^3 x_2 x_3^6 x_4^{13} x_5^{22}$ | 178. $x_1^3 x_2 x_3^6 x_4^{15} x_5^{20}$ | 179. $x_1^3 x_2 x_3^6 x_4^{28} x_5^7$    | 180. $x_1^3 x_2 x_3^6 x_4^{29} x_5^6$    |
| 181. $x_1^3 x_2 x_3^7 x_4^4 x_5^{30}$    | 182. $x_1^3 x_2 x_3^7 x_4^6 x_5^{28}$    | 183. $x_1^3 x_2 x_3^7 x_4^{12} x_5^{22}$ | 184. $x_1^3 x_2 x_3^7 x_4^{14} x_5^{20}$ |
| 185. $x_1^3 x_2 x_3^7 x_4^{28} x_5^6$    | 186. $x_1^3 x_2 x_3^7 x_4^{30} x_5^4$    | 187. $x_1^3 x_2 x_3^{12} x_4^6 x_5^{23}$ | 188. $x_1^3 x_2 x_3^{12} x_4^7 x_5^{22}$ |
| 189. $x_1^3 x_2 x_3^{12} x_4^{22} x_5^7$ | 190. $x_1^3 x_2 x_3^{12} x_4^{23} x_5^6$ | 191. $x_1^3 x_2 x_3^{13} x_4^6 x_5^{22}$ | 192. $x_1^3 x_2 x_3^{13} x_4^{22} x_5^6$ |
| 193. $x_1^3 x_2 x_3^{14} x_4^4 x_5^{23}$ | 194. $x_1^3 x_2 x_3^{14} x_4^5 x_5^{22}$ | 195. $x_1^3 x_2 x_3^{14} x_4^7 x_5^{20}$ | 196. $x_1^3 x_2 x_3^{14} x_4^{20} x_5^7$ |
| 197. $x_1^3 x_2 x_3^{14} x_4^{21} x_5^6$ | 198. $x_1^3 x_2 x_3^{14} x_4^{23} x_5^4$ | 199. $x_1^3 x_2 x_3^{15} x_4^4 x_5^{22}$ | 200. $x_1^3 x_2 x_3^{15} x_4^6 x_5^{20}$ |

201.  $x_1^3 x_2 x_3^{15} x_4^{20} x_5^6$       202.  $x_1^3 x_2 x_3^{15} x_4^{22} x_5^4$       203.  $x_1^3 x_2 x_3^{28} x_4^6 x_5^7$       204.  $x_1^3 x_2 x_3^{28} x_4^7 x_5^6$   
 205.  $x_1^3 x_2 x_3^{29} x_4^6 x_5^6$       206.  $x_1^3 x_2 x_3^{30} x_4^7 x_5^5$       207.  $x_1^3 x_2 x_3^{30} x_4^5 x_5^6$       208.  $x_1^3 x_2 x_3^{30} x_4^7 x_5^4$   
 209.  $x_1^3 x_2^3 x_3^4 x_4^5 x_5^{30}$       210.  $x_1^3 x_2^3 x_3^4 x_4^{13} x_5^{22}$       211.  $x_1^3 x_2^3 x_3^4 x_4^{29} x_5^6$       212.  $x_1^3 x_2^3 x_3^5 x_4^4 x_5^{30}$   
 213.  $x_1^3 x_2^3 x_3^6 x_4^4 x_5^{28}$       214.  $x_1^3 x_2^3 x_3^6 x_4^5 x_5^{22}$       215.  $x_1^3 x_2^3 x_3^6 x_4^{14} x_5^{20}$       216.  $x_1^3 x_2^3 x_3^6 x_4^{28} x_5^6$   
 217.  $x_1^3 x_2^3 x_3^6 x_4^4 x_5^4$       218.  $x_1^3 x_2^3 x_3^{12} x_4^5 x_5^{22}$       219.  $x_1^3 x_2^3 x_3^{12} x_4^{21} x_5^6$       220.  $x_1^3 x_2^3 x_3^{13} x_4^4 x_5^{22}$   
 221.  $x_1^3 x_2^3 x_3^{13} x_4^6 x_5^{20}$       222.  $x_1^3 x_2^3 x_3^{13} x_4^{20} x_5^6$       223.  $x_1^3 x_2^3 x_3^{13} x_4^{22} x_5^4$       224.  $x_1^3 x_2^3 x_3^{28} x_4^5 x_5^6$   
 225.  $x_1^3 x_2^3 x_3^{29} x_4^4 x_5^6$       226.  $x_1^3 x_2^3 x_3^{29} x_4^6 x_5^4$       227.  $x_1^3 x_2^4 x_3^3 x_4^5 x_5^{30}$       228.  $x_1^3 x_2^4 x_3^3 x_4^{13} x_5^{22}$   
 229.  $x_1^3 x_2^4 x_3^4 x_4^{29} x_5^6$       230.  $x_1^3 x_2^4 x_3^{11} x_4^5 x_5^{22}$       231.  $x_1^3 x_2^4 x_3^{11} x_4^{21} x_5^6$       232.  $x_1^3 x_2^4 x_3^{27} x_4^5 x_5^6$   
 233.  $x_1^3 x_2^5 x_3^4 x_4^6 x_5^{30}$       234.  $x_1^3 x_2^5 x_3^4 x_4^{14} x_5^{22}$       235.  $x_1^3 x_2^5 x_3^4 x_4^{30} x_5^6$       236.  $x_1^3 x_2^5 x_3^4 x_4^{30} x_5^6$   
 237.  $x_1^3 x_2^5 x_3^4 x_4^7 x_5^{28}$       238.  $x_1^3 x_2^5 x_3^4 x_4^{12} x_5^{23}$       239.  $x_1^3 x_2^5 x_3^4 x_4^{13} x_5^{22}$       240.  $x_1^3 x_2^5 x_3^4 x_4^{15} x_5^{20}$   
 241.  $x_1^3 x_2^5 x_3^4 x_4^{28} x_5^7$       242.  $x_1^3 x_2^5 x_3^4 x_4^{29} x_5^6$       243.  $x_1^3 x_2^5 x_3^4 x_4^{30} x_5^6$       244.  $x_1^3 x_2^5 x_3^4 x_4^{28} x_5^6$   
 245.  $x_1^3 x_2^5 x_3^4 x_4^{12} x_5^{22}$       246.  $x_1^3 x_2^5 x_3^4 x_4^{14} x_5^{20}$       247.  $x_1^3 x_2^5 x_3^4 x_4^{28} x_5^6$       248.  $x_1^3 x_2^5 x_3^4 x_4^{30} x_5^4$   
 249.  $x_1^3 x_2^5 x_3^4 x_4^{30} x_5^6$       250.  $x_1^3 x_2^5 x_3^4 x_4^{24} x_5^4$       251.  $x_1^3 x_2^5 x_3^4 x_4^{23} x_5^6$       252.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$   
 253.  $x_1^3 x_2^5 x_3^4 x_4^{15} x_5^{16}$       254.  $x_1^3 x_2^5 x_3^4 x_4^{24} x_5^7$       255.  $x_1^3 x_2^5 x_3^4 x_4^{25} x_5^6$       256.  $x_1^3 x_2^5 x_3^4 x_4^{28} x_5^6$   
 257.  $x_1^3 x_2^5 x_3^4 x_4^{24} x_5^6$       258.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^8$       259.  $x_1^3 x_2^5 x_3^4 x_4^{10} x_5^{20}$       260.  $x_1^3 x_2^5 x_3^4 x_4^{14} x_5^{16}$   
 261.  $x_1^3 x_2^5 x_3^4 x_4^{24} x_5^6$       262.  $x_1^3 x_2^5 x_3^4 x_4^{26} x_5^4$       263.  $x_1^3 x_2^5 x_3^4 x_4^{23} x_5^6$       264.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$   
 265.  $x_1^3 x_2^5 x_3^4 x_4^{20} x_5^{20}$       266.  $x_1^3 x_2^5 x_3^4 x_4^{20} x_5^7$       267.  $x_1^3 x_2^5 x_3^4 x_4^{21} x_5^6$       268.  $x_1^3 x_2^5 x_3^4 x_4^{23} x_5^4$   
 269.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^{22}$       270.  $x_1^3 x_2^5 x_3^4 x_4^{20} x_5^{20}$       271.  $x_1^3 x_2^5 x_3^4 x_4^{20} x_5^6$       272.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^4$   
 273.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^{22}$       274.  $x_1^3 x_2^5 x_3^4 x_4^{20} x_5^{20}$       275.  $x_1^3 x_2^5 x_3^4 x_4^{16} x_5^6$       276.  $x_1^3 x_2^5 x_3^4 x_4^{18} x_5^4$   
 277.  $x_1^3 x_2^5 x_3^4 x_4^{24} x_5^6$       278.  $x_1^3 x_2^5 x_3^4 x_4^{26} x_5^6$       279.  $x_1^3 x_2^5 x_3^4 x_4^{26} x_5^4$       280.  $x_1^3 x_2^5 x_3^4 x_4^{27} x_5^6$   
 281.  $x_1^3 x_2^5 x_3^4 x_4^{24} x_5^6$       282.  $x_1^3 x_2^5 x_3^4 x_4^{26} x_5^6$       283.  $x_1^3 x_2^5 x_3^4 x_4^{30} x_5^6$       284.  $x_1^3 x_2^5 x_3^4 x_4^{28} x_5^6$   
 285.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^{22}$       286.  $x_1^3 x_2^5 x_3^4 x_4^{20} x_5^{20}$       287.  $x_1^3 x_2^5 x_3^4 x_4^{28} x_5^6$       288.  $x_1^3 x_2^5 x_3^4 x_4^{30} x_5^4$   
 289.  $x_1^3 x_2^5 x_3^4 x_4^{28} x_5^{28}$       290.  $x_1^3 x_2^5 x_3^4 x_4^{24} x_5^6$       291.  $x_1^3 x_2^5 x_3^4 x_4^{25} x_5^6$       292.  $x_1^3 x_2^5 x_3^4 x_4^{10} x_5^{20}$   
 293.  $x_1^3 x_2^5 x_3^4 x_4^{14} x_5^{16}$       294.  $x_1^3 x_2^5 x_3^4 x_4^{24} x_5^6$       295.  $x_1^3 x_2^5 x_3^4 x_4^{26} x_5^4$       296.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^{22}$   
 297.  $x_1^3 x_2^5 x_3^4 x_4^{20} x_5^{20}$       298.  $x_1^3 x_2^5 x_3^4 x_4^{20} x_5^6$       299.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^4$       300.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^{20}$   
 301.  $x_1^3 x_2^5 x_3^4 x_4^{16} x_5^{16}$       302.  $x_1^3 x_2^5 x_3^4 x_4^{18} x_5^4$       303.  $x_1^3 x_2^5 x_3^4 x_4^{25} x_5^6$       304.  $x_1^3 x_2^5 x_3^4 x_4^{25} x_5^6$   
 305.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^{22}$       306.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       307.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       308.  $x_1^3 x_2^5 x_3^4 x_4^{23} x_5^6$   
 309.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^{22}$       310.  $x_1^3 x_2^5 x_3^4 x_4^{20} x_5^{20}$       311.  $x_1^3 x_2^5 x_3^4 x_4^{20} x_5^6$       312.  $x_1^3 x_2^5 x_3^4 x_4^{21} x_5^6$   
 313.  $x_1^3 x_2^5 x_3^4 x_4^{23} x_5^4$       314.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       315.  $x_1^3 x_2^5 x_3^4 x_4^{20} x_5^6$       316.  $x_1^3 x_2^5 x_3^4 x_4^{20} x_5^6$   
 317.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^4$       318.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       319.  $x_1^3 x_2^5 x_3^4 x_4^{20} x_5^6$       320.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$   
 321.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^4$       322.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       323.  $x_1^3 x_2^5 x_3^4 x_4^{20} x_5^6$       324.  $x_1^3 x_2^5 x_3^4 x_4^{20} x_5^6$   
 325.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^4$       326.  $x_1^3 x_2^5 x_3^4 x_4^{20} x_5^6$       327.  $x_1^3 x_2^5 x_3^4 x_4^{16} x_5^6$       328.  $x_1^3 x_2^5 x_3^4 x_4^{18} x_5^4$   
 329.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^4$       330.  $x_1^3 x_2^5 x_3^4 x_4^{20} x_5^6$       331.  $x_1^3 x_2^5 x_3^4 x_4^{20} x_5^6$       332.  $x_1^3 x_2^5 x_3^4 x_4^{20} x_5^6$   
 333.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^4$       334.  $x_1^3 x_2^5 x_3^4 x_4^{20} x_5^6$       335.  $x_1^3 x_2^5 x_3^4 x_4^{20} x_5^6$       336.  $x_1^3 x_2^5 x_3^4 x_4^{20} x_5^6$   
 337.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^4$       338.  $x_1^3 x_2^5 x_3^4 x_4^{20} x_5^6$       339.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       340.  $x_1^3 x_2^5 x_3^4 x_4^{20} x_5^6$   
 341.  $x_1^3 x_2^5 x_3^4 x_4^{20} x_5^6$       342.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       343.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       344.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$   
 345.  $x_1^3 x_2^5 x_3^4 x_4^{20} x_5^6$       346.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       347.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       348.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$   
 349.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       350.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       351.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       352.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$   
 353.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       354.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       355.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       356.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$   
 357.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       358.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       359.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       360.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$   
 361.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       362.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       363.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       364.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$   
 365.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       366.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       367.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       368.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$   
 369.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       370.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       371.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$       372.  $x_1^3 x_2^5 x_3^4 x_4^{22} x_5^6$

- |  |  |  |   |
|--|--|--|---|
| 373. $x_1^7 x_2 x_3^{26} x_4^5 x_5^6$    | 374. $x_1^7 x_2 x_3^{27} x_4^4 x_5^6$          | 375. $x_1^7 x_2 x_3^{27} x_4^6 x_5^4$    | 376. $x_1^7 x_2 x_3^{30} x_4 x_5^6$         |
| 377. $x_1^7 x_2^3 x_3 x_4^4 x_5^{30}$    | 378. $x_1^7 x_2^3 x_3 x_4^6 x_5^{28}$          | 379. $x_1^7 x_2^3 x_3 x_4^{12} x_5^{22}$ | 380. $x_1^7 x_2^3 x_3 x_4^{14} x_5^{20}$    |
| 381. $x_1^7 x_2^3 x_3 x_4^{28} x_5^6$    | 382. $x_1^7 x_2^3 x_3 x_4^{30} x_5^4$          | 383. $x_1^7 x_2^3 x_3 x_4^5 x_5^{28}$    | 384. $x_1^7 x_2^3 x_3 x_4^6 x_5^{24}$       |
| 385. $x_1^7 x_2^3 x_3 x_4^8 x_5^{22}$    | 386. $x_1^7 x_2^3 x_3 x_4^{10} x_5^{20}$       | 387. $x_1^7 x_2^3 x_3 x_4^{14} x_5^{16}$ | 388. $x_1^7 x_2^3 x_3 x_4^{24} x_5^6$       |
| 389. $x_1^7 x_2^3 x_3 x_4^{26} x_5^4$    | 390. $x_1^7 x_2^3 x_3 x_4^{22} x_5^6$          | 391. $x_1^7 x_2^3 x_3 x_4^6 x_5^{20}$    | 392. $x_1^7 x_2^3 x_3 x_4^{20} x_5^6$       |
| 393. $x_1^7 x_2^3 x_3 x_4^{22} x_5^4$    | 394. $x_1^7 x_2^3 x_3 x_4^{13} x_5^2 x_5^{20}$ | 395. $x_1^7 x_2^3 x_3 x_4^6 x_5^{16}$    | 396. $x_1^7 x_2^3 x_3 x_4^{18} x_5^{14}$    |
| 397. $x_1^7 x_2^3 x_3^{25} x_4^4 x_5^6$  | 398. $x_1^7 x_2^3 x_3^{25} x_4^6 x_5^4$        | 399. $x_1^7 x_2^3 x_3^{29} x_4^2 x_5^4$  | 400. $x_1^7 x_2^3 x_3 x_4^{28} x_5^6$       |
| 401. $x_1^7 x_2^3 x_3 x_4^6 x_5^{24}$    | 402. $x_1^7 x_2^3 x_3 x_4^8 x_5^{22}$          | 403. $x_1^7 x_2^3 x_3 x_4^{10} x_5^{20}$ | 404. $x_1^7 x_2^3 x_3 x_4^{14} x_5^{16}$    |
| 405. $x_1^7 x_2^3 x_3 x_4^{24} x_5^6$    | 406. $x_1^7 x_2^3 x_3 x_4^{26} x_5^4$          | 407. $x_1^7 x_2^3 x_3 x_4^8 x_5^{16}$    | 408. $x_1^7 x_2^3 x_3 x_4^2 x_5^{20}$       |
| 409. $x_1^7 x_2^3 x_3 x_4^6 x_5^{16}$    | 410. $x_1^7 x_2^3 x_3 x_4^{18} x_5^4$          | 411. $x_1^7 x_2^3 x_3 x_4^2 x_5^4$       | 412. $x_1^7 x_2^3 x_3 x_4^6 x_5^{20}$       |
| 413. $x_1^7 x_2^{11} x_3 x_4^4 x_5^{22}$ | 414. $x_1^7 x_2^{11} x_3 x_4^6 x_5^{20}$       | 415. $x_1^7 x_2^{11} x_3 x_4^{20} x_5^6$ | 416. $x_1^7 x_2^{11} x_3 x_4^{22} x_5^{14}$ |
| 417. $x_1^7 x_2^{11} x_3 x_4^5 x_5^{20}$ | 418. $x_1^7 x_2^{11} x_3 x_4^6 x_5^{16}$       | 419. $x_1^7 x_2^{11} x_3 x_4^{18} x_5^4$ | 420. $x_1^7 x_2^{11} x_3 x_4^{21} x_5^{14}$ |
| 421. $x_1^7 x_2^{15} x_3 x_4^4 x_5^{20}$ | 422. $x_1^7 x_2^{27} x_3 x_4^4 x_5^6$          | 423. $x_1^7 x_2^{27} x_3 x_4^6 x_5^4$    | 424. $x_1^7 x_2^{27} x_3 x_4^2 x_5^4$       |
| 425. $x_1^{15} x_2 x_3 x_4^6 x_5^{22}$   | 426. $x_1^{15} x_2 x_3 x_4^{22} x_5^6$         | 427. $x_1^{15} x_2 x_3 x_4^4 x_5^{23}$   | 428. $x_1^{15} x_2 x_3 x_4^5 x_5^{22}$      |
| 429. $x_1^{15} x_2 x_3 x_4^7 x_5^{20}$   | 430. $x_1^{15} x_2 x_3 x_4^{20} x_5^7$         | 431. $x_1^{15} x_2 x_3 x_4^{21} x_5^6$   | 432. $x_1^{15} x_2 x_3 x_4^{23} x_5^{14}$   |
| 433. $x_1^{15} x_2 x_3 x_4^4 x_5^{22}$   | 434. $x_1^{15} x_2 x_3 x_4^6 x_5^{20}$         | 435. $x_1^{15} x_2 x_3 x_4^{20} x_5^6$   | 436. $x_1^{15} x_2 x_3 x_4^{22} x_5^{14}$   |
| 437. $x_1^{15} x_2 x_3 x_4 x_5^{22}$     | 438. $x_1^{15} x_2 x_3 x_4^2 x_5^{20}$         | 439. $x_1^{15} x_2 x_3 x_4^2 x_5^6$      | 440. $x_1^{15} x_2 x_3 x_4^3 x_5^{14}$      |
| 441. $x_1^{15} x_2^3 x_3 x_4^4 x_5^{22}$ | 442. $x_1^{15} x_2^3 x_3 x_4^6 x_5^{20}$       | 443. $x_1^{15} x_2^3 x_3 x_4^{20} x_5^6$ | 444. $x_1^{15} x_2^3 x_3 x_4^2 x_5^{14}$    |
| 445. $x_1^{15} x_2^3 x_3 x_4^5 x_5^{20}$ | 446. $x_1^{15} x_2^3 x_3 x_4^6 x_5^{16}$       | 447. $x_1^{15} x_2^3 x_3 x_4^{18} x_5^4$ | 448. $x_1^{15} x_2^3 x_3 x_4^2 x_5^{14}$    |
| 449. $x_1^{15} x_2 x_3 x_4^2 x_5^{20}$   | 450. $x_1^{15} x_2^{23} x_3 x_4^2 x_5^4$       |  |   |

Then,  $B_5^0((3)^3|(1)^2) \cup \widetilde{B}_5^+((3)^3|(1)^2) \cup \{Y_t : 1 \leq t \leq 34\}$  is a set of generators for  $\text{Ker}(\widetilde{S}q_{(5,20)}^0)$ . Here, the monomials  $Y_t$  are given in Subsection 3.3.

## 6 The admissible monomials of degree $2^{d+1} + 2^d - 4$ for $3 \leq d \leq 5$

In this subsection we list the needed admissible monomials of degree  $m = 2^{d+1} + 2^d - 4$  in  $P_5$  for  $3 \leq d \leq 5$ . The elements in  $B_5^0(m)$  can be determined by using the results in [7], [4], [20] and Proposition 2.4.2. We have  $|B_5^0((4)^{d-2}|(2)|(1)^2)| = 225$ ,  $|B_5^0((4)^{d-2}|(2)|(3))| = 20$ ,  $|B_5^0((4)^{d-1}|(2))| = 30$ , hence  $|B_5^0(m)| = 275$  for any  $d \geq 3$ . So, we list only the elements in  $B_5^+(m)$ .

### 6.1 The case $d = 3$

$\widetilde{B}_5^+(4, 2, 1, 1) = \{a_t = a_{2,t} : 1 \leq t \leq 215\}$ , where

- |                                   |                                   |                                 |                                   |
|-----------------------------------|-----------------------------------|---------------------------------|-----------------------------------|
| 1. $x_1 x_2 x_3 x_4^3 x_5^{14}$   | 2. $x_1 x_2 x_3 x_4^6 x_5^{11}$   | 3. $x_1 x_2 x_3 x_4^7 x_5^{10}$ | 4. $x_1 x_2 x_3 x_4^{14} x_5^3$   |
| 5. $x_1 x_2 x_3^2 x_4^3 x_5^{13}$ | 6. $x_1 x_2 x_3^2 x_4^5 x_5^{11}$ | 7. $x_1 x_2 x_3^2 x_4^9 x_5^9$  | 8. $x_1 x_2 x_3^2 x_4^{13} x_5^3$ |

9.  $x_1x_2x_3^3x_4x_5^{14}$
13.  $x_1x_2x_3^3x_4^5x_5^{10}$
17.  $x_1x_2x_3^3x_4^{13}x_5^2$
21.  $x_1x_2x_3^3x_4^9x_5^3$
25.  $x_1x_2x_3^3x_4^8x_5^5$
29.  $x_1x_2x_3^{14}x_4x_5^3$
33.  $x_1x_2^2x_3^3x_4^7x_5^9$
37.  $x_1x_2^2x_3^3x_4^{13}x_5$
41.  $x_1x_2^2x_3^3x_4^{11}x_5$
45.  $x_1x_2^2x_3^{13}x_4^3x_5$
49.  $x_1x_2^3x_3^3x_4^4x_5^{11}$
53.  $x_1x_2^3x_3^3x_4^{12}x_5^3$
57.  $x_1x_2^3x_3^3x_4^5x_5^9$
61.  $x_1x_2^3x_3^3x_4^5x_5^8$
65.  $x_1x_2^3x_3^3x_4^9x_5^3$
69.  $x_1x_2^3x_3^3x_4^8x_5^5$
73.  $x_1x_2^3x_3^3x_4^9x_5^5$
77.  $x_1x_2^3x_3^{12}x_4x_5^3$
81.  $x_1x_2^3x_3^{14}x_4x_5$
85.  $x_1x_2^6x_3^3x_4^{11}x_5$
89.  $x_1x_2^6x_3^3x_4^3x_5$
93.  $x_1x_2^7x_3^3x_4^8x_5^5$
97.  $x_1x_2^7x_3^3x_4^9x_5^5$
101.  $x_1x_2^7x_3^3x_4x_5^3$
105.  $x_1x_2^7x_3^{10}x_4x_5$
109.  $x_1^3x_2x_3x_4x_5^{14}$
113.  $x_1^3x_2x_3x_4^5x_5^{10}$
117.  $x_1^3x_2x_3x_4^{13}x_5^2$
121.  $x_1^3x_2x_3^2x_4^{13}x_5$
125.  $x_1^3x_2x_3^2x_4^{12}x_5$
129.  $x_1^3x_2x_3^2x_4^{11}x_5$
133.  $x_1^3x_2x_3^2x_4^8x_5^3$
137.  $x_1^3x_2x_3^2x_4^9x_5^5$
141.  $x_1^3x_2x_3^{12}x_4^3x_5$
145.  $x_1^3x_2^3x_3x_4x_5^{12}$
149.  $x_1^3x_2^3x_3^4x_4x_5^9$
153.  $x_1^3x_2^3x_3^{12}x_4x_5^5$
157.  $x_1^3x_2^5x_3^3x_4x_5^9$
161.  $x_1^3x_2^5x_3^3x_4^2x_5^9$
165.  $x_1^3x_2^5x_3^3x_4^{10}x_5$
169.  $x_1^3x_2^5x_3^3x_4^8x_5^5$
173.  $x_1^3x_2^5x_3^3x_4^2x_5^5$
177.  $x_1^3x_2^7x_3^3x_4x_5^5$
10.  $x_1x_2x_3^3x_4^2x_5^{13}$
14.  $x_1x_2x_3^3x_4^6x_5^9$
18.  $x_1x_2x_3^3x_4^{14}x_5$
22.  $x_1x_2x_3^6x_4^{11}x_5$
26.  $x_1x_2x_3^7x_4^8x_5^5$
30.  $x_1x_2x_3^{14}x_4^3x_5$
34.  $x_1x_2^2x_3^3x_4^{13}x_5^3$
38.  $x_1x_2^2x_3^5x_4x_5^{11}$
42.  $x_1x_2^2x_3^7x_4x_5^9$
46.  $x_1x_2^2x_3^3x_4x_5^{14}$
50.  $x_1x_2^2x_3^5x_4x_5^{10}$
54.  $x_1x_2^2x_3^3x_4^{13}x_5^2$
58.  $x_1x_2^2x_3^3x_4^{12}x_5$
62.  $x_1x_2^2x_3^3x_4^{12}x_5$
66.  $x_1x_2^2x_3^4x_4^{11}x_5$
70.  $x_1x_2^2x_3^5x_4^8x_5^5$
74.  $x_1x_2^2x_3^6x_4^9x_5^5$
78.  $x_1x_2^2x_3^{12}x_4^3x_5$
82.  $x_1x_2^2x_3^3x_4x_5^{11}$
86.  $x_1x_2^2x_3^3x_4x_5^{11}$
90.  $x_1x_2^2x_3^{11}x_4x_5^5$
94.  $x_1x_2^2x_3^8x_4x_5^3$
98.  $x_1x_2^2x_3^9x_4x_5^5$
102.  $x_1x_2^7x_3^8x_4x_5^5$
106.  $x_1x_2^{14}x_3x_4x_5^3$
110.  $x_1^3x_2x_3x_4^2x_5^{13}$
114.  $x_1^3x_2x_3x_4^6x_5^9$
118.  $x_1^3x_2x_3x_4^{14}x_5$
122.  $x_1^3x_2x_3^3x_4x_5^{12}$
126.  $x_1^3x_2x_3^4x_4x_5^{11}$
130.  $x_1^3x_2x_3^5x_4x_5^{10}$
134.  $x_1^3x_2x_3^5x_4^9x_5^5$
138.  $x_1^3x_2x_3^7x_4x_5^8$
142.  $x_1^3x_2x_3^{13}x_4x_5^2$
146.  $x_1^3x_2^3x_3^3x_4^4x_5^9$
150.  $x_1^3x_2^3x_3^4x_4^9x_5^5$
154.  $x_1^3x_2^4x_3^3x_4x_5^{11}$
158.  $x_1^3x_2^4x_3^3x_4^9x_5^5$
162.  $x_1^3x_2^5x_3^3x_4^8x_5^5$
166.  $x_1^3x_2^5x_3^2x_4^9x_5^5$
170.  $x_1^3x_2^5x_3^8x_4x_5^3$
174.  $x_1^3x_2^5x_3^{10}x_4x_5^5$
178.  $x_1^3x_2^{13}x_3x_4x_5^2$
11.  $x_1x_2x_3^3x_4^2x_5^{12}$
15.  $x_1x_2x_3^3x_4^8x_5^5$
19.  $x_1x_2x_3^6x_4x_5^{11}$
23.  $x_1x_2x_3^7x_4x_5^{10}$
27.  $x_1x_2x_3^7x_4^9x_5^2$
31.  $x_1x_2^2x_3^3x_4^3x_5^{13}$
35.  $x_1x_2^2x_3^3x_4^3x_5^{13}$
39.  $x_1x_2^2x_3^5x_4^9x_5^5$
43.  $x_1x_2^2x_3^7x_4^9x_5^5$
47.  $x_1x_2^3x_3^3x_4^2x_5^{13}$
51.  $x_1x_2^3x_3^6x_4x_5^9$
55.  $x_1x_2^3x_3^4x_4^{14}x_5$
59.  $x_1x_2^3x_3^3x_4x_5^{12}$
63.  $x_1x_2^3x_3^4x_4x_5^{11}$
67.  $x_1x_2^3x_3^5x_4x_5^{10}$
71.  $x_1x_2^3x_3^5x_4^2x_5^5$
75.  $x_1x_2^3x_3^7x_4x_5^8$
79.  $x_1x_2^3x_3^{13}x_4x_5^2$
83.  $x_1x_2^6x_3^3x_4^9x_5^5$
87.  $x_1x_2^6x_3^3x_4^9x_5^5$
91.  $x_1x_2^7x_3^3x_4x_5^{10}$
95.  $x_1x_2^7x_3^6x_4x_5^2$
99.  $x_1x_2^7x_3^3x_4x_5^8$
103.  $x_1x_2^7x_3^9x_4x_5^2$
107.  $x_1x_2^{14}x_3x_3^3x_4x_5$
111.  $x_1^3x_2x_3x_3^3x_4^2x_5^{12}$
115.  $x_1^3x_2x_3x_3^4x_4^8x_5^5$
119.  $x_1^3x_2x_3^2x_4x_5^{13}$
123.  $x_1^3x_2x_3^3x_4^9x_5^5$
127.  $x_1^3x_2x_3^4x_4^9x_5^5$
131.  $x_1^3x_2x_3^5x_4^9x_5^5$
135.  $x_1^3x_2x_3^5x_4^{10}x_5$
139.  $x_1^3x_2x_3^7x_4^8x_5^5$
143.  $x_1^3x_2x_3^{13}x_4^2x_5$
147.  $x_1^3x_2^3x_3^3x_4^5x_5^8$
151.  $x_1^3x_2^3x_3^5x_4x_5^8$
155.  $x_1^3x_2^4x_3^3x_4^9x_5^5$
159.  $x_1^3x_2^5x_3^{11}x_4x_5^5$
163.  $x_1^3x_2^5x_3^8x_4^3x_5^5$
167.  $x_1^3x_2^5x_3^2x_4^9x_5^5$
171.  $x_1^3x_2^5x_3^8x_4^3x_5^5$
175.  $x_1^3x_2^7x_3^3x_4x_5^8$
179.  $x_1^3x_2^{13}x_3x_3^2x_4x_5$
12.  $x_1x_2x_3^3x_4^4x_5^{11}$
16.  $x_1x_2x_3^3x_4^{12}x_5^3$
20.  $x_1x_2x_3^6x_4^3x_5^9$
24.  $x_1x_2x_3^7x_4^2x_5^9$
28.  $x_1x_2x_3^7x_4^{10}x_5$
32.  $x_1x_2^2x_3^3x_4^5x_5^{11}$
36.  $x_1x_2^2x_3^3x_4^5x_5^9$
40.  $x_1x_2^2x_3^3x_4^9x_5^3$
44.  $x_1x_2^2x_3^{13}x_4x_5^3$
48.  $x_1x_2^3x_3^3x_4^3x_5^{12}$
52.  $x_1x_2^3x_3^3x_4^7x_5^8$
56.  $x_1x_2^3x_3^3x_4x_5^{13}$
60.  $x_1x_2^3x_3^3x_4^9x_5^5$
64.  $x_1x_2^3x_3^3x_4^9x_5^5$
68.  $x_1x_2^3x_3^3x_4^9x_5^5$
72.  $x_1x_2^3x_3^3x_4^{10}x_5$
76.  $x_1x_2^3x_3^3x_4^8x_5^5$
80.  $x_1x_2^3x_3^{13}x_4^2x_5$
84.  $x_1x_2^6x_3^3x_4^9x_5^3$
88.  $x_1x_2^6x_3^3x_4^9x_5^3$
92.  $x_1x_2^7x_3^3x_4^9x_5^5$
96.  $x_1x_2^7x_3^3x_4^{10}x_5$
100.  $x_1x_2^7x_3^3x_4^8x_5^5$
104.  $x_1x_2^7x_3^9x_4^2x_5$
108.  $x_1x_2^{14}x_3^3x_4x_5$
112.  $x_1^3x_2x_3x_3^4x_4^3x_5^{11}$
116.  $x_1^3x_2x_3x_3^4x_4^{12}x_5^3$
120.  $x_1^3x_2x_3^2x_4^5x_5^9$
124.  $x_1^3x_2x_3^3x_4^5x_5^9$
128.  $x_1^3x_2x_3^4x_4^9x_5^3$
132.  $x_1^3x_2x_3^5x_4^8x_5^5$
136.  $x_1^3x_2x_3^6x_4x_5^9$
140.  $x_1^3x_2x_3^{13}x_4x_5^3$
144.  $x_1^3x_2x_3^{14}x_4x_5$
148.  $x_1^3x_2^3x_3^3x_4^{12}x_5$
152.  $x_1^3x_2^3x_3^5x_4^8x_5^5$
156.  $x_1^3x_2^4x_3^3x_4^{11}x_5$
160.  $x_1^3x_2^5x_3^3x_4x_5^{10}$
164.  $x_1^3x_2^5x_3^3x_4^9x_5^2$
168.  $x_1^3x_2^5x_3^3x_4^8x_5^5$
172.  $x_1^3x_2^5x_3^9x_4x_5^2$
176.  $x_1^3x_2^7x_3^3x_4^8x_5^5$
180.  $x_1^3x_2^{13}x_3^2x_4x_5$



- |                                   |                                   |                                   |                                   |
|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| 181. $x_1^7 x_2 x_3 x_4 x_5^{10}$ | 182. $x_1^7 x_2 x_3 x_4^2 x_5^9$  | 183. $x_1^7 x_2 x_3 x_4^3 x_5^8$  | 184. $x_1^7 x_2 x_3 x_4^4 x_5^7$  |
| 185. $x_1^7 x_2 x_3 x_4^5 x_5^6$  | 186. $x_1^7 x_2 x_3 x_4^6 x_5^5$  | 187. $x_1^7 x_2 x_3^2 x_4 x_5^9$  | 188. $x_1^7 x_2 x_3^3 x_4 x_5^8$  |
| 189. $x_1^7 x_2 x_3^3 x_4 x_5^8$  | 190. $x_1^7 x_2 x_3^4 x_4 x_5^7$  | 191. $x_1^7 x_2 x_3^4 x_4 x_5^3$  | 192. $x_1^7 x_2 x_3^3 x_4 x_5^3$  |
| 193. $x_1^7 x_2 x_3^3 x_4 x_5^2$  | 194. $x_1^7 x_2 x_3^2 x_4 x_5^2$  | 195. $x_1^7 x_2 x_3^{10} x_4 x_5$ | 196. $x_1^7 x_3^3 x_4 x_5^8$      |
| 197. $x_1^7 x_2^3 x_3 x_4 x_5^8$  | 198. $x_1^7 x_2^3 x_3^2 x_4 x_5$  | 199. $x_1^7 x_2^3 x_3 x_4 x_5^2$  | 200. $x_1^7 x_2^3 x_3 x_4 x_5^2$  |
| 201. $x_1 x_2 x_3 x_4^{15} x_5^2$ | 202. $x_1 x_2 x_3 x_4^2 x_5^{15}$ | 203. $x_1 x_2 x_3^{15} x_4 x_5^2$ | 204. $x_1 x_2 x_3^{15} x_4 x_5^2$ |
| 205. $x_1 x_2 x_3^2 x_4 x_5^{15}$ | 206. $x_1 x_2 x_3^3 x_4 x_5^{15}$ | 207. $x_1 x_2^{15} x_3 x_4 x_5^2$ | 208. $x_1 x_2^{15} x_3 x_4 x_5^2$ |
| 209. $x_1 x_2^{15} x_3^2 x_4 x_5$ | 210. $x_1 x_2^2 x_3 x_4 x_5^{15}$ | 211. $x_1 x_2^2 x_3 x_4^{15} x_5$ | 212. $x_1 x_2^2 x_3^{15} x_4 x_5$ |
| 213. $x_1^{15} x_2 x_3 x_4 x_5^2$ | 214. $x_1^{15} x_2 x_3 x_4 x_5$   | 215. $x_1^{15} x_2 x_3 x_4 x_5$   |                                   |

We have  $|B_5^+(4, 2, 3)| = 50$  and its elements have been explicitly determined in [8].

$B_5^+(4, 4, 2) = \{\bar{a}_t : 1 \leq t \leq 91\}$ , where

- |                                     |                                     |                                     |                                     |
|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|
| 1. $x_1 x_2^2 x_3^3 x_4^7 x_5^7$    | 2. $x_1 x_2^2 x_3^7 x_4^3 x_5^7$    | 3. $x_1 x_2^2 x_3^7 x_4^3 x_5^3$    | 4. $x_1 x_2^3 x_3^2 x_4^7 x_5^7$    |
| 5. $x_1 x_2^3 x_3^6 x_4^7 x_5^7$    | 6. $x_1 x_2^3 x_3^3 x_4^6 x_5^6$    | 7. $x_1 x_2^3 x_3^6 x_4^3 x_5^7$    | 8. $x_1 x_2^3 x_3^6 x_4^7 x_5^3$    |
| 9. $x_1 x_2^3 x_3^7 x_4^7 x_5^7$    | 10. $x_1 x_2^3 x_3^7 x_4^6 x_5^6$   | 11. $x_1 x_2^3 x_3^7 x_4^6 x_5^3$   | 12. $x_1 x_2^3 x_3^7 x_4^7 x_5^2$   |
| 13. $x_1 x_2^6 x_3^3 x_4^7 x_5^7$   | 14. $x_1 x_2^6 x_3^3 x_4^3 x_5^3$   | 15. $x_1 x_2^6 x_3^7 x_4^3 x_5^3$   | 16. $x_1 x_2^7 x_3^2 x_4^3 x_5^7$   |
| 17. $x_1 x_2^7 x_3^2 x_4^7 x_5^3$   | 18. $x_1 x_2^7 x_3^2 x_4^7 x_5^7$   | 19. $x_1 x_2^7 x_3^3 x_4^6 x_5^3$   | 20. $x_1 x_2^7 x_3^3 x_4^6 x_5^3$   |
| 21. $x_1 x_2^7 x_3^3 x_4^7 x_5^2$   | 22. $x_1 x_2^7 x_3^6 x_4^3 x_5^3$   | 23. $x_1 x_2^7 x_3^7 x_4^2 x_5^3$   | 24. $x_1 x_2^7 x_3^7 x_4^3 x_5^2$   |
| 25. $x_1^3 x_2 x_3^2 x_4^7 x_5^7$   | 26. $x_1^3 x_2 x_3^6 x_4^7 x_5^7$   | 27. $x_1^3 x_2 x_3^3 x_4^6 x_5^3$   | 28. $x_1^3 x_2 x_3^6 x_4^3 x_5^7$   |
| 29. $x_1^3 x_2 x_3^6 x_4^7 x_5^3$   | 30. $x_1^3 x_2 x_3^7 x_4^7 x_5^7$   | 31. $x_1^3 x_2 x_3^7 x_4^6 x_5^3$   | 32. $x_1^3 x_2 x_3^7 x_4^6 x_5^3$   |
| 33. $x_1^3 x_2 x_3^7 x_4^7 x_5^2$   | 34. $x_1^3 x_2^3 x_3^6 x_4^7 x_5^7$ | 35. $x_1^3 x_2^3 x_3^7 x_4^6 x_5^3$ | 36. $x_1^3 x_2^3 x_3^3 x_4^7 x_5^7$ |
| 37. $x_1^3 x_2^3 x_3^5 x_4^6 x_5^6$ | 38. $x_1^3 x_2^3 x_3^3 x_4^4 x_5^4$ | 39. $x_1^3 x_2^3 x_3^4 x_4^3 x_5^7$ | 40. $x_1^3 x_2^3 x_3^4 x_4^7 x_5^3$ |
| 41. $x_1^3 x_2^3 x_3^5 x_4^7 x_5^7$ | 42. $x_1^3 x_2^3 x_3^5 x_4^6 x_5^6$ | 43. $x_1^3 x_2^3 x_3^5 x_4^6 x_5^3$ | 44. $x_1^3 x_2^3 x_3^5 x_4^7 x_5^2$ |
| 45. $x_1^3 x_2^3 x_3^7 x_4^6 x_5^6$ | 46. $x_1^3 x_2^3 x_3^7 x_4^4 x_5^4$ | 47. $x_1^3 x_2^3 x_3^7 x_4^3 x_5^3$ | 48. $x_1^3 x_2^3 x_3^7 x_4^5 x_5^2$ |
| 49. $x_1^3 x_2^5 x_3^2 x_4^7 x_5^7$ | 50. $x_1^3 x_2^5 x_3^2 x_4^3 x_5^3$ | 51. $x_1^3 x_2^5 x_3^3 x_4^2 x_5^7$ | 52. $x_1^3 x_2^5 x_3^3 x_4^2 x_5^3$ |
| 53. $x_1^3 x_2^5 x_3^3 x_4^6 x_5^3$ | 54. $x_1^3 x_2^5 x_3^3 x_4^2 x_5^2$ | 55. $x_1^3 x_2^5 x_3^6 x_4^3 x_5^3$ | 56. $x_1^3 x_2^5 x_3^7 x_4^2 x_5^3$ |
| 57. $x_1^3 x_2^5 x_3^7 x_4^2 x_5^2$ | 58. $x_1^3 x_2^7 x_3^2 x_4^7 x_5^7$ | 59. $x_1^3 x_2^7 x_3^3 x_4^6 x_5^3$ | 60. $x_1^3 x_2^7 x_3^3 x_4^6 x_5^3$ |
| 61. $x_1^3 x_2^7 x_3^3 x_4^7 x_5^2$ | 62. $x_1^3 x_2^7 x_3^3 x_4^6 x_5^6$ | 63. $x_1^3 x_2^7 x_3^3 x_4^4 x_5^4$ | 64. $x_1^3 x_2^7 x_3^3 x_4^4 x_5^3$ |
| 65. $x_1^3 x_2^7 x_3^5 x_4^2 x_5^2$ | 66. $x_1^3 x_2^7 x_3^5 x_4^2 x_5^3$ | 67. $x_1^3 x_2^7 x_3^5 x_4^2 x_5^2$ | 68. $x_1^3 x_2^7 x_3^7 x_4^2 x_5^2$ |
| 69. $x_1^7 x_2 x_3^2 x_4^7 x_5^7$   | 70. $x_1^7 x_2 x_3^2 x_4^7 x_5^3$   | 71. $x_1^7 x_2 x_3^3 x_4^2 x_5^7$   | 72. $x_1^7 x_2 x_3^3 x_4^2 x_5^6$   |
| 73. $x_1^7 x_2 x_3^3 x_4^6 x_5^3$   | 74. $x_1^7 x_2 x_3^3 x_4^2 x_5^2$   | 75. $x_1^7 x_2 x_3^6 x_4^3 x_5^3$   | 76. $x_1^7 x_2 x_3^7 x_4^2 x_5^3$   |
| 77. $x_1^7 x_2 x_3^7 x_4^2 x_5^2$   | 78. $x_1^7 x_2^3 x_3^2 x_4^7 x_5^7$ | 79. $x_1^7 x_2^3 x_3^3 x_4^6 x_5^3$ | 80. $x_1^7 x_2^3 x_3^3 x_4^6 x_5^3$ |
| 81. $x_1^7 x_2^3 x_3^7 x_4^2 x_5^2$ | 82. $x_1^7 x_2^3 x_3^3 x_4^6 x_5^6$ | 83. $x_1^7 x_2^3 x_3^3 x_4^4 x_5^4$ | 84. $x_1^7 x_2^3 x_3^3 x_4^4 x_5^3$ |
| 85. $x_1^7 x_2^3 x_3^5 x_4^2 x_5^2$ | 86. $x_1^7 x_2^3 x_3^5 x_4^2 x_5^3$ | 87. $x_1^7 x_2^3 x_3^5 x_4^2 x_5^2$ | 88. $x_1^7 x_2^3 x_3^7 x_4^2 x_5^2$ |
| 89. $x_1^7 x_2^7 x_3^2 x_4^2 x_5^3$ | 90. $x_1^7 x_2^7 x_3^3 x_4^2 x_5^2$ | 91. $x_1^7 x_2^7 x_3^3 x_4^2 x_5^2$ |                                     |

## 6.2 The case $d = 4$

$\tilde{B}_5^+((4)^2(2)|(1)^2) = \{b_t : 1 \leq t \leq 870\}$ , where

- |                                     |                                     |                                     |                                  |
|-------------------------------------|-------------------------------------|-------------------------------------|----------------------------------|
| 1. $x_1 x_2^2 x_3^3 x_4^7 x_5^{31}$ | 2. $x_1 x_2^2 x_3^3 x_4^3 x_5^7$    | 3. $x_1 x_2^2 x_3^7 x_4^3 x_5^{31}$ | 4. $x_1 x_2^2 x_3^7 x_4^3 x_5^3$ |
| 5. $x_1 x_2^2 x_3^{31} x_4^7 x_5^7$ | 6. $x_1 x_2^2 x_3^{31} x_4^3 x_5^3$ | 7. $x_1 x_2^2 x_3^2 x_4^3 x_5^{31}$ | 8. $x_1 x_2^2 x_3^2 x_4^3 x_5^7$ |

9.  $x_1 x_2^3 x_3^7 x_4^2 x_5^{31}$
10.  $x_1 x_2^3 x_3^7 x_4^3 x_5^{25}$
11.  $x_1 x_2^3 x_3^{31} x_4^2 x_5^7$
12.  $x_1 x_2^3 x_3^{31} x_4^7 x_5^2$
13.  $x_1 x_2^7 x_3^2 x_4^3 x_5^{31}$
14.  $x_1 x_2^7 x_3^3 x_4^3 x_5^{31}$
15.  $x_1 x_2^7 x_3^3 x_4^4 x_5^{31}$
16.  $x_1 x_2^7 x_3^3 x_4^7 x_5^3$
17.  $x_1 x_2^7 x_3^3 x_4^7 x_5^3$
18.  $x_1 x_2^7 x_3^{31} x_4^2 x_5^3$
19.  $x_1 x_2^{31} x_3^2 x_4^3 x_5^7$
20.  $x_1 x_2^{31} x_3^2 x_4^7 x_5^3$
21.  $x_1 x_2^{31} x_3^3 x_4^2 x_5^7$
22.  $x_1 x_2^{31} x_3^3 x_4^7 x_5^3$
23.  $x_1 x_2^{31} x_3^3 x_4^7 x_5^3$
24.  $x_1 x_2^{31} x_3^7 x_4^2 x_5^3$
25.  $x_1^3 x_2^2 x_3^7 x_4^2 x_5^3$
26.  $x_1^3 x_2^2 x_3^7 x_4^3 x_5^3$
27.  $x_1^3 x_2^2 x_3^7 x_4^4 x_5^3$
28.  $x_1^3 x_2^2 x_3^7 x_4^7 x_5^3$
29.  $x_1^3 x_2^2 x_3^{31} x_4^2 x_5^7$
30.  $x_1^3 x_2^2 x_3^{31} x_4^3 x_5^7$
31.  $x_1^3 x_2^2 x_3^7 x_4^2 x_5^{31}$
32.  $x_1^3 x_2^2 x_3^7 x_4^3 x_5^{31}$
33.  $x_1^3 x_2^2 x_3^{31} x_4^2 x_5^7$
34.  $x_1^3 x_2^2 x_3^{31} x_4^3 x_5^7$
35.  $x_1^3 x_2^{31} x_3^7 x_4^2 x_5^3$
36.  $x_1^3 x_2^{31} x_3^7 x_4^3 x_5^3$
37.  $x_1^7 x_2^2 x_3^3 x_4^3 x_5^{31}$
38.  $x_1^7 x_2^2 x_3^3 x_4^4 x_5^{31}$
39.  $x_1^7 x_2^2 x_3^3 x_4^7 x_5^{31}$
40.  $x_1^7 x_2^2 x_3^3 x_4^7 x_5^3$
41.  $x_1^7 x_2^2 x_3^{31} x_4^2 x_5^3$
42.  $x_1^7 x_2^2 x_3^{31} x_4^3 x_5^3$
43.  $x_1^7 x_2^2 x_3^3 x_4^2 x_5^{31}$
44.  $x_1^7 x_2^2 x_3^3 x_4^3 x_5^{31}$
45.  $x_1^7 x_2^2 x_3^{31} x_4^3 x_5^3$
46.  $x_1^7 x_2^{31} x_3^2 x_4^3 x_5^3$
47.  $x_1^7 x_2^{31} x_3^3 x_4^2 x_5^3$
48.  $x_1^7 x_2^{31} x_3^3 x_4^3 x_5^3$
49.  $x_1^{31} x_2^2 x_3^2 x_4^3 x_5^7$
50.  $x_1^{31} x_2^2 x_3^2 x_4^7 x_5^3$
51.  $x_1^{31} x_2^2 x_3^3 x_4^2 x_5^7$
52.  $x_1^{31} x_2^2 x_3^3 x_4^7 x_5^3$
53.  $x_1^{31} x_2^2 x_3^7 x_4^2 x_5^3$
54.  $x_1^{31} x_2^2 x_3^7 x_4^3 x_5^3$
55.  $x_1^{31} x_2^2 x_3^3 x_4^2 x_5^7$
56.  $x_1^{31} x_2^2 x_3^3 x_4^7 x_5^3$
57.  $x_1^{31} x_2^3 x_3^2 x_4^2 x_5^3$
58.  $x_1^{31} x_2^3 x_3^2 x_4^3 x_5^3$
59.  $x_1^{31} x_2^3 x_3^3 x_4^2 x_5^3$
60.  $x_1^{31} x_2^3 x_3^3 x_4^3 x_5^3$
61.  $x_1 x_2^2 x_3^3 x_4^{19} x_5^{23}$
62.  $x_1 x_2^2 x_3^{15} x_4^3 x_5^{23}$
63.  $x_1 x_2^2 x_3^{15} x_4^{23} x_5^3$
64.  $x_1 x_2^2 x_3^2 x_4^{15} x_5^{23}$
65.  $x_1 x_2^2 x_3^{15} x_4^2 x_5^{23}$
66.  $x_1 x_2^2 x_3^{15} x_4^3 x_5^3$
67.  $x_1 x_2^{15} x_3^3 x_4^2 x_5^3$
68.  $x_1 x_2^{15} x_3^2 x_4^{23} x_5^3$
69.  $x_1 x_2^{15} x_3^3 x_4^2 x_5^{23}$
70.  $x_1 x_2^{15} x_3^3 x_4^3 x_5^3$
71.  $x_1 x_2^{15} x_3^3 x_4^2 x_5^3$
72.  $x_1 x_2^{15} x_3^3 x_4^3 x_5^3$
73.  $x_1^3 x_2^2 x_3^2 x_4^{19} x_5^{23}$
74.  $x_1^3 x_2^2 x_3^{15} x_4^2 x_5^{23}$
75.  $x_1^3 x_2^2 x_3^{15} x_4^3 x_5^3$
76.  $x_1^3 x_2^{15} x_3^2 x_4^2 x_5^{23}$
77.  $x_1^3 x_2^{15} x_3^3 x_4^2 x_5^3$
78.  $x_1^3 x_2^{15} x_3^3 x_4^3 x_5^3$
79.  $x_1^{15} x_2^2 x_3^2 x_4^2 x_5^{23}$
80.  $x_1^{15} x_2^2 x_3^2 x_4^3 x_5^3$
81.  $x_1^{15} x_2^2 x_3^3 x_4^2 x_5^{23}$
82.  $x_1^{15} x_2^2 x_3^3 x_4^3 x_5^3$
83.  $x_1^{15} x_2^2 x_3^3 x_4^2 x_5^3$
84.  $x_1^{15} x_2^2 x_3^3 x_4^3 x_5^3$
85.  $x_1^{15} x_2^3 x_3^2 x_4^2 x_5^{23}$
86.  $x_1^{15} x_2^3 x_3^2 x_4^3 x_5^3$
87.  $x_1^{15} x_2^3 x_3^3 x_4^2 x_5^3$
88.  $x_1^{15} x_2^3 x_3^3 x_4^3 x_5^3$
89.  $x_1^{15} x_2^3 x_3^3 x_4^3 x_5^3$
90.  $x_1^{15} x_2^3 x_3^3 x_4^2 x_5^3$
91.  $x_1 x_2^2 x_3^7 x_4^7 x_5^{27}$
92.  $x_1 x_2^2 x_3^7 x_4^7 x_5^3$
93.  $x_1 x_2^7 x_3^2 x_4^7 x_5^{27}$
94.  $x_1 x_2^7 x_3^2 x_4^7 x_5^3$
95.  $x_1 x_2^7 x_3^3 x_4^7 x_5^{27}$
96.  $x_1 x_2^7 x_3^3 x_4^7 x_5^3$
97.  $x_1 x_2^7 x_3^7 x_4^2 x_5^7$
98.  $x_1 x_2^7 x_3^7 x_4^3 x_5^7$
99.  $x_1 x_2^7 x_3^7 x_4^4 x_5^{27}$
100.  $x_1 x_2^7 x_3^7 x_4^7 x_5^7$
101.  $x_1^7 x_2^2 x_3^2 x_4^2 x_5^{27}$
102.  $x_1^7 x_2^2 x_3^3 x_4^2 x_5^7$
103.  $x_1^7 x_2^2 x_3^7 x_4^2 x_5^7$
104.  $x_1^7 x_2^2 x_3^7 x_4^3 x_5^7$
105.  $x_1^7 x_2^2 x_3^7 x_4^4 x_5^{27}$
106.  $x_1^7 x_2^2 x_3^7 x_4^7 x_5^7$
107.  $x_1^7 x_2^2 x_3^7 x_4^7 x_5^7$
108.  $x_1^7 x_2^7 x_3^2 x_4^7 x_5^7$
109.  $x_1^7 x_2^7 x_3^3 x_4^7 x_5^7$
110.  $x_1^7 x_2^7 x_3^3 x_4^7 x_5^7$
111.  $x_1 x_2^2 x_3^7 x_4^1 x_5^{23}$
112.  $x_1 x_2^2 x_3^7 x_4^{11} x_5^{23}$
113.  $x_1 x_2^2 x_3^{11} x_4^2 x_5^{23}$
114.  $x_1 x_2^2 x_3^{11} x_4^3 x_5^{23}$
115.  $x_1 x_2^2 x_3^7 x_4^1 x_5^{23}$
116.  $x_1 x_2^2 x_3^{11} x_4^2 x_5^{23}$
117.  $x_1^7 x_2^2 x_3^{11} x_4^2 x_5^3$
118.  $x_1^7 x_2^{11} x_3^2 x_4^2 x_5^{23}$
119.  $x_1^7 x_2^{11} x_3^2 x_4^3 x_5^3$
120.  $x_1^7 x_2^{11} x_3^3 x_4^2 x_5^3$
121.  $x_1 x_2^2 x_3^7 x_4^{15} x_5^{19}$
122.  $x_1 x_2^2 x_3^{15} x_4^2 x_5^{19}$
123.  $x_1 x_2^2 x_3^{15} x_4^3 x_5^{19}$
124.  $x_1 x_2^2 x_3^7 x_4^{15} x_5^{19}$
125.  $x_1 x_2^2 x_3^{15} x_4^2 x_5^{19}$
126.  $x_1 x_2^2 x_3^{15} x_4^3 x_5^{19}$
127.  $x_1 x_2^{15} x_3^2 x_4^7 x_5^{19}$
128.  $x_1 x_2^{15} x_3^2 x_4^9 x_5^7$
129.  $x_1 x_2^{15} x_3^2 x_4^9 x_5^{19}$
130.  $x_1 x_2^{15} x_3^3 x_4^9 x_5^3$
131.  $x_1 x_2^{15} x_3^3 x_4^9 x_5^3$
132.  $x_1 x_2^{15} x_3^3 x_4^9 x_5^3$
133.  $x_1^7 x_2^2 x_3^2 x_4^{15} x_5^{19}$
134.  $x_1^7 x_2^2 x_3^{15} x_4^2 x_5^{19}$
135.  $x_1^7 x_2^2 x_3^{15} x_4^3 x_5^{19}$
136.  $x_1^7 x_2^{15} x_3^2 x_4^9 x_5^{19}$
137.  $x_1^7 x_2^{15} x_3^3 x_4^9 x_5^3$
138.  $x_1^7 x_2^{15} x_3^3 x_4^9 x_5^3$
139.  $x_1^{15} x_2^2 x_3^2 x_4^7 x_5^{19}$
140.  $x_1^{15} x_2^2 x_3^2 x_4^9 x_5^7$
141.  $x_1^{15} x_2^2 x_3^2 x_4^9 x_5^{19}$
142.  $x_1^{15} x_2^2 x_3^3 x_4^9 x_5^3$
143.  $x_1^{15} x_2^2 x_3^3 x_4^9 x_5^3$
144.  $x_1^{15} x_2^2 x_3^3 x_4^9 x_5^3$
145.  $x_1^{15} x_2^2 x_3^3 x_4^9 x_5^3$
146.  $x_1^{15} x_2^2 x_3^3 x_4^9 x_5^3$
147.  $x_1^{15} x_2^2 x_3^3 x_4^9 x_5^3$
148.  $x_1^{15} x_2^{19} x_3^2 x_4^7 x_5^3$
149.  $x_1^{15} x_2^{19} x_3^2 x_4^7 x_5^3$
150.  $x_1^{15} x_2^{19} x_3^3 x_4^7 x_5^3$
151.  $x_1 x_2^3 x_3^3 x_4^6 x_5^{31}$
152.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
153.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
154.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
155.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
156.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
157.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
158.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
159.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
160.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
161.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
162.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
163.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
164.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
165.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
166.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
167.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
168.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
169.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
170.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
171.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
172.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
173.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
174.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
175.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
176.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
177.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
178.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
179.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$
180.  $x_1 x_2^3 x_3^3 x_4^6 x_5^3$

- 181.  $x_1 x_2^3 x_3^3 x_4^7 x_5^{30}$
- 185.  $x_1 x_2^3 x_3^{30} x_4^3 x_5^7$
- 189.  $x_1 x_2^3 x_3^{30} x_4^3 x_5^3$
- 193.  $x_1^3 x_2 x_3^3 x_4^7 x_5^{30}$
- 197.  $x_1^3 x_2 x_3^{30} x_4^3 x_5^7$
- 201.  $x_1^3 x_2^3 x_3^7 x_4^3 x_5^{30}$
- 205.  $x_1^7 x_2 x_3^3 x_4^3 x_5^{30}$
- 209.  $x_1^7 x_2^3 x_3^4 x_4^3 x_5^3$
- 213.  $x_1 x_2^3 x_3^{14} x_4^{23} x_5^3$
- 217.  $x_1^3 x_2^3 x_3^{14} x_4^3 x_5^{23}$
- 221.  $x_1 x_2^3 x_3^4 x_4^{15} x_5^{22}$
- 225.  $x_1 x_2^{15} x_3^3 x_4^3 x_5^3$
- 229.  $x_1^3 x_2 x_3^{15} x_4^{22} x_5^3$
- 233.  $x_1^3 x_2^{15} x_3^3 x_4^3 x_5^3$
- 237.  $x_1^{15} x_2 x_3^2 x_4^3 x_5^3$
- 241.  $x_1 x_2^3 x_3^6 x_4^7 x_5^{27}$
- 245.  $x_1 x_2^6 x_3^3 x_4^7 x_5^{27}$
- 249.  $x_1 x_2^6 x_3^2 x_4^7 x_5^{27}$
- 253.  $x_1 x_2^6 x_3^3 x_4^7 x_5^{27}$
- 257.  $x_1^3 x_2 x_3^6 x_4^7 x_5^{27}$
- 261.  $x_1^3 x_2^7 x_3 x_4^6 x_5^{27}$
- 265.  $x_1^7 x_2 x_3^3 x_4^{27} x_5^6$
- 269.  $x_1^7 x_2 x_3^2 x_4^6 x_5^3$
- 273.  $x_1^7 x_2^7 x_3 x_4^6 x_5^3$
- 277.  $x_1 x_2^6 x_3^3 x_4^{11} x_5^{23}$
- 281.  $x_1 x_2^3 x_3^6 x_4^{15} x_5^{19}$
- 285.  $x_1 x_2^6 x_3^{15} x_4^3 x_5^{19}$
- 289.  $x_1 x_2^{15} x_3^6 x_4^3 x_5^{19}$
- 293.  $x_1^3 x_2 x_3^6 x_4^{15} x_5^{19}$
- 297.  $x_1^3 x_2^{15} x_3 x_4^{19} x_5^6$
- 301.  $x_1^{15} x_2 x_3^6 x_4^3 x_5^{19}$
- 305.  $x_1^{15} x_2^3 x_3 x_4^6 x_5^{19}$
- 309.  $x_1^{15} x_2^{19} x_3 x_4^6 x_5^3$
- 313.  $x_1 x_2^7 x_3^3 x_4^7 x_5^{26}$
- 317.  $x_1 x_2^7 x_3^6 x_4^3 x_5^7$
- 321.  $x_1^3 x_2^7 x_3 x_4^7 x_5^{26}$
- 325.  $x_1^7 x_2 x_3^3 x_4^{26} x_5^7$
- 329.  $x_1^7 x_2 x_3^6 x_4^3 x_5^3$
- 333.  $x_1^7 x_2^7 x_3 x_4^3 x_5^{26}$
- 337.  $x_1 x_2^7 x_3^3 x_4^{10} x_5^{23}$
- 341.  $x_1^3 x_2^7 x_3 x_4^{10} x_5^{23}$
- 345.  $x_1^7 x_2^3 x_3 x_4^{10} x_5^{23}$
- 349.  $x_1 x_2^7 x_3^{11} x_4^{22} x_5^3$
- 182.  $x_1 x_2^3 x_3^3 x_4^{30} x_5^7$
- 186.  $x_1 x_2^3 x_3^{30} x_4^3 x_5^3$
- 190.  $x_1 x_2^{30} x_3^3 x_4^3 x_5^7$
- 194.  $x_1^3 x_2 x_3^3 x_4^{30} x_5^7$
- 198.  $x_1^3 x_2 x_3^{30} x_4^3 x_5^3$
- 202.  $x_1^3 x_2^7 x_3 x_4^3 x_5^{30}$
- 206.  $x_1^7 x_2 x_3^3 x_4^{30} x_5^3$
- 210.  $x_1^7 x_2^3 x_3^3 x_4^3 x_5^{30}$
- 214.  $x_1 x_2^{14} x_3^3 x_4^3 x_5^{23}$
- 218.  $x_1^3 x_2 x_3^{14} x_4^3 x_5^{23}$
- 222.  $x_1 x_2^3 x_3^{15} x_4^3 x_5^{22}$
- 226.  $x_1 x_2^{15} x_3^2 x_4^3 x_5^3$
- 230.  $x_1^3 x_2^3 x_3 x_4^{15} x_5^{22}$
- 234.  $x_1^3 x_2^{15} x_3^3 x_4^3 x_5^{22}$
- 238.  $x_1^{15} x_2^3 x_3^3 x_4^3 x_5^{22}$
- 242.  $x_1 x_2^3 x_3^6 x_4^2 x_5^{27}$
- 246.  $x_1 x_2^6 x_3^3 x_4^2 x_5^{27}$
- 250.  $x_1 x_2^6 x_3^2 x_4^2 x_5^{27}$
- 254.  $x_1 x_2^6 x_3^3 x_4^2 x_5^{27}$
- 258.  $x_1^3 x_2 x_3^6 x_4^2 x_5^{27}$
- 262.  $x_1^3 x_2^7 x_3 x_4^2 x_5^6$
- 266.  $x_1^7 x_2 x_3^6 x_4^3 x_5^{27}$
- 270.  $x_1^7 x_2^3 x_3^6 x_4^2 x_5^{27}$
- 274.  $x_1^7 x_2^7 x_3 x_4^6 x_5^3$
- 278.  $x_1 x_2^6 x_3^{11} x_4^3 x_5^{23}$
- 282.  $x_1 x_2^3 x_3^{15} x_4^6 x_5^{19}$
- 286.  $x_1 x_2^6 x_3^{15} x_4^3 x_5^{19}$
- 290.  $x_1 x_2^{15} x_3^6 x_4^3 x_5^{19}$
- 294.  $x_1^3 x_2 x_3^{15} x_4^6 x_5^{19}$
- 298.  $x_1^3 x_2^{15} x_3^{19} x_4 x_5^6$
- 302.  $x_1^{15} x_2 x_3^6 x_4^{19} x_5^3$
- 306.  $x_1^{15} x_2^3 x_3 x_4^{19} x_5^6$
- 310.  $x_1^{15} x_2^{19} x_3^3 x_4^6 x_5^3$
- 314.  $x_1 x_2^7 x_3^3 x_4^{26} x_5^7$
- 318.  $x_1 x_2^7 x_3^6 x_4^3 x_5^7$
- 322.  $x_1^3 x_2^7 x_3 x_4^6 x_5^7$
- 326.  $x_1^7 x_2 x_3^3 x_4^6 x_5^3$
- 330.  $x_1^7 x_2^3 x_3 x_4^6 x_5^3$
- 334.  $x_1^7 x_2^7 x_3 x_4^6 x_5^3$
- 338.  $x_1 x_2^7 x_3^{10} x_4^3 x_5^{23}$
- 342.  $x_1^3 x_2^7 x_3 x_4^{10} x_5^{23}$
- 346.  $x_1 x_2^3 x_3^7 x_4^{11} x_5^{22}$
- 350.  $x_1^3 x_2 x_3^7 x_4^{11} x_5^{22}$
- 183.  $x_1 x_2^3 x_3^7 x_4^3 x_5^{30}$
- 187.  $x_1 x_2^7 x_3^3 x_4^3 x_5^{30}$
- 191.  $x_1 x_2^{30} x_3^3 x_4^3 x_5^3$
- 195.  $x_1^3 x_2 x_3^7 x_4^3 x_5^{30}$
- 199.  $x_1^3 x_2^3 x_3^7 x_4^3 x_5^{30}$
- 203.  $x_1^3 x_2^7 x_3 x_4^{30} x_5^3$
- 207.  $x_1^7 x_2 x_3^{30} x_4^3 x_5^3$
- 211.  $x_1 x_2^3 x_3^{14} x_4^3 x_5^{23}$
- 215.  $x_1 x_2^{14} x_3^3 x_4^{23} x_5^3$
- 219.  $x_1^3 x_2 x_3^{14} x_4^{23} x_5^3$
- 223.  $x_1 x_2^3 x_3^{15} x_4^2 x_5^{22}$
- 227.  $x_1^3 x_2 x_3^3 x_4^{15} x_5^{22}$
- 231.  $x_1^3 x_2^3 x_3^{15} x_4^2 x_5^{22}$
- 235.  $x_1^{15} x_2 x_3^3 x_4^3 x_5^{22}$
- 239.  $x_1^{15} x_2^3 x_3 x_4^2 x_5^3$
- 243.  $x_1 x_2^3 x_3^6 x_4^2 x_5^{27}$
- 247.  $x_1 x_2^6 x_3^3 x_4^2 x_5^{27}$
- 251.  $x_1 x_2^6 x_3^2 x_4^2 x_5^{27}$
- 255.  $x_1 x_2^6 x_3^3 x_4^2 x_5^{27}$
- 259.  $x_1^3 x_2 x_3^6 x_4^2 x_5^{27}$
- 263.  $x_1^3 x_2^7 x_3 x_4^2 x_5^6$
- 267.  $x_1^7 x_2 x_3^6 x_4^2 x_5^3$
- 271.  $x_1^7 x_2^3 x_3 x_4^2 x_5^6$
- 275.  $x_1^7 x_2^7 x_3^3 x_4^2 x_5^6$
- 279.  $x_1 x_2^6 x_3^{11} x_4^2 x_5^3$
- 283.  $x_1 x_2^3 x_3^{15} x_4^{19} x_5^6$
- 287.  $x_1 x_2^{15} x_3^6 x_4^{19} x_5^6$
- 291.  $x_1 x_2^{15} x_3^9 x_4^3 x_5^6$
- 295.  $x_1^3 x_2 x_3^{15} x_4^{19} x_5^6$
- 299.  $x_1^{15} x_2 x_3^6 x_4^{19} x_5^6$
- 303.  $x_1^{15} x_2 x_3^9 x_4^3 x_5^6$
- 307.  $x_1^{15} x_2^3 x_3^9 x_4^3 x_5^6$
- 311.  $x_1 x_2^3 x_3^7 x_4^2 x_5^6$
- 315.  $x_1 x_2^7 x_3^3 x_4^2 x_5^6$
- 319.  $x_1^3 x_2 x_3^7 x_4^2 x_5^6$
- 323.  $x_1^3 x_2^7 x_3 x_4^2 x_5^6$
- 327.  $x_1^7 x_2 x_3^3 x_4^2 x_5^3$
- 331.  $x_1^7 x_2^3 x_3 x_4^2 x_5^6$
- 335.  $x_1^7 x_2^7 x_3^3 x_4^2 x_5^6$
- 339.  $x_1 x_2^7 x_3^{10} x_4^2 x_5^3$
- 343.  $x_1^3 x_2 x_3^{10} x_4^2 x_5^3$
- 347.  $x_1 x_2^7 x_3^3 x_4^{11} x_5^{22}$
- 351.  $x_1^3 x_2^7 x_3 x_4^{11} x_5^{22}$
- 184.  $x_1 x_2^3 x_3^7 x_4^{30} x_5^3$
- 188.  $x_1 x_2^7 x_3^3 x_4^{30} x_5^3$
- 192.  $x_1 x_2^{30} x_3^3 x_4^3 x_5^3$
- 196.  $x_1^3 x_2 x_3^7 x_4^{30} x_5^3$
- 200.  $x_1^3 x_2^3 x_3^7 x_4^{30} x_5^3$
- 204.  $x_1^3 x_2^7 x_3^3 x_4^{30} x_5^3$
- 208.  $x_1^7 x_2 x_3^3 x_4^{30} x_5^3$
- 212.  $x_1 x_2^3 x_3^{14} x_4^3 x_5^{23}$
- 216.  $x_1 x_2^{14} x_3^3 x_4^3 x_5^3$
- 220.  $x_1^3 x_2 x_3^{14} x_4^3 x_5^3$
- 224.  $x_1 x_2^{15} x_3^3 x_4^3 x_5^{22}$
- 228.  $x_1^3 x_2 x_3^{15} x_4^3 x_5^{22}$
- 232.  $x_1^3 x_2^{15} x_3^3 x_4^3 x_5^{22}$
- 236.  $x_1^{15} x_2 x_3^3 x_4^2 x_5^3$
- 240.  $x_1^{15} x_2^3 x_3 x_4^2 x_5^3$
- 244.  $x_1 x_2^3 x_3^6 x_4^2 x_5^{27}$
- 248.  $x_1 x_2^6 x_3^3 x_4^2 x_5^{27}$
- 252.  $x_1 x_2^6 x_3^2 x_4^2 x_5^{27}$
- 256.  $x_1 x_2^6 x_3^3 x_4^2 x_5^{27}$
- 260.  $x_1^3 x_2 x_3^6 x_4^2 x_5^{27}$
- 264.  $x_1^3 x_2^7 x_3 x_4^2 x_5^6$
- 268.  $x_1^7 x_2 x_3^6 x_4^2 x_5^3$
- 272.  $x_1^7 x_2^3 x_3 x_4^2 x_5^6$
- 276.  $x_1 x_2^3 x_3^6 x_4^{11} x_5^{23}$
- 280.  $x_1^3 x_2 x_3^6 x_4^{11} x_5^{23}$
- 284.  $x_1 x_2^6 x_3^3 x_4^{15} x_5^{19}$
- 288.  $x_1 x_2^{15} x_3^3 x_4^{19} x_5^6$
- 292.  $x_1 x_2^{15} x_3^{19} x_4^6 x_5^3$
- 296.  $x_1^3 x_2^{15} x_3 x_4^6 x_5^{19}$
- 300.  $x_1^{15} x_2 x_3^3 x_4^{19} x_5^6$
- 304.  $x_1^{15} x_2 x_3^{19} x_4^6 x_5^3$
- 308.  $x_1^{15} x_2^{19} x_3 x_4^6 x_5^3$
- 312.  $x_1 x_2^3 x_3^7 x_4^{26} x_5^7$
- 316.  $x_1 x_2^7 x_3^3 x_4^{26} x_5^3$
- 320.  $x_1^3 x_2 x_3^7 x_4^{26} x_5^7$
- 324.  $x_1^7 x_2 x_3^3 x_4^{26} x_5^3$
- 328.  $x_1^7 x_2 x_3^6 x_4^3 x_5^7$
- 332.  $x_1^7 x_2^3 x_3^3 x_4^2 x_5^6$
- 336.  $x_1 x_2^3 x_3^{10} x_4^2 x_5^{23}$
- 340.  $x_1^3 x_2 x_3^{10} x_4^2 x_5^{23}$
- 344.  $x_1^7 x_2 x_3^{10} x_4^2 x_5^3$
- 348.  $x_1 x_2^7 x_3^{11} x_4^2 x_5^{22}$
- 352.  $x_1^3 x_2^7 x_3^{11} x_4^2 x_5^{22}$

353.  $x_1^7 x_2 x_3 x_4^{11} x_5^{22}$  354.  $x_1^7 x_2 x_3^{11} x_4^3 x_5^{22}$  355.  $x_1^7 x_2 x_3^{11} x_4^{22} x_5^3$  356.  $x_1^7 x_2^3 x_3 x_4^{11} x_5^{22}$   
 357.  $x_1^7 x_2^3 x_3^{11} x_4 x_5^{22}$  358.  $x_1^7 x_2^{11} x_3 x_4^3 x_5^{22}$  359.  $x_1^7 x_2^{11} x_3 x_4^{22} x_5^3$  360.  $x_1^7 x_2^{11} x_3^3 x_4 x_5^{22}$   
 361.  $x_1 x_2^3 x_3^7 x_4^{14} x_5^{19}$  362.  $x_1 x_2^3 x_3^{14} x_4^7 x_5^{19}$  363.  $x_1 x_2^3 x_3^{14} x_4^{19} x_5^{19}$  364.  $x_1 x_2^7 x_3^3 x_4^{14} x_5^{19}$   
 365.  $x_1 x_2^7 x_3^{14} x_4^3 x_5^{19}$  366.  $x_1 x_2^7 x_3^{14} x_4^9 x_5^3$  367.  $x_1 x_2^{14} x_3^3 x_4^7 x_5^{19}$  368.  $x_1 x_2^{14} x_3^3 x_4^{19} x_5^7$   
 369.  $x_1 x_2^{14} x_3^7 x_4^3 x_5^{19}$  370.  $x_1 x_2^{14} x_3^7 x_4^9 x_5^3$  371.  $x_1 x_2^{14} x_3^3 x_4^9 x_5^7$  372.  $x_1 x_2^{14} x_3^{19} x_4^3 x_5^3$   
 373.  $x_1^3 x_2 x_3^7 x_4^{14} x_5^{19}$  374.  $x_1^3 x_2 x_3^{14} x_4^7 x_5^{19}$  375.  $x_1^3 x_2 x_3^{14} x_4^{19} x_5^7$  376.  $x_1^3 x_2^7 x_3 x_4^{14} x_5^{19}$   
 377.  $x_1^7 x_2 x_3^3 x_4^{14} x_5^{19}$  378.  $x_1^7 x_2 x_3^{14} x_4^3 x_5^{19}$  379.  $x_1^7 x_2 x_3^{14} x_4^{19} x_5^3$  380.  $x_1^7 x_2^3 x_3 x_4^{14} x_5^{19}$   
 381.  $x_1 x_2^3 x_3^7 x_4^{15} x_5^{18}$  382.  $x_1 x_2^3 x_3^{15} x_4^7 x_5^{18}$  383.  $x_1 x_2^3 x_3^{15} x_4^{18} x_5^7$  384.  $x_1 x_2^7 x_3^3 x_4^{15} x_5^{18}$   
 385.  $x_1 x_2^3 x_3^{15} x_4^3 x_5^{18}$  386.  $x_1 x_2^3 x_3^{15} x_4^{18} x_5^3$  387.  $x_1 x_2^{15} x_3^3 x_4^7 x_5^{18}$  388.  $x_1 x_2^{15} x_3^3 x_4^{18} x_5^7$   
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 393.  $x_1^3 x_2 x_3^7 x_4^{15} x_5^{18}$  394.  $x_1^3 x_2 x_3^{15} x_4^7 x_5^{18}$  395.  $x_1^3 x_2 x_3^{15} x_4^{18} x_5^7$  396.  $x_1^3 x_2^7 x_3 x_4^{15} x_5^{18}$   
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 401.  $x_1^7 x_2 x_3^3 x_4^{15} x_5^{18}$  402.  $x_1^7 x_2 x_3^{15} x_4^3 x_5^{18}$  403.  $x_1^7 x_2 x_3^{15} x_4^{18} x_5^3$  404.  $x_1^7 x_2^3 x_3^3 x_4^{15} x_5^{18}$   
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 413.  $x_1^{15} x_2 x_3^{18} x_4^3 x_5^7$  414.  $x_1^{15} x_2 x_3^{18} x_4^3 x_5^3$  415.  $x_1^{15} x_2^3 x_3 x_4^7 x_5^{18}$  416.  $x_1^{15} x_2^3 x_3 x_4^{18} x_5^7$   
 417.  $x_1^{15} x_2^3 x_3^7 x_4 x_5^{18}$  418.  $x_1^{15} x_2^3 x_3^3 x_4^7 x_5^{18}$  419.  $x_1^{15} x_2^3 x_3^3 x_4^{18} x_5^3$  420.  $x_1^{15} x_2^3 x_3^3 x_4^{18} x_5^7$   
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 465.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^3$  466.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^3$  467.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^3$  468.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^3$   
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 477.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^3$  478.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^3$  479.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^3$  480.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^3$   
 481.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^3$  482.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^3$  483.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^3$  484.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^3$   
 485.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^3$  486.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^3$  487.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^3$  488.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^3$   
 489.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^3$  490.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^3$  491.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^3$  492.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^3$   
 493.  $x_1^{15} x_2^3 x_3^3 x_4^3 x_5^2$  494.  $x_1^{15} x_2^3 x_3^3 x_4^3 x_5^2$  495.  $x_1^{15} x_2^3 x_3^3 x_4^3 x_5^2$  496.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^2$   
 497.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^2$  498.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^2$  499.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^2$  500.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^2$   
 501.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^2$  502.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^2$  503.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^2$  504.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^2$   
 505.  $x_1^7 x_2^3 x_3^3 x_4^3 x_5^2$  506.  $x_1^7 x_2^3 x_3^3 x_4^3 x_5^2$  507.  $x_1^7 x_2^3 x_3^3 x_4^3 x_5^2$  508.  $x_1^7 x_2^3 x_3^3 x_4^3 x_5^2$   
 509.  $x_1^7 x_2^3 x_3^3 x_4^3 x_5^2$  510.  $x_1^7 x_2^3 x_3^3 x_4^3 x_5^2$  511.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^2$  512.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^2$   
 513.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^2$  514.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^2$  515.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^2$  516.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^2$   
 517.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^2$  518.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^2$  519.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^2$  520.  $x_1^3 x_2^3 x_3^3 x_4^3 x_5^2$   
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525.  $x_1^3 x_2^3 x_3^{25} x_4^2 x_5^7$  526.  $x_1^3 x_2^7 x_3^{25} x_4^2 x_5^2$  527.  $x_1^7 x_2^3 x_3^7 x_4^{25} x_5^2$  528.  $x_1^7 x_2^3 x_3^{25} x_4^2 x_5^7$   
 529.  $x_1^7 x_2^3 x_3^{25} x_4^2 x_5^2$  530.  $x_1^7 x_2^7 x_3^3 x_4^{25} x_5^2$  531.  $x_1^7 x_2^7 x_3^{25} x_4^2 x_5^3$  532.  $x_1^7 x_2^7 x_3^{25} x_4^2 x_5^2$   
 533.  $x_1^3 x_2^7 x_3^9 x_4^2 x_5^{23}$  534.  $x_1^3 x_2^7 x_3^9 x_4^{23} x_5^2$  535.  $x_1^7 x_2^3 x_3^9 x_4^2 x_5^{23}$  536.  $x_1^7 x_2^3 x_3^9 x_4^2 x_5^2$   
 537.  $x_1^7 x_2^9 x_3^3 x_4^2 x_5^{23}$  538.  $x_1^7 x_2^9 x_3^3 x_4^2 x_5^2$  539.  $x_1^7 x_2^9 x_3^3 x_4^2 x_5^{23}$  540.  $x_1^7 x_2^9 x_3^3 x_4^2 x_5^2$   
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 545.  $x_1^7 x_2^{11} x_3^3 x_4^{21} x_5^2$  546.  $x_1^7 x_2^{11} x_3^{11} x_4^2 x_5^3$  547.  $x_1^7 x_2^{11} x_3^{11} x_4^2 x_5^2$  548.  $x_1^3 x_2^7 x_3^{13} x_4^2 x_5^{19}$   
 549.  $x_1^3 x_2^3 x_3^{13} x_4^{19} x_5^2$  550.  $x_1^3 x_2^{13} x_3^2 x_4^7 x_5^{19}$  551.  $x_1^3 x_2^{13} x_3^2 x_4^{19} x_5^2$  552.  $x_1^3 x_2^{13} x_3^2 x_4^2 x_5^{19}$   
 553.  $x_1^3 x_2^{13} x_3^7 x_4^{19} x_5^2$  554.  $x_1^3 x_2^{13} x_3^{19} x_4^2 x_5^7$  555.  $x_1^3 x_2^{13} x_3^{19} x_4^2 x_5^2$  556.  $x_1^3 x_2^{13} x_3^{19} x_4^2 x_5^{19}$   
 557.  $x_1^3 x_2^{13} x_3^{19} x_4^{19} x_5^2$  558.  $x_1^3 x_2^7 x_3^{15} x_4^{17} x_5^2$  559.  $x_1^3 x_2^{15} x_3^7 x_4^{17} x_5^2$  560.  $x_1^3 x_2^{15} x_3^{17} x_4^2 x_5^7$   
 561.  $x_1^3 x_2^{15} x_3^{17} x_4^2 x_5^2$  562.  $x_1^7 x_2^3 x_3^{15} x_4^{17} x_5^2$  563.  $x_1^7 x_2^{15} x_3^3 x_4^{17} x_5^2$  564.  $x_1^3 x_2^{15} x_3^{17} x_4^2 x_5^3$   
 565.  $x_1^7 x_2^{15} x_3^{17} x_4^2 x_5^2$  566.  $x_1^{15} x_2^3 x_3^7 x_4^{17} x_5^2$  567.  $x_1^{15} x_2^3 x_3^{17} x_4^2 x_5^2$  568.  $x_1^{15} x_2^3 x_3^{17} x_4^2 x_5^2$   
 569.  $x_1^{15} x_2^7 x_3^3 x_4^{17} x_5^2$  570.  $x_1^{15} x_2^7 x_3^{17} x_4^2 x_5^3$  571.  $x_1^{15} x_2^7 x_3^{17} x_4^2 x_5^2$  572.  $x_1^7 x_2^{11} x_3^3 x_4^2 x_5^{19}$   
 573.  $x_1^7 x_2^{11} x_3^5 x_4^{19} x_5^2$  574.  $x_1^7 x_2^{11} x_3^{19} x_4^2 x_5^2$  575.  $x_1^7 x_2^7 x_3^9 x_4^2 x_5^{19}$  576.  $x_1^7 x_2^9 x_3^9 x_4^{19} x_5^2$   
 577.  $x_1^7 x_2^9 x_3^7 x_4^2 x_5^{19}$  578.  $x_1^7 x_2^9 x_3^7 x_4^2 x_5^{19}$  579.  $x_1^7 x_2^9 x_3^7 x_4^{19} x_5^2$  580.  $x_1^7 x_2^7 x_3^{11} x_4^{17} x_5^2$   
 581.  $x_1^7 x_2^{11} x_3^7 x_4^{17} x_5^2$  582.  $x_1^7 x_2^{11} x_3^{17} x_4^2 x_5^7$  583.  $x_1^7 x_2^{11} x_3^{17} x_4^2 x_5^2$  584.  $x_1^3 x_2^3 x_3^3 x_4^4 x_5^{31}$   
 585.  $x_1^3 x_2^3 x_3^3 x_4^4 x_5^4$  586.  $x_1^3 x_2^3 x_3^4 x_4^3 x_5^{31}$  587.  $x_1^3 x_2^3 x_3^4 x_4^3 x_5^4$  588.  $x_1^3 x_2^3 x_3^3 x_4^4 x_5^4$   
 589.  $x_1^3 x_2^3 x_3^{31} x_4^4 x_5^3$  590.  $x_1^3 x_2^{31} x_3^3 x_4^4 x_5^3$  591.  $x_1^3 x_2^{31} x_3^3 x_4^4 x_5^3$  592.  $x_1^3 x_2^3 x_3^3 x_4^4 x_5^4$   
 593.  $x_1^{31} x_2^3 x_3^3 x_4^4 x_5^3$  594.  $x_1^3 x_2^3 x_3^3 x_4^5 x_5^{30}$  595.  $x_1^3 x_2^3 x_3^5 x_4^3 x_5^{30}$  596.  $x_1^3 x_2^3 x_3^5 x_4^3 x_5^3$   
 597.  $x_1^3 x_2^5 x_3^3 x_4^4 x_5^{30}$  598.  $x_1^3 x_2^5 x_3^3 x_4^{30} x_5^3$  599.  $x_1^3 x_2^5 x_3^{30} x_4^3 x_5^3$  600.  $x_1^3 x_2^3 x_3^3 x_4^2 x_5^6$   
 601.  $x_1^3 x_2^3 x_3^{29} x_4^2 x_5^6$  602.  $x_1^3 x_2^3 x_3^{29} x_4^6 x_5^3$  603.  $x_1^3 x_2^9 x_3^3 x_4^3 x_5^6$  604.  $x_1^3 x_2^9 x_3^3 x_4^6 x_5^3$   
 605.  $x_1^3 x_2^9 x_3^6 x_4^3 x_5^3$  606.  $x_1^3 x_2^3 x_3^7 x_4^2 x_5^8$  607.  $x_1^3 x_2^3 x_3^7 x_4^2 x_5^8$  608.  $x_1^3 x_2^3 x_3^7 x_4^2 x_5^8$   
 609.  $x_1^3 x_2^3 x_3^7 x_4^2 x_5^3$  610.  $x_1^3 x_2^3 x_3^8 x_4^3 x_5^7$  611.  $x_1^3 x_2^3 x_3^8 x_4^3 x_5^7$  612.  $x_1^3 x_2^3 x_3^8 x_4^3 x_5^8$   
 613.  $x_1^3 x_2^3 x_3^8 x_4^3 x_5^3$  614.  $x_1^7 x_2^3 x_3^3 x_4^2 x_5^8$  615.  $x_1^7 x_2^3 x_3^3 x_4^2 x_5^8$  616.  $x_1^3 x_2^3 x_3^3 x_4^{12} x_5^{23}$   
 617.  $x_1^3 x_2^3 x_3^{12} x_4^2 x_5^{23}$  618.  $x_1^3 x_2^3 x_3^{12} x_4^2 x_5^3$  619.  $x_1^3 x_2^3 x_3^{13} x_4^3 x_5^{22}$  620.  $x_1^3 x_2^3 x_3^{13} x_4^3 x_5^{22}$   
 621.  $x_1^3 x_2^3 x_3^{13} x_4^2 x_5^3$  622.  $x_1^3 x_2^{13} x_3^3 x_4^2 x_5^{22}$  623.  $x_1^3 x_2^{13} x_3^3 x_4^2 x_5^3$  624.  $x_1^3 x_2^{13} x_3^{22} x_4^3 x_5^3$   
 625.  $x_1^3 x_2^3 x_3^{15} x_4^2 x_5^{20}$  626.  $x_1^3 x_2^3 x_3^{15} x_4^2 x_5^{20}$  627.  $x_1^3 x_2^3 x_3^{15} x_4^{20} x_5^3$  628.  $x_1^3 x_2^{15} x_3^3 x_4^3 x_5^{20}$   
 629.  $x_1^3 x_2^{15} x_3^3 x_4^{20} x_5^3$  630.  $x_1^{15} x_2^3 x_3^3 x_4^2 x_5^{20}$  631.  $x_1^{15} x_2^3 x_3^3 x_4^2 x_5^3$  632.  $x_1^3 x_2^3 x_3^4 x_4^2 x_5^{27}$   
 633.  $x_1^3 x_2^3 x_3^4 x_4^2 x_5^7$  634.  $x_1^3 x_2^3 x_3^7 x_4^4 x_5^{27}$  635.  $x_1^3 x_2^3 x_3^7 x_4^4 x_5^{27}$  636.  $x_1^3 x_2^3 x_3^7 x_4^4 x_5^{27}$   
 637.  $x_1^3 x_2^5 x_3^3 x_4^2 x_5^4$  638.  $x_1^3 x_2^5 x_3^7 x_4^4 x_5^4$  639.  $x_1^3 x_2^5 x_3^7 x_4^4 x_5^3$  640.  $x_1^7 x_2^3 x_3^3 x_4^4 x_5^{27}$   
 641.  $x_1^7 x_2^3 x_3^3 x_4^2 x_5^4$  642.  $x_1^7 x_2^3 x_3^7 x_4^4 x_5^4$  643.  $x_1^7 x_2^3 x_3^7 x_4^4 x_5^3$  644.  $x_1^7 x_2^{27} x_3^3 x_4^4 x_5^4$   
 645.  $x_1^7 x_2^{27} x_3^3 x_4^4 x_5^3$  646.  $x_1^3 x_2^3 x_3^4 x_4^{11} x_5^{23}$  647.  $x_1^3 x_2^3 x_3^4 x_4^{15} x_5^{19}$  648.  $x_1^3 x_2^3 x_3^{15} x_4^4 x_5^{19}$   
 649.  $x_1^3 x_2^3 x_3^{15} x_4^{19} x_5^4$  650.  $x_1^3 x_2^{15} x_3^3 x_4^4 x_5^{19}$  651.  $x_1^3 x_2^{15} x_3^3 x_4^{19} x_5^4$  652.  $x_1^3 x_2^{15} x_3^3 x_4^4 x_5^4$   
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 657.  $x_1^3 x_2^3 x_3^5 x_4^6 x_5^{27}$  658.  $x_1^3 x_2^3 x_3^5 x_4^2 x_5^6$  659.  $x_1^3 x_2^5 x_3^3 x_4^6 x_5^{27}$  660.  $x_1^3 x_2^5 x_3^3 x_4^2 x_5^6$   
 661.  $x_1^3 x_2^3 x_3^6 x_4^2 x_5^{27}$  662.  $x_1^3 x_2^3 x_3^6 x_4^2 x_5^3$  663.  $x_1^3 x_2^5 x_3^7 x_4^3 x_5^6$  664.  $x_1^3 x_2^5 x_3^7 x_4^3 x_5^3$   
 665.  $x_1^3 x_2^3 x_3^6 x_4^2 x_5^6$  666.  $x_1^3 x_2^3 x_3^6 x_4^2 x_5^7$  667.  $x_1^3 x_2^3 x_3^6 x_4^2 x_5^6$  668.  $x_1^3 x_2^5 x_3^7 x_4^3 x_5^6$   
 669.  $x_1^3 x_2^5 x_3^6 x_4^2 x_5^7$  670.  $x_1^3 x_2^5 x_3^7 x_4^3 x_5^6$  671.  $x_1^3 x_2^5 x_3^7 x_4^3 x_5^3$  672.  $x_1^3 x_2^5 x_3^6 x_4^3 x_5^7$   
 673.  $x_1^3 x_2^5 x_3^6 x_4^3 x_5^3$  674.  $x_1^3 x_2^7 x_3^3 x_4^2 x_5^6$  675.  $x_1^3 x_2^7 x_3^3 x_4^2 x_5^6$  676.  $x_1^3 x_2^7 x_3^3 x_4^2 x_5^3$   
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 681.  $x_1^3 x_2^5 x_3^3 x_4^{10} x_5^{23}$  682.  $x_1^3 x_2^5 x_3^{10} x_4^3 x_5^{23}$  683.  $x_1^3 x_2^5 x_3^{10} x_4^2 x_5^3$  684.  $x_1^3 x_2^3 x_3^3 x_4^{11} x_5^{22}$   
 685.  $x_1^3 x_2^5 x_3^3 x_4^{11} x_5^{22}$  686.  $x_1^3 x_2^5 x_3^{11} x_4^3 x_5^{22}$  687.  $x_1^3 x_2^5 x_3^{11} x_4^2 x_5^3$  688.  $x_1^3 x_2^3 x_3^3 x_4^{14} x_5^{19}$   
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 693.  $x_1^3 x_2^3 x_3^{15} x_4^3 x_5^{18}$  694.  $x_1^3 x_2^5 x_3^4 x_4^{15} x_5^{18}$  695.  $x_1^3 x_2^5 x_3^{15} x_4^3 x_5^{18}$  696.  $x_1^3 x_2^3 x_3^{15} x_4^3 x_5^3$

697.  $x_1^3 x_2^{15} x_3^3 x_4^5 x_5^{18}$   
 701.  $x_1^{15} x_2^3 x_3^3 x_4^3 x_5^{18}$   
 705.  $x_1^3 x_2^7 x_3^{25} x_4^6 x_5^6$   
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 713.  $x_1^3 x_2^{13} x_3^4 x_4^{19} x_5^6$   
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 725.  $x_1^3 x_2^3 x_3^7 x_4^7 x_5^{24}$   
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 753.  $x_1^3 x_2^3 x_3^9 x_4^{22} x_5^3$   
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 777.  $x_1^3 x_2^3 x_3^{13} x_4^{18} x_5^3$   
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 785.  $x_1^3 x_2^3 x_3^{15} x_4^{16} x_5^7$   
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 793.  $x_1^3 x_2^3 x_3^{15} x_4^{16} x_5^3$   
 797.  $x_1^3 x_2^3 x_3^{15} x_4^{16} x_5^3$   
 801.  $x_1^{15} x_2^3 x_3^3 x_4^{16} x_5^3$   
 805.  $x_1^3 x_2^7 x_3^{11} x_4^{19} x_5^4$   
 809.  $x_1^3 x_2^{11} x_3^3 x_4^{19} x_5^4$   
 813.  $x_1^3 x_2^3 x_3^{11} x_4^{19} x_5^6$   
 817.  $x_1^3 x_2^3 x_3^5 x_4^{10} x_5^{19}$   
 821.  $x_1^3 x_2^3 x_3^{11} x_4^{18} x_5^7$   
 825.  $x_1^3 x_2^3 x_3^{11} x_4^{18} x_5^3$   
 829.  $x_1^3 x_2^3 x_3^6 x_4^{19} x_5^3$   
 833.  $x_1^3 x_2^3 x_3^6 x_4^{19} x_5^3$   
 837.  $x_1^3 x_2^{11} x_3^{17} x_4^6 x_5^6$   
 841.  $x_1^3 x_2^3 x_3^7 x_4^8 x_5^{19}$   
 845.  $x_1^3 x_2^3 x_3^{19} x_4^3 x_5^3$   
 849.  $x_1^3 x_2^3 x_3^9 x_4^{18} x_5^3$   
 853.  $x_1^3 x_2^3 x_3^9 x_4^{18} x_5^3$   
 857.  $x_1^3 x_2^3 x_3^9 x_4^{18} x_5^3$   
 861.  $x_1^3 x_2^3 x_3^{11} x_4^{16} x_5^3$   
 865.  $x_1^3 x_2^3 x_3^{11} x_4^{16} x_5^3$   
 869.  $x_1^3 x_2^{11} x_3^3 x_4^6 x_5^3$
698.  $x_1^3 x_2^{15} x_3^5 x_4^3 x_5^{18}$   
 702.  $x_1^{15} x_2^3 x_3^5 x_4^3 x_5^{18}$   
 706.  $x_1^3 x_2^7 x_3^{25} x_4^6 x_5^3$   
 710.  $x_1^3 x_2^3 x_3^{13} x_4^6 x_5^{19}$   
 714.  $x_1^3 x_2^{13} x_3^6 x_4^6 x_5^{19}$   
 718.  $x_1^3 x_2^3 x_3^{15} x_4^{17} x_5^6$   
 722.  $x_1^{15} x_2^3 x_3^4 x_4^{17} x_5^6$   
 726.  $x_1^3 x_2^3 x_3^7 x_4^{24} x_5^7$   
 730.  $x_1^3 x_2^3 x_3^7 x_4^{24} x_5^3$   
 734.  $x_1^3 x_2^3 x_3^7 x_4^{24} x_5^3$   
 738.  $x_1^3 x_2^3 x_3^8 x_4^5 x_5^3$   
 742.  $x_1^3 x_2^3 x_3^8 x_4^5 x_5^3$   
 746.  $x_1^3 x_2^3 x_3^9 x_4^5 x_5^{22}$   
 750.  $x_1^3 x_2^3 x_3^9 x_4^5 x_5^{22}$   
 754.  $x_1^3 x_2^3 x_3^{11} x_4^{20} x_5^3$   
 758.  $x_1^3 x_2^3 x_3^{11} x_4^{20} x_5^3$   
 762.  $x_1^3 x_2^{11} x_3^3 x_4^{20} x_5^3$   
 766.  $x_1^3 x_2^3 x_3^{12} x_4^{19} x_5^3$   
 770.  $x_1^3 x_2^3 x_3^{13} x_4^{18} x_5^7$   
 774.  $x_1^3 x_2^3 x_3^{13} x_4^{18} x_5^3$   
 778.  $x_1^3 x_2^3 x_3^{13} x_4^{18} x_5^3$   
 782.  $x_1^3 x_2^3 x_3^{13} x_4^{18} x_5^3$   
 786.  $x_1^3 x_2^3 x_3^{15} x_4^{16} x_5^3$   
 790.  $x_1^3 x_2^3 x_3^{15} x_4^{16} x_5^3$   
 794.  $x_1^3 x_2^3 x_3^{15} x_4^{16} x_5^3$   
 798.  $x_1^{15} x_2^3 x_3^3 x_4^{16} x_5^3$   
 802.  $x_1^{15} x_2^3 x_3^3 x_4^{16} x_5^3$   
 806.  $x_1^3 x_2^7 x_3^{11} x_4^{19} x_5^4$   
 810.  $x_1^3 x_2^{11} x_3^3 x_4^{19} x_5^4$   
 814.  $x_1^3 x_2^3 x_3^{11} x_4^{19} x_5^3$   
 818.  $x_1^3 x_2^3 x_3^5 x_4^{10} x_5^{19}$   
 822.  $x_1^3 x_2^3 x_3^{11} x_4^{18} x_5^3$   
 826.  $x_1^3 x_2^{11} x_3^3 x_4^{18} x_5^3$   
 830.  $x_1^3 x_2^3 x_3^9 x_4^{19} x_5^6$   
 834.  $x_1^3 x_2^3 x_3^{11} x_4^{17} x_5^6$   
 838.  $x_1^3 x_2^{11} x_3^{17} x_4^6 x_5^3$   
 842.  $x_1^3 x_2^3 x_3^8 x_4^7 x_5^{19}$   
 846.  $x_1^3 x_2^3 x_3^9 x_4^{18} x_5^3$   
 850.  $x_1^3 x_2^3 x_3^9 x_4^{18} x_5^3$   
 854.  $x_1^3 x_2^3 x_3^9 x_4^{18} x_5^3$   
 858.  $x_1^3 x_2^3 x_3^{11} x_4^{16} x_5^3$   
 862.  $x_1^3 x_2^3 x_3^{11} x_4^{16} x_5^3$   
 866.  $x_1^3 x_2^3 x_3^{11} x_4^{16} x_5^3$   
 870.  $x_1^3 x_2^{11} x_3^3 x_4^6 x_5^3$
699.  $x_1^3 x_2^{15} x_3^5 x_4^{18} x_5^3$   
 703.  $x_1^3 x_2^3 x_3^7 x_4^{25} x_5^6$   
 707.  $x_1^3 x_2^3 x_3^7 x_4^{25} x_5^6$   
 711.  $x_1^3 x_2^3 x_3^{13} x_4^{19} x_5^6$   
 715.  $x_1^3 x_2^{13} x_3^6 x_4^{19} x_5^3$   
 719.  $x_1^3 x_2^{15} x_3^3 x_4^{17} x_5^6$   
 723.  $x_1^{15} x_2^3 x_3^{17} x_4^6 x_5^6$   
 727.  $x_1^3 x_2^3 x_3^7 x_4^{24} x_5^7$   
 731.  $x_1^3 x_2^3 x_3^7 x_4^{24} x_5^3$   
 735.  $x_1^3 x_2^3 x_3^7 x_4^{24} x_5^3$   
 739.  $x_1^3 x_2^3 x_3^8 x_4^5 x_5^3$   
 743.  $x_1^3 x_2^3 x_3^8 x_4^5 x_5^3$   
 747.  $x_1^3 x_2^3 x_3^9 x_4^5 x_5^{22}$   
 751.  $x_1^3 x_2^3 x_3^9 x_4^5 x_5^{22}$   
 755.  $x_1^3 x_2^3 x_3^{11} x_4^{20} x_5^3$   
 759.  $x_1^3 x_2^3 x_3^{11} x_4^{20} x_5^3$   
 763.  $x_1^3 x_2^3 x_3^{11} x_4^{20} x_5^3$   
 767.  $x_1^3 x_2^3 x_3^{12} x_4^{19} x_5^3$   
 771.  $x_1^3 x_2^3 x_3^{13} x_4^{18} x_5^7$   
 775.  $x_1^3 x_2^3 x_3^{13} x_4^{18} x_5^3$   
 779.  $x_1^3 x_2^3 x_3^{13} x_4^{18} x_5^3$   
 783.  $x_1^3 x_2^3 x_3^{15} x_4^{16} x_5^3$   
 787.  $x_1^3 x_2^3 x_3^{15} x_4^{16} x_5^3$   
 791.  $x_1^3 x_2^3 x_3^{15} x_4^{16} x_5^3$   
 795.  $x_1^3 x_2^3 x_3^{15} x_4^{16} x_5^3$   
 799.  $x_1^{15} x_2^3 x_3^3 x_4^{16} x_5^3$   
 803.  $x_1^{15} x_2^3 x_3^3 x_4^{16} x_5^3$   
 807.  $x_1^3 x_2^7 x_3^{11} x_4^{19} x_5^4$   
 811.  $x_1^3 x_2^3 x_3^3 x_4^{19} x_5^3$   
 815.  $x_1^3 x_2^3 x_3^{10} x_4^7 x_5^{19}$   
 819.  $x_1^3 x_2^3 x_3^7 x_4^{11} x_5^{18}$   
 823.  $x_1^3 x_2^3 x_3^{11} x_4^5 x_5^{18}$   
 827.  $x_1^3 x_2^{11} x_3^3 x_4^5 x_5^{18}$   
 831.  $x_1^3 x_2^3 x_3^9 x_4^{19} x_5^6$   
 835.  $x_1^3 x_2^3 x_3^{11} x_4^{17} x_5^6$   
 839.  $x_1^3 x_2^3 x_3^8 x_4^{19} x_5^3$   
 843.  $x_1^3 x_2^3 x_3^8 x_4^{19} x_5^3$   
 847.  $x_1^3 x_2^3 x_3^9 x_4^{18} x_5^3$   
 851.  $x_1^3 x_2^3 x_3^9 x_4^{18} x_5^3$   
 855.  $x_1^3 x_2^3 x_3^9 x_4^{18} x_5^3$   
 859.  $x_1^3 x_2^3 x_3^{11} x_4^{16} x_5^3$   
 863.  $x_1^3 x_2^3 x_3^{11} x_4^{16} x_5^3$   
 867.  $x_1^3 x_2^{11} x_3^3 x_4^6 x_5^3$
700.  $x_1^{15} x_2^3 x_3^3 x_4^5 x_5^{18}$   
 704.  $x_1^3 x_2^7 x_3^3 x_4^{25} x_5^6$   
 708.  $x_1^3 x_2^3 x_3^{25} x_4^6 x_5^6$   
 712.  $x_1^3 x_2^{13} x_3^3 x_4^6 x_5^{19}$   
 716.  $x_1^3 x_2^{13} x_3^3 x_4^6 x_5^{19}$   
 720.  $x_1^3 x_2^{15} x_3^{17} x_4^6 x_5^3$   
 724.  $x_1^{15} x_2^3 x_3^{17} x_4^6 x_5^3$   
 728.  $x_1^3 x_2^3 x_3^7 x_4^{24} x_5^7$   
 732.  $x_1^3 x_2^3 x_3^7 x_4^{24} x_5^3$   
 736.  $x_1^3 x_2^3 x_3^7 x_4^{24} x_5^3$   
 740.  $x_1^3 x_2^3 x_3^8 x_4^5 x_5^3$   
 744.  $x_1^3 x_2^3 x_3^8 x_4^5 x_5^3$   
 748.  $x_1^3 x_2^3 x_3^9 x_4^5 x_5^{22}$   
 752.  $x_1^3 x_2^3 x_3^9 x_4^5 x_5^{22}$   
 756.  $x_1^3 x_2^3 x_3^{11} x_4^{20} x_5^3$   
 760.  $x_1^3 x_2^3 x_3^{11} x_4^{20} x_5^3$   
 764.  $x_1^3 x_2^3 x_3^{12} x_4^{19} x_5^3$   
 768.  $x_1^3 x_2^3 x_3^{13} x_4^{18} x_5^7$   
 772.  $x_1^3 x_2^3 x_3^{13} x_4^{18} x_5^3$   
 776.  $x_1^3 x_2^3 x_3^{13} x_4^{18} x_5^3$   
 780.  $x_1^3 x_2^3 x_3^{13} x_4^{18} x_5^3$   
 784.  $x_1^3 x_2^3 x_3^{15} x_4^{16} x_5^3$   
 788.  $x_1^3 x_2^3 x_3^{15} x_4^{16} x_5^3$   
 792.  $x_1^3 x_2^3 x_3^{15} x_4^{16} x_5^3$   
 796.  $x_1^3 x_2^3 x_3^{15} x_4^{16} x_5^3$   
 800.  $x_1^{15} x_2^3 x_3^3 x_4^{16} x_5^3$   
 804.  $x_1^3 x_2^7 x_3^{11} x_4^{19} x_5^4$   
 808.  $x_1^3 x_2^{11} x_3^3 x_4^{19} x_5^4$   
 812.  $x_1^3 x_2^3 x_3^{11} x_4^{19} x_5^3$   
 816.  $x_1^3 x_2^3 x_3^{10} x_4^7 x_5^{19}$   
 820.  $x_1^3 x_2^3 x_3^{11} x_4^5 x_5^{18}$   
 824.  $x_1^3 x_2^3 x_3^{11} x_4^5 x_5^{18}$   
 828.  $x_1^3 x_2^{11} x_3^3 x_4^5 x_5^{18}$   
 832.  $x_1^3 x_2^3 x_3^9 x_4^{19} x_5^6$   
 836.  $x_1^3 x_2^{11} x_3^3 x_4^{17} x_5^6$   
 840.  $x_1^3 x_2^3 x_3^8 x_4^{19} x_5^3$   
 844.  $x_1^3 x_2^3 x_3^8 x_4^{19} x_5^3$   
 848.  $x_1^3 x_2^3 x_3^9 x_4^{18} x_5^3$   
 852.  $x_1^3 x_2^3 x_3^9 x_4^{18} x_5^3$   
 856.  $x_1^3 x_2^3 x_3^9 x_4^{18} x_5^3$   
 860.  $x_1^3 x_2^3 x_3^{11} x_4^{16} x_5^3$   
 864.  $x_1^3 x_2^3 x_3^{11} x_4^{16} x_5^3$   
 868.  $x_1^3 x_2^{11} x_3^3 x_4^6 x_5^3$

$B_5^+((4)|^2|(2)|(3)) = \{\bar{b}_t : 1 \leq t \leq 90\}$ , where

- |  |  |  |  |
|--|--|--|--|
| 1. $x_1x_2^6x_3^{11}x_4^{11}x_5^{15}$    | 2. $x_1x_2^6x_3^{11}x_4^{15}x_5^{11}$    | 3. $x_1x_2^6x_3^{15}x_4^{11}x_5^{11}$    | 4. $x_1x_2^7x_3^{10}x_4^{11}x_5^{15}$    |
| 5. $x_1x_2^7x_3^{10}x_4^{15}x_5^{11}$    | 6. $x_1x_2^7x_3^{11}x_4^{10}x_5^{15}$    | 7. $x_1x_2^7x_3^{11}x_4^{11}x_5^{14}$    | 8. $x_1x_2^7x_3^{11}x_4^{14}x_5^{11}$    |
| 9. $x_1x_2^7x_3^{11}x_4^{15}x_5^{10}$    | 10. $x_1x_2^7x_3^{14}x_4^{11}x_5^{11}$   | 11. $x_1x_2^7x_3^{15}x_4^{10}x_5^{11}$   | 12. $x_1x_2^7x_3^{15}x_4^{11}x_5^{10}$   |
| 13. $x_1x_2^{14}x_3^7x_4^{11}x_5^{11}$   | 14. $x_1x_2^{15}x_3^6x_4^{11}x_5^{11}$   | 15. $x_1x_2^{15}x_3^7x_4^{10}x_5^{11}$   | 16. $x_1x_2^{15}x_3^7x_4^{11}x_5^{10}$   |
| 17. $x_1^3x_2^3x_3^{10}x_4^{11}x_5^{15}$ | 18. $x_1^3x_2^3x_3^{10}x_4^{15}x_5^{11}$ | 19. $x_1^3x_2^3x_3^{11}x_4^{10}x_5^{15}$ | 20. $x_1^3x_2^3x_3^{11}x_4^{11}x_5^{14}$ |
| 21. $x_1^3x_2^3x_3^{11}x_4^{14}x_5^{11}$ | 22. $x_1^3x_2^3x_3^{11}x_4^{15}x_5^{10}$ | 23. $x_1^3x_2^3x_3^{14}x_4^{11}x_5^{11}$ | 24. $x_1^3x_2^3x_3^{15}x_4^{10}x_5^{11}$ |
| 25. $x_1^3x_2^3x_3^{15}x_4^{11}x_5^{10}$ | 26. $x_1^3x_2^7x_3^9x_4^{10}x_5^{15}$    | 27. $x_1^3x_2^7x_3^9x_4^{11}x_5^{14}$    | 28. $x_1^3x_2^7x_3^9x_4^{14}x_5^{11}$    |
| 29. $x_1^3x_2^7x_3^9x_4^{15}x_5^{10}$    | 30. $x_1^3x_2^7x_3^{11}x_4^9x_5^{14}$    | 31. $x_1^3x_2^7x_3^{11}x_4^{13}x_5^{10}$ | 32. $x_1^3x_2^7x_3^{13}x_4^{10}x_5^{11}$ |
| 33. $x_1^3x_2^7x_3^{13}x_4^{11}x_5^{10}$ | 34. $x_1^3x_2^7x_3^{15}x_4^9x_5^{10}$    | 35. $x_1^3x_2^{13}x_3^6x_4^{11}x_5^{11}$ | 36. $x_1^3x_2^{13}x_3^7x_4^{10}x_5^{11}$ |
| 37. $x_1^3x_2^{13}x_3^7x_4^{11}x_5^{10}$ | 38. $x_1^3x_2^{15}x_3^5x_4^{10}x_5^{11}$ | 39. $x_1^3x_2^{15}x_3^5x_4^{11}x_5^{10}$ | 40. $x_1^3x_2^{15}x_3^7x_4^9x_5^{10}$    |
| 41. $x_1^7x_2x_3^{10}x_4^{11}x_5^{15}$   | 42. $x_1^7x_2x_3^{10}x_4^{15}x_5^{11}$   | 43. $x_1^7x_2x_3^{11}x_4^{10}x_5^{15}$   | 44. $x_1^7x_2x_3^{11}x_4^{11}x_5^{14}$   |
| 45. $x_1^7x_2x_3^{11}x_4^{14}x_5^{11}$   | 46. $x_1^7x_2x_3^{11}x_4^{15}x_5^{10}$   | 47. $x_1^7x_2x_3^{14}x_4^{11}x_5^{11}$   | 48. $x_1^7x_2x_3^{15}x_4^{10}x_5^{11}$   |
| 49. $x_1^7x_2x_3^{15}x_4^{11}x_5^{10}$   | 50. $x_1^7x_2^3x_3^9x_4^{10}x_5^{15}$    | 51. $x_1^7x_2^3x_3^9x_4^{11}x_5^{14}$    | 52. $x_1^7x_2^3x_3^9x_4^{14}x_5^{11}$    |
| 53. $x_1^7x_2^3x_3^9x_4^{15}x_5^{10}$    | 54. $x_1^7x_2^3x_3^{11}x_4^9x_5^{14}$    | 55. $x_1^7x_2^3x_3^{11}x_4^{13}x_5^{10}$ | 56. $x_1^7x_2^3x_3^{13}x_4^{10}x_5^{11}$ |
| 57. $x_1^7x_2^3x_3^{13}x_4^{11}x_5^{10}$ | 58. $x_1^7x_2^3x_3^{15}x_4^9x_5^{10}$    | 59. $x_1^7x_2^7x_3^8x_4^{11}x_5^{11}$    | 60. $x_1^7x_2^7x_3^9x_4^{10}x_5^{11}$    |
| 61. $x_1^7x_2^7x_3^9x_4^{11}x_5^{10}$    | 62. $x_1^7x_2^7x_3^{11}x_4^8x_5^{11}$    | 63. $x_1^7x_2^7x_3^{11}x_4^9x_5^{10}$    | 64. $x_1^7x_2^7x_3^{11}x_4^{11}x_5^8$    |
| 65. $x_1^7x_2^{11}x_3x_4^{10}x_5^{15}$   | 66. $x_1^7x_2^{11}x_3x_4^{11}x_5^{14}$   | 67. $x_1^7x_2^{11}x_3x_4^{14}x_5^{11}$   | 68. $x_1^7x_2^{11}x_3x_4^{15}x_5^{10}$   |
| 69. $x_1^7x_2^{11}x_3^3x_4^{14}x_5^{15}$ | 70. $x_1^7x_2^{11}x_3^3x_4^{13}x_5^{10}$ | 71. $x_1^7x_2^{11}x_3^5x_4^{10}x_5^{11}$ | 72. $x_1^7x_2^{11}x_3^5x_4^{11}x_5^{10}$ |
| 73. $x_1^7x_2^{11}x_3^7x_4^{10}x_5^{15}$ | 74. $x_1^7x_2^{11}x_3^{11}x_4x_5^{14}$   | 75. $x_1^7x_2^{11}x_3^{11}x_4^7x_5^8$    | 76. $x_1^7x_2^{11}x_3^{15}x_4x_5^{10}$   |
| 77. $x_1^7x_2^{15}x_3x_4^{10}x_5^{11}$   | 78. $x_1^7x_2^{15}x_3x_4^{11}x_5^{10}$   | 79. $x_1^7x_2^{15}x_3^3x_4^9x_5^{10}$    | 80. $x_1^7x_2^{15}x_3^{11}x_4x_5^{10}$   |
| 81. $x_1^{15}x_2x_3^6x_4^{11}x_5^{11}$   | 82. $x_1^{15}x_2x_3^7x_4^{10}x_5^{11}$   | 83. $x_1^{15}x_2x_3^7x_4^{11}x_5^{10}$   | 84. $x_1^{15}x_2x_3^5x_4^{10}x_5^{11}$   |
| 85. $x_1^{15}x_2x_3^5x_4^{11}x_5^{10}$   | 86. $x_1^{15}x_2x_3^7x_4^9x_5^{10}$      | 87. $x_1^{15}x_2x_3x_4^{10}x_5^{11}$     | 88. $x_1^{15}x_2x_3x_4^{11}x_5^{10}$     |
| 89. $x_1^{15}x_2x_3^3x_4^9x_5^{10}$      | 90. $x_1^{15}x_2x_3^{11}x_4x_5^{10}$     |  |  |

$B_5^+((4)|^3|(2)) = \{\tilde{b}_t : 1 \leq t \leq 146\}$ , where

- |  |  |  |  |
|--|--|--|--|
| 1. $x_1x_2^6x_3^7x_4^{15}x_5^{15}$     | 2. $x_1x_2^6x_3^{15}x_4^7x_5^{15}$     | 3. $x_1x_2^6x_3^{15}x_4^{15}x_5^7$     | 4. $x_1x_2^7x_3^6x_4^{15}x_5^{15}$       |
| 5. $x_1x_2^7x_3^7x_4^{14}x_5^{15}$     | 6. $x_1x_2^7x_3^7x_4^{15}x_5^{14}$     | 7. $x_1x_2^7x_3^{14}x_4^7x_5^{15}$     | 8. $x_1x_2^7x_3^{14}x_4^{15}x_5^7$       |
| 9. $x_1x_2^7x_3^{15}x_4^6x_5^{15}$     | 10. $x_1x_2^7x_3^{15}x_4^7x_5^{14}$    | 11. $x_1x_2^7x_3^{15}x_4^{14}x_5^7$    | 12. $x_1x_2^7x_3^{15}x_4^{15}x_5^6$      |
| 13. $x_1x_2^{14}x_3^7x_4^{15}x_5^{15}$ | 14. $x_1x_2^{14}x_3^7x_4^{15}x_5^7$    | 15. $x_1x_2^{14}x_3^{15}x_4^7x_5^7$    | 16. $x_1x_2^{15}x_3^6x_4^7x_5^{15}$      |
| 17. $x_1x_2^{15}x_3^6x_4^{15}x_5^7$    | 18. $x_1x_2^{15}x_3^6x_4^{15}x_5^{15}$ | 19. $x_1x_2^{15}x_3^7x_4^{15}x_5^{14}$ | 20. $x_1x_2^{15}x_3^7x_4^{14}x_5^7$      |
| 21. $x_1x_2^{15}x_3^7x_4^{15}x_5^6$    | 22. $x_1x_2^{15}x_3^{14}x_4^7x_5^7$    | 23. $x_1x_2^{15}x_3^{15}x_4^6x_5^7$    | 24. $x_1x_2^{15}x_3^{15}x_4^7x_5^6$      |
| 25. $x_1^3x_2^5x_3^6x_4^{15}x_5^{15}$  | 26. $x_1^3x_2^5x_3^7x_4^{14}x_5^{15}$  | 27. $x_1^3x_2^5x_3^7x_4^{15}x_5^{14}$  | 28. $x_1^3x_2^5x_3^{14}x_4^7x_5^{15}$    |
| 29. $x_1^3x_2^5x_3^{14}x_4^{15}x_5^7$  | 30. $x_1^3x_2^5x_3^{15}x_4^6x_5^{15}$  | 31. $x_1^3x_2^5x_3^{15}x_4^7x_5^{14}$  | 32. $x_1^3x_2^5x_3^{15}x_4^{14}x_5^7$    |
| 33. $x_1^3x_2^5x_3^{15}x_4^{15}x_5^6$  | 34. $x_1^3x_2^7x_3^5x_4^{14}x_5^{15}$  | 35. $x_1^3x_2^7x_3^5x_4^{15}x_5^{14}$  | 36. $x_1^3x_2^7x_3^{13}x_4^{13}x_5^{14}$ |
| 37. $x_1^3x_2^7x_3^{13}x_4^6x_5^{15}$  | 38. $x_1^3x_2^7x_3^{13}x_4^7x_5^{14}$  | 39. $x_1^3x_2^7x_3^{13}x_4^{14}x_5^7$  | 40. $x_1^3x_2^7x_3^{13}x_4^{15}x_5^6$    |
| 41. $x_1^3x_2^7x_3^{15}x_4^5x_5^{14}$  | 42. $x_1^3x_2^7x_3^{15}x_4^{13}x_5^6$  | 43. $x_1^3x_2^{13}x_3^6x_4^7x_5^{15}$  | 44. $x_1^3x_2^{13}x_3^6x_4^{15}x_5^7$    |
| 45. $x_1^3x_2^{13}x_3^6x_4^{15}x_5^7$  | 46. $x_1^3x_2^{13}x_3^7x_4^7x_5^{14}$  | 47. $x_1^3x_2^{13}x_3^7x_4^{14}x_5^7$  | 48. $x_1^3x_2^{13}x_3^7x_4^{15}x_5^6$    |
| 49. $x_1^3x_2^{13}x_3^{14}x_4^7x_5^7$  | 50. $x_1^3x_2^{13}x_3^{15}x_4^6x_5^7$  | 51. $x_1^3x_2^{13}x_3^{15}x_4^7x_5^6$  | 52. $x_1^3x_2^{15}x_3^5x_4^9x_5^{15}$    |
| 53. $x_1^3x_2^{15}x_3^3x_4^7x_5^{14}$  | 54. $x_1^3x_2^{15}x_3^5x_4^{14}x_5^7$  | 55. $x_1^3x_2^{15}x_3^6x_4^{15}x_5^6$  | 56. $x_1^3x_2^{15}x_3^7x_4^9x_5^{14}$    |
| 57. $x_1^3x_2^{15}x_3^7x_4^{13}x_5^6$  | 58. $x_1^3x_2^{15}x_3^{13}x_4^6x_5^7$  | 59. $x_1^3x_2^{15}x_3^{13}x_4^7x_5^6$  | 60. $x_1^3x_2^{15}x_3^{15}x_4^5x_5^6$    |
| 61. $x_1^7x_2x_3^6x_4^{15}x_5^{15}$    | 62. $x_1^7x_2x_3^7x_4^{14}x_5^{15}$    | 63. $x_1^7x_2x_3^7x_4^{15}x_5^{14}$    | 64. $x_1^7x_2x_3^{14}x_4^7x_5^{15}$      |

- |  |  |  |  |
|--|--|--|--|
| 65. $x_1^7 x_2 x_3^{14} x_4^{15} x_5^7$      | 66. $x_1^7 x_2 x_3^{15} x_4^6 x_5^{15}$      | 67. $x_1^7 x_2 x_3^{15} x_4^7 x_5^{14}$      | 68. $x_1^7 x_2 x_3^{15} x_4^{14} x_5^7$      |
| 69. $x_1^7 x_2 x_3^{15} x_4^{15} x_5^6$      | 70. $x_1^7 x_2^3 x_3^3 x_4^{14} x_5^{15}$    | 71. $x_1^7 x_2^3 x_3^5 x_4^{15} x_5^{14}$    | 72. $x_1^7 x_2^3 x_3^7 x_4^{13} x_5^{14}$    |
| 73. $x_1^7 x_2^3 x_3^{13} x_4^6 x_5^{15}$    | 74. $x_1^7 x_2^3 x_3^{13} x_4^7 x_5^{14}$    | 75. $x_1^7 x_2^3 x_3^{13} x_4^{14} x_5^7$    | 76. $x_1^7 x_2^3 x_3^{13} x_4^{15} x_5^6$    |
| 77. $x_1^7 x_2^3 x_3^{15} x_4^5 x_5^{14}$    | 78. $x_1^7 x_2^3 x_3^{15} x_4^{13} x_5^6$    | 79. $x_1^7 x_2^3 x_3^{14} x_4^{15} x_5^{15}$ | 80. $x_1^7 x_2^3 x_3^{14} x_4^{15} x_5^{14}$ |
| 81. $x_1^7 x_2^3 x_3^{13} x_4^{13} x_5^{14}$ | 82. $x_1^7 x_2^3 x_3^9 x_4^8 x_5^{15}$       | 83. $x_1^7 x_2^3 x_3^9 x_4^9 x_5^{14}$       | 84. $x_1^7 x_2^3 x_3^{14} x_4^{15} x_5^8$    |
| 85. $x_1^7 x_2^3 x_3^6 x_4^6 x_5^{15}$       | 86. $x_1^7 x_2^3 x_3^7 x_4^7 x_5^{14}$       | 87. $x_1^7 x_2^3 x_3^{14} x_4^7 x_5^7$       | 88. $x_1^7 x_2^3 x_3^9 x_4^{15} x_5^6$       |
| 89. $x_1^7 x_2^3 x_3^{11} x_4^9 x_5^{14}$    | 90. $x_1^7 x_2^3 x_3^{11} x_4^{13} x_5^6$    | 91. $x_1^7 x_2^3 x_3^{15} x_4^{15} x_5^{14}$ | 92. $x_1^7 x_2^3 x_3^{15} x_4^8 x_5^8$       |
| 93. $x_1^7 x_2^3 x_3^{15} x_4^9 x_5^6$       | 94. $x_1^7 x_2^{11} x_3^5 x_4^6 x_5^{15}$    | 95. $x_1^7 x_2^{11} x_3^7 x_4^7 x_5^{14}$    | 96. $x_1^7 x_2^{11} x_3^5 x_4^{14} x_5^7$    |
| 97. $x_1^7 x_2^{11} x_3^5 x_4^{15} x_5^6$    | 98. $x_1^7 x_2^{11} x_3^5 x_4^{15} x_5^{14}$ | 99. $x_1^7 x_2^{11} x_3^7 x_4^{13} x_5^6$    | 100. $x_1^7 x_2^{11} x_3^{13} x_4^6 x_5^7$   |
| 101. $x_1^7 x_2^{11} x_3^{13} x_4^7 x_5^6$   | 102. $x_1^7 x_2^{11} x_3^{15} x_4^5 x_5^6$   | 103. $x_1^7 x_2^{15} x_3^6 x_4^5 x_5^{15}$   | 104. $x_1^7 x_2^{15} x_3^7 x_4^7 x_5^{14}$   |
| 105. $x_1^7 x_2^{15} x_3^3 x_4^{14} x_5^7$   | 106. $x_1^7 x_2^{15} x_3^3 x_4^{15} x_5^6$   | 107. $x_1^7 x_2^{15} x_3^5 x_4^5 x_5^{14}$   | 108. $x_1^7 x_2^{15} x_3^5 x_4^{13} x_5^6$   |
| 109. $x_1^7 x_2^{15} x_3^7 x_4^8 x_5^{14}$   | 110. $x_1^7 x_2^{15} x_3^7 x_4^8 x_5^8$      | 111. $x_1^7 x_2^{15} x_3^9 x_4^6 x_5^6$      | 112. $x_1^7 x_2^{15} x_3^{11} x_4^5 x_5^6$   |
| 113. $x_1^7 x_2^{15} x_3^{15} x_4^6 x_5^6$   | 114. $x_1^7 x_2^{15} x_3^6 x_4^7 x_5^{15}$   | 115. $x_1^7 x_2^{15} x_3^6 x_4^{15} x_5^7$   | 116. $x_1^7 x_2^{15} x_3^7 x_4^6 x_5^{15}$   |
| 117. $x_1^7 x_2^3 x_3^7 x_4^{14} x_5^{14}$   | 118. $x_1^7 x_2^3 x_3^7 x_4^{14} x_5^7$      | 119. $x_1^7 x_2^3 x_3^{15} x_4^{15} x_5^6$   | 120. $x_1^7 x_2^3 x_3^{14} x_4^7 x_5^7$      |
| 121. $x_1^7 x_2^3 x_3^{15} x_4^6 x_5^7$      | 122. $x_1^7 x_2^3 x_3^{15} x_4^6 x_5^6$      | 123. $x_1^7 x_2^3 x_3^5 x_4^6 x_5^{15}$      | 124. $x_1^7 x_2^3 x_3^7 x_4^{14} x_5^{14}$   |
| 125. $x_1^7 x_2^3 x_3^5 x_4^{14} x_5^7$      | 126. $x_1^7 x_2^3 x_3^5 x_4^{15} x_5^6$      | 127. $x_1^7 x_2^3 x_3^5 x_4^{14} x_5^{14}$   | 128. $x_1^7 x_2^3 x_3^7 x_4^{13} x_5^6$      |
| 129. $x_1^7 x_2^3 x_3^{13} x_4^6 x_5^7$      | 130. $x_1^7 x_2^3 x_3^{13} x_4^6 x_5^6$      | 131. $x_1^7 x_2^3 x_3^{15} x_4^5 x_5^6$      | 132. $x_1^7 x_2^3 x_3^6 x_4^{15} x_5^{15}$   |
| 133. $x_1^7 x_2^3 x_3^7 x_4^{14} x_5^{14}$   | 134. $x_1^7 x_2^3 x_3^7 x_4^{14} x_5^7$      | 135. $x_1^7 x_2^3 x_3^{15} x_4^5 x_5^6$      | 136. $x_1^7 x_2^3 x_3^5 x_4^{14} x_5^{14}$   |
| 137. $x_1^7 x_2^3 x_3^{13} x_4^{13} x_5^6$   | 138. $x_1^7 x_2^3 x_3^7 x_4^{14} x_5^{14}$   | 139. $x_1^7 x_2^3 x_3^7 x_4^8 x_5^8$         | 140. $x_1^7 x_2^3 x_3^9 x_4^6 x_5^6$         |
| 141. $x_1^7 x_2^3 x_3^{11} x_4^5 x_5^6$      | 142. $x_1^7 x_2^3 x_3^{15} x_4^6 x_5^6$      | 143. $x_1^7 x_2^{15} x_3^6 x_4^5 x_5^7$      | 144. $x_1^7 x_2^{15} x_3^7 x_4^6 x_5^6$      |
| 145. $x_1^7 x_2^{15} x_3^5 x_4^6 x_5^6$      | 146. $x_1^7 x_2^{15} x_3^7 x_4^6 x_5^6$      |  |  |

### 6.3 The case $d = 5$

$\tilde{B}_5^+((4)^3(2)|(1)^2) = \{x_i^{63}\theta_{J_i}(w) \mid w \in B_4^+(29), 1 \leq i \leq 5\} \cup \{c_t : 1 \leq t \leq 1005\}$ , where  $B_4^+(29) = \{\tilde{w}_u : 1 \leq u \leq 33\}$  with  $\tilde{w}_u$  the monomials determined as follows:

- |                                  |                                  |                                  |                                  |                                  |
|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| 1. $x_1 x_2^6 x_3^7 x_4^{15}$    | 2. $x_1 x_2^6 x_3^{15} x_4^7$    | 3. $x_1 x_2^7 x_3^6 x_4^{15}$    | 4. $x_1 x_2^7 x_3^7 x_4^{14}$    | 5. $x_1 x_2^7 x_3^{14} x_4^7$    |
| 6. $x_1 x_2^7 x_3^{15} x_4^6$    | 7. $x_1 x_2^{14} x_3^7 x_4^7$    | 8. $x_1 x_2^{15} x_3^6 x_4^7$    | 9. $x_1 x_2^{15} x_3^7 x_4^6$    | 10. $x_1^3 x_2^5 x_3^6 x_4^{15}$ |
| 11. $x_1^3 x_2^5 x_3^7 x_4^{14}$ | 12. $x_1^3 x_2^5 x_3^{14} x_4^7$ | 13. $x_1^3 x_2^5 x_3^{15} x_4^6$ | 14. $x_1^3 x_2^7 x_3^5 x_4^{14}$ | 15. $x_1^3 x_2^7 x_3^{13} x_4^6$ |
| 16. $x_1^3 x_2^{13} x_3^6 x_4^7$ | 17. $x_1^3 x_2^{13} x_3^7 x_4^6$ | 18. $x_1^3 x_2^{15} x_3^5 x_4^6$ | 19. $x_1^3 x_2^6 x_3^4 x_4^{15}$ | 20. $x_1^3 x_2^7 x_3^4 x_4^{14}$ |
| 21. $x_1^7 x_2 x_3^{14} x_4^7$   | 22. $x_1^7 x_2 x_3^{15} x_4^6$   | 23. $x_1^7 x_2^3 x_3^5 x_4^{14}$ | 24. $x_1^7 x_2^3 x_3^{13} x_4^6$ | 25. $x_1^7 x_2^7 x_3^4 x_4^{14}$ |
| 26. $x_1^7 x_2^7 x_3^8 x_4^8$    | 27. $x_1^7 x_2^7 x_3^9 x_4^6$    | 28. $x_1^7 x_2^{11} x_3^5 x_4^6$ | 29. $x_1^7 x_2^{15} x_3 x_4^6$   | 30. $x_1^{15} x_2 x_3^9 x_4^7$   |
| 31. $x_1^{15} x_2 x_3^7 x_4^6$   | 32. $x_1^{15} x_2^3 x_3^5 x_4^6$ | 33. $x_1^{15} x_2^7 x_3 x_4^6$   |                                  |                                  |

The monomials  $c_t$  are determined as follows:

- |   |   |  |
|---|---|--|
| 1. $x_1 x_2^6 x_3^4 x_4^{31} x_5^{47}$    | 2. $x_1 x_2^6 x_3^{15} x_4^{15} x_5^{55}$ | 3. $x_1 x_2^6 x_3^{15} x_4^{23} x_5^{47}$  |
| 4. $x_1 x_2^6 x_3^{15} x_4^{31} x_5^{39}$ | 5. $x_1 x_2^6 x_3^{15} x_4^{55} x_5^{15}$ | 6. $x_1 x_2^6 x_3^{31} x_4^7 x_5^{47}$     |
| 7. $x_1 x_2^6 x_3^{31} x_4^{15} x_5^{39}$ | 8. $x_1 x_2^6 x_3^{31} x_4^{39} x_5^{15}$ | 9. $x_1 x_2^6 x_3^{31} x_4^{47} x_5^7$     |
| 10. $x_1 x_2^7 x_3^6 x_4^{31} x_5^{47}$   | 11. $x_1 x_2^7 x_3^7 x_4^{15} x_5^{62}$   | 12. $x_1 x_2^7 x_3^7 x_4^{30} x_5^{47}$    |
| 13. $x_1 x_2^7 x_3^7 x_4^{31} x_5^{46}$   | 14. $x_1 x_2^7 x_3^6 x_4^{62} x_5^{15}$   | 15. $x_1 x_2^7 x_3^{14} x_4^{15} x_5^{55}$ |



16.  $x_1 x_2^7 x_3^{14} x_4^{23} x_5^{47}$
17.  $x_1 x_2^7 x_3^{14} x_4^{31} x_5^{39}$
18.  $x_1 x_2^7 x_3^{14} x_4^{55} x_5^{15}$
19.  $x_1 x_2^7 x_3^{15} x_4^{62} x_5^{54}$
20.  $x_1 x_2^7 x_3^{15} x_4^{14} x_5^{55}$
21.  $x_1 x_2^7 x_3^{15} x_4^{15} x_5^{54}$
22.  $x_1 x_2^7 x_3^{15} x_4^{22} x_5^{47}$
23.  $x_1 x_2^7 x_3^{15} x_4^{23} x_5^{46}$
24.  $x_1 x_2^7 x_3^{15} x_4^{30} x_5^{39}$
25.  $x_1 x_2^7 x_3^{15} x_4^{31} x_5^{38}$
26.  $x_1 x_2^7 x_3^{15} x_4^{54} x_5^{15}$
27.  $x_1 x_2^7 x_3^{15} x_4^{55} x_5^{14}$
28.  $x_1 x_2^7 x_3^{15} x_4^{62} x_5^{7}$
29.  $x_1 x_2^7 x_3^{30} x_4^{47} x_5^{39}$
30.  $x_1 x_2^7 x_3^{30} x_4^{15} x_5^{39}$
31.  $x_1 x_2^7 x_3^{30} x_4^{39} x_5^{15}$
32.  $x_1 x_2^7 x_3^{30} x_4^{47} x_5^{7}$
33.  $x_1 x_2^7 x_3^{31} x_4^{6} x_5^{47}$
34.  $x_1 x_2^7 x_3^{31} x_4^{7} x_5^{46}$
35.  $x_1 x_2^7 x_3^{31} x_4^{14} x_5^{39}$
36.  $x_1 x_2^7 x_3^{31} x_4^{15} x_5^{38}$
37.  $x_1 x_2^7 x_3^{31} x_4^{38} x_5^{15}$
38.  $x_1 x_2^7 x_3^{31} x_4^{39} x_5^{14}$
39.  $x_1 x_2^7 x_3^{31} x_4^{46} x_5^{7}$
40.  $x_1 x_2^7 x_3^{31} x_4^{47} x_5^{6}$
41.  $x_1 x_2^7 x_3^{62} x_4^{7} x_5^{15}$
42.  $x_1 x_2^7 x_3^{62} x_4^{15} x_5^{7}$
43.  $x_1 x_2^{14} x_3^{15} x_4^{15} x_5^{55}$
44.  $x_1 x_2^{14} x_3^{23} x_4^{23} x_5^{47}$
45.  $x_1 x_2^{14} x_3^{31} x_4^{31} x_5^{39}$
46.  $x_1 x_2^{14} x_3^{55} x_4^{15} x_5^{15}$
47.  $x_1 x_2^{14} x_3^{15} x_4^{7} x_5^{55}$
48.  $x_1 x_2^{14} x_3^{15} x_4^{23} x_5^{39}$
49.  $x_1 x_2^{14} x_3^{15} x_4^{55} x_5^{7}$
50.  $x_1 x_2^{14} x_3^{23} x_4^{7} x_5^{47}$
51.  $x_1 x_2^{14} x_3^{23} x_4^{15} x_5^{39}$
52.  $x_1 x_2^{14} x_3^{23} x_4^{39} x_5^{15}$
53.  $x_1 x_2^{14} x_3^{23} x_4^{47} x_5^{39}$
54.  $x_1 x_2^{14} x_3^{31} x_4^{7} x_5^{39}$
55.  $x_1 x_2^{14} x_3^{31} x_4^{39} x_5^{7}$
56.  $x_1 x_2^{14} x_3^{55} x_4^{15} x_5^{7}$
57.  $x_1 x_2^{14} x_3^{55} x_4^{15} x_5^{7}$
58.  $x_1 x_2^{15} x_3^{15} x_4^{15} x_5^{55}$
59.  $x_1 x_2^{15} x_3^{23} x_4^{47} x_5^{39}$
60.  $x_1 x_2^{15} x_3^{31} x_4^{31} x_5^{39}$
61.  $x_1 x_2^{15} x_3^{55} x_4^{15} x_5^{55}$
62.  $x_1 x_2^{15} x_3^{7} x_4^{62} x_5^{55}$
63.  $x_1 x_2^{15} x_3^{14} x_4^{14} x_5^{55}$
64.  $x_1 x_2^{15} x_3^{15} x_4^{15} x_5^{54}$
65.  $x_1 x_2^{15} x_3^{22} x_4^{47} x_5^{46}$
66.  $x_1 x_2^{15} x_3^{23} x_4^{23} x_5^{46}$
67.  $x_1 x_2^{15} x_3^{30} x_4^{39} x_5^{15}$
68.  $x_1 x_2^{15} x_3^{31} x_4^{38} x_5^{15}$
69.  $x_1 x_2^{15} x_3^{54} x_4^{5} x_5^{15}$
70.  $x_1 x_2^{15} x_3^{55} x_4^{14} x_5^{15}$
71.  $x_1 x_2^{15} x_3^{62} x_4^{7} x_5^{15}$
72.  $x_1 x_2^{15} x_3^{14} x_4^{7} x_5^{55}$
73.  $x_1 x_2^{15} x_3^{14} x_4^{23} x_5^{39}$
74.  $x_1 x_2^{15} x_3^{14} x_4^{55} x_5^{7}$
75.  $x_1 x_2^{15} x_3^{15} x_4^{6} x_5^{55}$
76.  $x_1 x_2^{15} x_3^{15} x_4^{7} x_5^{54}$
77.  $x_1 x_2^{15} x_3^{15} x_4^{22} x_5^{39}$
78.  $x_1 x_2^{15} x_3^{15} x_4^{23} x_5^{38}$
79.  $x_1 x_2^{15} x_3^{15} x_4^{7} x_5^{54}$
80.  $x_1 x_2^{15} x_3^{15} x_4^{55} x_5^{6}$
81.  $x_1 x_2^{15} x_3^{22} x_4^{7} x_5^{47}$
82.  $x_1 x_2^{15} x_3^{22} x_4^{15} x_5^{39}$
83.  $x_1 x_2^{15} x_3^{22} x_4^{39} x_5^{15}$
84.  $x_1 x_2^{15} x_3^{22} x_4^{47} x_5^{7}$
85.  $x_1 x_2^{15} x_3^{23} x_4^{47} x_5^{46}$
86.  $x_1 x_2^{15} x_3^{23} x_4^{7} x_5^{46}$
87.  $x_1 x_2^{15} x_3^{23} x_4^{14} x_5^{39}$
88.  $x_1 x_2^{15} x_3^{23} x_4^{19} x_5^{38}$
89.  $x_1 x_2^{15} x_3^{23} x_4^{38} x_5^{15}$
90.  $x_1 x_2^{15} x_3^{23} x_4^{39} x_5^{14}$
91.  $x_1 x_2^{15} x_3^{23} x_4^{46} x_5^{7}$
92.  $x_1 x_2^{15} x_3^{23} x_4^{47} x_5^{6}$
93.  $x_1 x_2^{15} x_3^{30} x_4^{7} x_5^{39}$
94.  $x_1 x_2^{15} x_3^{30} x_4^{39} x_5^{7}$
95.  $x_1 x_2^{15} x_3^{31} x_4^{6} x_5^{39}$
96.  $x_1 x_2^{15} x_3^{31} x_4^{7} x_5^{38}$
97.  $x_1 x_2^{15} x_3^{31} x_4^{38} x_5^{15}$
98.  $x_1 x_2^{15} x_3^{31} x_4^{39} x_5^{6}$
99.  $x_1 x_2^{15} x_3^{34} x_4^{7} x_5^{15}$
100.  $x_1 x_2^{15} x_3^{54} x_4^{15} x_5^{7}$
101.  $x_1 x_2^{15} x_3^{55} x_4^{6} x_5^{15}$
102.  $x_1 x_2^{15} x_3^{55} x_4^{7} x_5^{14}$
103.  $x_1 x_2^{15} x_3^{55} x_4^{14} x_5^{7}$
104.  $x_1 x_2^{15} x_3^{55} x_4^{15} x_5^{6}$
105.  $x_1 x_2^{15} x_3^{62} x_4^{7} x_5^{7}$
106.  $x_1 x_2^{30} x_3^{7} x_4^{47} x_5^{15}$
107.  $x_1 x_2^{30} x_3^{15} x_4^{39} x_5^{15}$
108.  $x_1 x_2^{30} x_3^{15} x_4^{39} x_5^{15}$
109.  $x_1 x_2^{30} x_3^{15} x_4^{47} x_5^{7}$
110.  $x_1 x_2^{30} x_3^{15} x_4^{7} x_5^{39}$
111.  $x_1 x_2^{30} x_3^{15} x_4^{39} x_5^{7}$
112.  $x_1 x_2^{30} x_3^{39} x_4^{7} x_5^{15}$
113.  $x_1 x_2^{30} x_3^{39} x_4^{15} x_5^{7}$
114.  $x_1 x_2^{30} x_3^{47} x_4^{7} x_5^{15}$
115.  $x_1 x_2^{31} x_3^{6} x_4^{7} x_5^{15}$
116.  $x_1 x_2^{31} x_3^{6} x_4^{15} x_5^{39}$
117.  $x_1 x_2^{31} x_3^{6} x_4^{39} x_5^{15}$
118.  $x_1 x_2^{31} x_3^{6} x_4^{47} x_5^{15}$
119.  $x_1 x_2^{31} x_3^{6} x_4^{47} x_5^{46}$
120.  $x_1 x_2^{31} x_3^{7} x_4^{4} x_5^{46}$
121.  $x_1 x_2^{31} x_3^{14} x_4^{39} x_5^{15}$
122.  $x_1 x_2^{31} x_3^{15} x_4^{5} x_5^{38}$
123.  $x_1 x_2^{31} x_3^{15} x_4^{38} x_5^{15}$
124.  $x_1 x_2^{31} x_3^{39} x_4^{5} x_5^{14}$
125.  $x_1 x_2^{31} x_3^{46} x_4^{7} x_5^{6}$
126.  $x_1 x_2^{31} x_3^{47} x_4^{6} x_5^{6}$
127.  $x_1 x_2^{31} x_3^{4} x_4^{7} x_5^{39}$
128.  $x_1 x_2^{31} x_3^{4} x_4^{39} x_5^{7}$
129.  $x_1 x_2^{31} x_3^{5} x_4^{6} x_5^{39}$
130.  $x_1 x_2^{31} x_3^{5} x_4^{39} x_5^{6}$
131.  $x_1 x_2^{31} x_3^{5} x_4^{39} x_5^{6}$
132.  $x_1 x_2^{31} x_3^{5} x_4^{39} x_5^{6}$
133.  $x_1 x_2^{31} x_3^{8} x_4^{15} x_5^{15}$
134.  $x_1 x_2^{31} x_3^{8} x_4^{15} x_5^{7}$
135.  $x_1 x_2^{31} x_3^{9} x_4^{15} x_5^{15}$
136.  $x_1 x_2^{31} x_3^{9} x_4^{14} x_5^{14}$
137.  $x_1 x_2^{31} x_3^{9} x_4^{14} x_5^{7}$
138.  $x_1 x_2^{31} x_3^{9} x_4^{15} x_5^{6}$
139.  $x_1 x_2^{31} x_3^{46} x_4^{7} x_5^{6}$
140.  $x_1 x_2^{31} x_3^{47} x_4^{7} x_5^{6}$
141.  $x_1 x_2^{31} x_3^{47} x_4^{7} x_5^{6}$
142.  $x_1 x_2^{62} x_3^{7} x_4^{15} x_5^{7}$
143.  $x_1 x_2^{62} x_3^{15} x_4^{15} x_5^{7}$
144.  $x_1 x_2^{62} x_3^{5} x_4^{7} x_5^{7}$

145.  $x_1^3 x_2^5 x_3^6 x_4^3 x_5^4 x_7^{47}$   
 148.  $x_1^3 x_2^5 x_3^7 x_4^3 x_5^4 x_7^{46}$   
 151.  $x_1^3 x_2^5 x_3^{14} x_4^{23} x_5^{47}$   
 154.  $x_1^3 x_2^5 x_3^{15} x_4^7 x_5^{62}$   
 157.  $x_1^3 x_2^5 x_3^{15} x_4^{22} x_5^{47}$   
 160.  $x_1^3 x_2^5 x_3^{15} x_4^{31} x_5^{38}$   
 163.  $x_1^3 x_2^5 x_3^{15} x_4^{62} x_7^{47}$   
 166.  $x_1^3 x_2^5 x_3^{30} x_4^{39} x_5^{15}$   
 169.  $x_1^3 x_2^5 x_3^{31} x_4^7 x_5^{46}$   
 172.  $x_1^3 x_2^5 x_3^{31} x_4^{38} x_5^{15}$   
 175.  $x_1^3 x_2^5 x_3^{31} x_4^{47} x_5^6$   
 178.  $x_1^3 x_2^7 x_3^5 x_4^{15} x_5^{62}$   
 181.  $x_1^3 x_2^7 x_3^5 x_4^{62} x_5^{15}$   
 184.  $x_1^3 x_2^7 x_3^5 x_4^{61} x_5^{14}$   
 187.  $x_1^3 x_2^7 x_3^{13} x_4^{15} x_5^{54}$   
 190.  $x_1^3 x_2^7 x_3^{13} x_4^{30} x_5^{39}$   
 193.  $x_1^3 x_2^7 x_3^{13} x_4^{55} x_5^{14}$   
 196.  $x_1^3 x_2^7 x_3^{15} x_4^{13} x_5^{54}$   
 199.  $x_1^3 x_2^7 x_3^{15} x_4^{53} x_5^{14}$   
 202.  $x_1^3 x_2^7 x_3^{29} x_4^7 x_5^{46}$   
 205.  $x_1^3 x_2^7 x_3^{29} x_4^{38} x_5^{15}$   
 208.  $x_1^3 x_2^7 x_3^{29} x_4^{47} x_5^6$   
 211.  $x_1^3 x_2^7 x_3^{31} x_4^{37} x_5^{14}$   
 214.  $x_1^3 x_2^7 x_3^{61} x_4^7 x_5^{14}$   
 217.  $x_1^3 x_2^{13} x_3^6 x_4^{15} x_5^{55}$   
 220.  $x_1^3 x_2^{13} x_3^6 x_4^{55} x_5^{15}$   
 223.  $x_1^3 x_2^{13} x_3^7 x_4^{15} x_5^{54}$   
 226.  $x_1^3 x_2^{13} x_3^7 x_4^{30} x_5^{39}$   
 229.  $x_1^3 x_2^{13} x_3^7 x_4^{55} x_5^{14}$   
 232.  $x_1^3 x_2^{13} x_3^{14} x_4^{23} x_5^{39}$   
 235.  $x_1^3 x_2^{13} x_3^{15} x_4^7 x_5^{54}$   
 238.  $x_1^3 x_2^{13} x_3^{15} x_4^{54} x_7^5$   
 241.  $x_1^3 x_2^{13} x_3^{22} x_4^{15} x_5^{39}$   
 244.  $x_1^3 x_2^{13} x_3^{23} x_4^6 x_5^{47}$   
 247.  $x_1^3 x_2^{13} x_3^{23} x_4^{15} x_5^{38}$   
 250.  $x_1^3 x_2^{13} x_3^{23} x_4^{46} x_7^5$   
 253.  $x_1^3 x_2^{13} x_3^{30} x_4^{39} x_7^5$   
 256.  $x_1^3 x_2^{13} x_3^{31} x_4^{38} x_7^5$   
 259.  $x_1^3 x_2^{13} x_3^{34} x_4^{15} x_7^5$   
 262.  $x_1^3 x_2^{13} x_3^{55} x_4^{14} x_7^5$   
 265.  $x_1^3 x_2^{15} x_3^5 x_4^7 x_5^{62}$   
 268.  $x_1^3 x_2^{15} x_3^5 x_4^{22} x_5^{47}$   
 271.  $x_1^3 x_2^{15} x_3^5 x_4^{31} x_5^{38}$   
 146.  $x_1^3 x_2^5 x_3^7 x_4^{15} x_5^{62}$   
 149.  $x_1^3 x_2^5 x_3^7 x_4^{62} x_5^{15}$   
 152.  $x_1^3 x_2^5 x_3^{14} x_4^{31} x_5^{39}$   
 155.  $x_1^3 x_2^5 x_3^{15} x_4^{14} x_5^{55}$   
 158.  $x_1^3 x_2^5 x_3^{15} x_4^{23} x_5^{46}$   
 161.  $x_1^3 x_2^5 x_3^{15} x_4^{54} x_5^{15}$   
 164.  $x_1^3 x_2^5 x_3^{30} x_4^7 x_5^{47}$   
 167.  $x_1^3 x_2^5 x_3^{30} x_4^{47} x_7^5$   
 170.  $x_1^3 x_2^5 x_3^{31} x_4^{14} x_5^{39}$   
 173.  $x_1^3 x_2^5 x_3^{31} x_4^{39} x_5^{14}$   
 176.  $x_1^3 x_2^5 x_3^{62} x_4^7 x_5^{15}$   
 179.  $x_1^3 x_2^7 x_3^5 x_4^{30} x_5^{47}$   
 182.  $x_1^3 x_2^7 x_3^5 x_4^{13} x_5^{62}$   
 185.  $x_1^3 x_2^7 x_3^{13} x_4^7 x_5^{62}$   
 188.  $x_1^3 x_2^7 x_3^{13} x_4^{22} x_5^{47}$   
 191.  $x_1^3 x_2^7 x_3^{13} x_4^{31} x_5^{38}$   
 194.  $x_1^3 x_2^7 x_3^{13} x_4^{62} x_7^5$   
 197.  $x_1^3 x_2^7 x_3^{15} x_4^{21} x_5^{46}$   
 200.  $x_1^3 x_2^7 x_3^{15} x_4^{61} x_6^5$   
 203.  $x_1^3 x_2^7 x_3^{29} x_4^{14} x_5^{39}$   
 206.  $x_1^3 x_2^7 x_3^{29} x_4^{39} x_5^{14}$   
 209.  $x_1^3 x_2^7 x_3^{31} x_4^5 x_5^{46}$   
 212.  $x_1^3 x_2^7 x_3^{31} x_4^{45} x_5^6$   
 215.  $x_1^3 x_2^7 x_3^{61} x_4^{14} x_7^5$   
 218.  $x_1^3 x_2^{13} x_3^6 x_4^{23} x_5^{47}$   
 221.  $x_1^3 x_2^{13} x_3^7 x_4^{14} x_5^{62}$   
 224.  $x_1^3 x_2^{13} x_3^7 x_4^{22} x_5^{47}$   
 227.  $x_1^3 x_2^{13} x_3^7 x_4^{31} x_5^{38}$   
 230.  $x_1^3 x_2^{13} x_3^7 x_4^{62} x_7^5$   
 233.  $x_1^3 x_2^{13} x_3^{14} x_4^{55} x_7^5$   
 236.  $x_1^3 x_2^{13} x_3^{15} x_4^{22} x_5^{39}$   
 239.  $x_1^3 x_2^{13} x_3^{15} x_4^{55} x_5^6$   
 242.  $x_1^3 x_2^{13} x_3^{22} x_4^{39} x_5^{15}$   
 245.  $x_1^3 x_2^{13} x_3^{23} x_4^7 x_5^{46}$   
 248.  $x_1^3 x_2^{13} x_3^{23} x_4^{38} x_5^{15}$   
 251.  $x_1^3 x_2^{13} x_3^{23} x_4^{47} x_6^5$   
 254.  $x_1^3 x_2^{13} x_3^{31} x_4^6 x_5^{39}$   
 257.  $x_1^3 x_2^{13} x_3^{31} x_4^{39} x_5^6$   
 260.  $x_1^3 x_2^{13} x_3^{55} x_4^6 x_5^{15}$   
 263.  $x_1^3 x_2^{13} x_3^{55} x_4^{15} x_5^6$   
 266.  $x_1^3 x_2^{15} x_3^5 x_4^{14} x_5^{55}$   
 269.  $x_1^3 x_2^{15} x_3^5 x_4^{23} x_5^{46}$   
 272.  $x_1^3 x_2^{15} x_3^5 x_4^{54} x_5^{15}$   
 147.  $x_1^3 x_2^5 x_3^7 x_4^{30} x_5^{47}$   
 150.  $x_1^3 x_2^5 x_3^{14} x_4^{15} x_5^{55}$   
 153.  $x_1^3 x_2^5 x_3^{14} x_4^{55} x_5^{15}$   
 156.  $x_1^3 x_2^5 x_3^{15} x_4^{15} x_5^{54}$   
 159.  $x_1^3 x_2^5 x_3^{15} x_4^{30} x_5^{39}$   
 162.  $x_1^3 x_2^5 x_3^{15} x_4^{55} x_5^{14}$   
 165.  $x_1^3 x_2^5 x_3^{30} x_4^{15} x_5^{39}$   
 168.  $x_1^3 x_2^5 x_3^{31} x_4^6 x_5^{47}$   
 171.  $x_1^3 x_2^5 x_3^{31} x_4^{15} x_5^{38}$   
 174.  $x_1^3 x_2^5 x_3^{31} x_4^{46} x_7^5$   
 177.  $x_1^3 x_2^5 x_3^{62} x_4^{15} x_5^6$   
 180.  $x_1^3 x_2^7 x_3^5 x_4^{31} x_5^{46}$   
 183.  $x_1^3 x_2^7 x_3^5 x_4^{29} x_5^{46}$   
 186.  $x_1^3 x_2^7 x_3^{13} x_4^{14} x_5^{55}$   
 189.  $x_1^3 x_2^7 x_3^{13} x_4^{23} x_5^{46}$   
 192.  $x_1^3 x_2^7 x_3^{13} x_4^{54} x_5^{15}$   
 195.  $x_1^3 x_2^7 x_3^{15} x_4^5 x_5^{62}$   
 198.  $x_1^3 x_2^7 x_3^{15} x_4^{29} x_5^{38}$   
 201.  $x_1^3 x_2^7 x_3^{29} x_4^6 x_5^{47}$   
 204.  $x_1^3 x_2^7 x_3^{29} x_4^{15} x_5^{38}$   
 207.  $x_1^3 x_2^7 x_3^{29} x_4^{46} x_7^5$   
 210.  $x_1^3 x_2^7 x_3^{31} x_4^{13} x_5^{38}$   
 213.  $x_1^3 x_2^7 x_3^{61} x_4^6 x_5^{15}$   
 216.  $x_1^3 x_2^7 x_3^{61} x_4^{15} x_6^5$   
 219.  $x_1^3 x_2^{13} x_3^6 x_4^{31} x_5^{39}$   
 222.  $x_1^3 x_2^{13} x_3^7 x_4^{14} x_5^{55}$   
 225.  $x_1^3 x_2^{13} x_3^7 x_4^{23} x_5^{46}$   
 228.  $x_1^3 x_2^{13} x_3^7 x_4^{54} x_5^{15}$   
 231.  $x_1^3 x_2^{13} x_3^{14} x_4^7 x_5^{55}$   
 234.  $x_1^3 x_2^{13} x_3^{15} x_4^6 x_5^{55}$   
 237.  $x_1^3 x_2^{13} x_3^{15} x_4^{23} x_5^{38}$   
 240.  $x_1^3 x_2^{13} x_3^{22} x_4^7 x_5^{47}$   
 243.  $x_1^3 x_2^{13} x_3^{22} x_4^{47} x_7^5$   
 246.  $x_1^3 x_2^{13} x_3^{23} x_4^{14} x_5^{39}$   
 249.  $x_1^3 x_2^{13} x_3^{23} x_4^{39} x_5^{14}$   
 252.  $x_1^3 x_2^{13} x_3^{30} x_4^7 x_5^{39}$   
 255.  $x_1^3 x_2^{13} x_3^{31} x_4^7 x_5^{38}$   
 258.  $x_1^3 x_2^{13} x_3^{34} x_4^7 x_5^{15}$   
 261.  $x_1^3 x_2^{13} x_3^{55} x_4^7 x_5^{14}$   
 264.  $x_1^3 x_2^{13} x_3^{62} x_4^7 x_5^6$   
 267.  $x_1^3 x_2^{15} x_3^5 x_4^{15} x_5^{54}$   
 270.  $x_1^3 x_2^{15} x_3^5 x_4^{30} x_5^{39}$   
 273.  $x_1^3 x_2^{15} x_3^5 x_4^{55} x_5^{14}$

- 274.  $x_1^3 x_2^{15} x_3^5 x_4^{62} x_5^7$
- 277.  $x_1^3 x_2^{15} x_3^7 x_4^{21} x_5^{46}$
- 280.  $x_1^3 x_2^{15} x_3^7 x_4^{61} x_5^6$
- 283.  $x_1^3 x_2^{15} x_3^{13} x_4^{22} x_5^{39}$
- 286.  $x_1^3 x_2^{15} x_3^{13} x_4^{55} x_5^6$
- 289.  $x_1^3 x_2^{15} x_3^{15} x_4^{53} x_5^6$
- 292.  $x_1^3 x_2^{15} x_3^{21} x_4^{14} x_5^{39}$
- 295.  $x_1^3 x_2^{15} x_3^{21} x_4^{39} x_5^{14}$
- 298.  $x_1^3 x_2^{15} x_3^{23} x_4^5 x_5^{46}$
- 301.  $x_1^3 x_2^{15} x_3^{23} x_4^{45} x_5^6$
- 304.  $x_1^3 x_2^{15} x_3^{29} x_4^{38} x_5^7$
- 307.  $x_1^3 x_2^{15} x_3^{31} x_4^7 x_5^6$
- 310.  $x_1^3 x_2^{15} x_3^{53} x_4^{14} x_5^7$
- 313.  $x_1^3 x_2^{15} x_3^{55} x_4^{13} x_5^6$
- 316.  $x_1^3 x_2^{29} x_3^6 x_4^7 x_5^{47}$
- 319.  $x_1^3 x_2^{29} x_3^6 x_4^{47} x_5^7$
- 322.  $x_1^3 x_2^{29} x_3^7 x_4^{14} x_5^{39}$
- 325.  $x_1^3 x_2^{29} x_3^7 x_4^{39} x_5^{14}$
- 328.  $x_1^3 x_2^{29} x_3^{14} x_4^7 x_5^{39}$
- 331.  $x_1^3 x_2^{29} x_3^{15} x_4^7 x_5^{38}$
- 334.  $x_1^3 x_2^{29} x_3^{38} x_4^7 x_5^{15}$
- 337.  $x_1^3 x_2^{29} x_3^{39} x_4^7 x_5^{14}$
- 340.  $x_1^3 x_2^{29} x_3^{46} x_4^7 x_5^6$
- 343.  $x_1^3 x_2^{31} x_3^6 x_4^7 x_5^{47}$
- 346.  $x_1^3 x_2^{31} x_3^6 x_4^{15} x_5^{38}$
- 349.  $x_1^3 x_2^{31} x_3^6 x_4^{46} x_5^7$
- 352.  $x_1^3 x_2^{31} x_3^{13} x_4^{13} x_5^{38}$
- 355.  $x_1^3 x_2^{31} x_3^{13} x_4^6 x_5^{39}$
- 358.  $x_1^3 x_2^{31} x_3^{13} x_4^{39} x_5^6$
- 361.  $x_1^3 x_2^{31} x_3^{37} x_4^6 x_5^{15}$
- 364.  $x_1^3 x_2^{31} x_3^{37} x_4^{15} x_5^6$
- 367.  $x_1^3 x_2^{31} x_3^{45} x_4^6 x_5^7$
- 370.  $x_1^3 x_2^{61} x_3^6 x_4^7 x_5^{15}$
- 373.  $x_1^3 x_2^{61} x_3^7 x_4^7 x_5^{14}$
- 376.  $x_1^3 x_2^{61} x_3^{14} x_4^7 x_5^7$
- 379.  $x_1^7 x_2 x_3^6 x_4^{31} x_5^{47}$
- 382.  $x_1^7 x_2 x_3^6 x_4^{31} x_5^{46}$
- 385.  $x_1^7 x_2 x_3^{14} x_4^{23} x_5^{47}$
- 388.  $x_1^7 x_2 x_3^{15} x_4^7 x_5^{62}$
- 391.  $x_1^7 x_2 x_3^{15} x_4^{22} x_5^{47}$
- 394.  $x_1^7 x_2 x_3^{15} x_4^{31} x_5^{38}$
- 397.  $x_1^7 x_2 x_3^{15} x_4^{62} x_5^5$
- 400.  $x_1^7 x_2 x_3^{30} x_4^{39} x_5^{15}$
- 275.  $x_1^3 x_2^{15} x_3^7 x_4^5 x_5^{62}$
- 278.  $x_1^3 x_2^{15} x_3^7 x_4^{29} x_5^{38}$
- 281.  $x_1^3 x_2^{15} x_3^{13} x_4^6 x_5^{55}$
- 284.  $x_1^3 x_2^{15} x_3^{13} x_4^{23} x_5^{38}$
- 287.  $x_1^3 x_2^{15} x_3^{15} x_4^5 x_5^{54}$
- 290.  $x_1^3 x_2^{15} x_3^{21} x_4^6 x_5^{47}$
- 293.  $x_1^3 x_2^{15} x_3^{21} x_4^{15} x_5^{38}$
- 296.  $x_1^3 x_2^{15} x_3^{21} x_4^{46} x_5^7$
- 299.  $x_1^3 x_2^{15} x_3^{23} x_4^{13} x_5^{38}$
- 302.  $x_1^3 x_2^{15} x_3^{29} x_4^6 x_5^{39}$
- 305.  $x_1^3 x_2^{15} x_3^{29} x_4^{39} x_5^6$
- 308.  $x_1^3 x_2^{15} x_3^{53} x_4^6 x_5^{15}$
- 311.  $x_1^3 x_2^{15} x_3^{53} x_4^{15} x_5^6$
- 314.  $x_1^3 x_2^{15} x_3^{61} x_4^6 x_5^7$
- 317.  $x_1^3 x_2^{29} x_3^6 x_4^{15} x_5^{39}$
- 320.  $x_1^3 x_2^{29} x_3^7 x_4^6 x_5^{47}$
- 323.  $x_1^3 x_2^{29} x_3^7 x_4^{15} x_5^{38}$
- 326.  $x_1^3 x_2^{29} x_3^7 x_4^{46} x_5^7$
- 329.  $x_1^3 x_2^{29} x_3^{14} x_4^{39} x_5^7$
- 332.  $x_1^3 x_2^{29} x_3^{15} x_4^{38} x_5^7$
- 335.  $x_1^3 x_2^{29} x_3^{38} x_4^{15} x_5^7$
- 338.  $x_1^3 x_2^{29} x_3^{39} x_4^{14} x_5^7$
- 341.  $x_1^3 x_2^{29} x_3^{47} x_4^6 x_5^7$
- 344.  $x_1^3 x_2^{31} x_3^5 x_4^7 x_5^{46}$
- 347.  $x_1^3 x_2^{31} x_3^5 x_4^{38} x_5^{15}$
- 350.  $x_1^3 x_2^{31} x_3^5 x_4^{47} x_5^6$
- 353.  $x_1^3 x_2^{31} x_3^7 x_4^7 x_5^{14}$
- 356.  $x_1^3 x_2^{31} x_3^{13} x_4^7 x_5^{38}$
- 359.  $x_1^3 x_2^{31} x_3^{15} x_4^7 x_5^{38}$
- 362.  $x_1^3 x_2^{31} x_3^{37} x_4^7 x_5^{14}$
- 365.  $x_1^3 x_2^{31} x_3^{39} x_4^7 x_5^{14}$
- 368.  $x_1^3 x_2^{31} x_3^{45} x_4^7 x_5^6$
- 371.  $x_1^3 x_2^{61} x_3^6 x_4^{15} x_5^7$
- 374.  $x_1^3 x_2^{61} x_3^7 x_4^{14} x_5^7$
- 377.  $x_1^3 x_2^{61} x_3^{15} x_4^6 x_5^7$
- 380.  $x_1^7 x_2 x_3^7 x_4^{15} x_5^{62}$
- 383.  $x_1^7 x_2 x_3^7 x_4^{62} x_5^{15}$
- 386.  $x_1^7 x_2 x_3^{14} x_4^{31} x_5^{39}$
- 389.  $x_1^7 x_2 x_3^{15} x_4^{14} x_5^{55}$
- 392.  $x_1^7 x_2 x_3^{15} x_4^{23} x_5^{46}$
- 395.  $x_1^7 x_2 x_3^{15} x_4^{54} x_5^{15}$
- 398.  $x_1^7 x_2 x_3^{30} x_4^7 x_5^{47}$
- 401.  $x_1^7 x_2 x_3^{30} x_4^{47} x_5^7$
- 276.  $x_1^3 x_2^{15} x_3^7 x_4^{13} x_5^{54}$
- 279.  $x_1^3 x_2^{15} x_3^7 x_4^{53} x_5^{14}$
- 282.  $x_1^3 x_2^{15} x_3^{13} x_4^7 x_5^{54}$
- 285.  $x_1^3 x_2^{15} x_3^{13} x_4^{54} x_5^7$
- 288.  $x_1^3 x_2^{15} x_3^{15} x_4^{21} x_5^{38}$
- 291.  $x_1^3 x_2^{15} x_3^{21} x_4^7 x_5^{46}$
- 294.  $x_1^3 x_2^{15} x_3^{21} x_4^{38} x_5^{15}$
- 297.  $x_1^3 x_2^{15} x_3^{21} x_4^{47} x_5^6$
- 300.  $x_1^3 x_2^{15} x_3^{23} x_4^{37} x_5^{14}$
- 303.  $x_1^3 x_2^{15} x_3^{29} x_4^7 x_5^{38}$
- 306.  $x_1^3 x_2^{15} x_3^{31} x_4^5 x_5^{38}$
- 309.  $x_1^3 x_2^{15} x_3^{53} x_4^7 x_5^{14}$
- 312.  $x_1^3 x_2^{15} x_3^{55} x_4^5 x_5^{14}$
- 315.  $x_1^3 x_2^{15} x_3^{61} x_4^7 x_5^6$
- 318.  $x_1^3 x_2^{29} x_3^6 x_4^{39} x_5^{15}$
- 321.  $x_1^3 x_2^{29} x_3^7 x_4^7 x_5^{46}$
- 324.  $x_1^3 x_2^{29} x_3^7 x_4^{38} x_5^{15}$
- 327.  $x_1^3 x_2^{29} x_3^7 x_4^{47} x_5^6$
- 330.  $x_1^3 x_2^{29} x_3^{15} x_4^6 x_5^{39}$
- 333.  $x_1^3 x_2^{29} x_3^{15} x_4^{39} x_5^6$
- 336.  $x_1^3 x_2^{29} x_3^{39} x_4^6 x_5^{15}$
- 339.  $x_1^3 x_2^{29} x_3^{39} x_4^{15} x_5^6$
- 342.  $x_1^3 x_2^{29} x_3^{47} x_4^7 x_5^6$
- 345.  $x_1^3 x_2^{31} x_3^5 x_4^{14} x_5^{39}$
- 348.  $x_1^3 x_2^{31} x_3^5 x_4^{39} x_5^{14}$
- 351.  $x_1^3 x_2^{31} x_3^5 x_4^{54} x_5^{46}$
- 354.  $x_1^3 x_2^{31} x_3^7 x_4^{45} x_5^6$
- 357.  $x_1^3 x_2^{31} x_3^{13} x_4^{38} x_5^7$
- 360.  $x_1^3 x_2^{31} x_3^{15} x_4^{37} x_5^6$
- 363.  $x_1^3 x_2^{31} x_3^{37} x_4^{14} x_5^7$
- 366.  $x_1^3 x_2^{31} x_3^{39} x_4^{13} x_5^6$
- 369.  $x_1^3 x_2^{31} x_3^{47} x_4^5 x_5^6$
- 372.  $x_1^3 x_2^{61} x_3^6 x_4^{15} x_5^{15}$
- 375.  $x_1^3 x_2^{61} x_3^7 x_4^{15} x_5^6$
- 378.  $x_1^3 x_2^{61} x_3^{15} x_4^7 x_5^6$
- 381.  $x_1^7 x_2 x_3^7 x_4^{30} x_5^{47}$
- 384.  $x_1^7 x_2 x_3^{14} x_4^{15} x_5^{55}$
- 387.  $x_1^7 x_2 x_3^{14} x_4^{55} x_5^{15}$
- 390.  $x_1^7 x_2 x_3^{15} x_4^{15} x_5^{54}$
- 393.  $x_1^7 x_2 x_3^{15} x_4^{30} x_5^{39}$
- 396.  $x_1^7 x_2 x_3^{15} x_4^{55} x_5^{14}$
- 399.  $x_1^7 x_2 x_3^{30} x_4^{15} x_5^{39}$
- 402.  $x_1^7 x_2 x_3^{31} x_4^6 x_5^{47}$

403.  $x_1^7 x_2 x_3^3 x_4^7 x_5^{46}$     404.  $x_1^7 x_2 x_3^3 x_4^{14} x_5^{39}$     405.  $x_1^7 x_2 x_3^3 x_4^{15} x_5^{38}$   
 406.  $x_1^7 x_2 x_3^3 x_4^{38} x_5^{15}$     407.  $x_1^7 x_2 x_3^3 x_4^{39} x_5^{14}$     408.  $x_1^7 x_2 x_3^3 x_4^{46} x_5^7$   
 409.  $x_1^7 x_2 x_3^3 x_4^{47} x_5^6$     410.  $x_1^7 x_2 x_3^6 x_4^7 x_5^{15}$     411.  $x_1^7 x_2 x_3^6 x_4^{15} x_5^7$   
 412.  $x_1^7 x_2^3 x_3^4 x_4^{15} x_5^{62}$     413.  $x_1^7 x_2^3 x_3^4 x_4^{30} x_5^{47}$     414.  $x_1^7 x_2^3 x_3^4 x_4^{31} x_5^{46}$   
 415.  $x_1^7 x_2^3 x_3^4 x_4^{62} x_5^{15}$     416.  $x_1^7 x_2^3 x_3^7 x_4^{13} x_5^{62}$     417.  $x_1^7 x_2^3 x_3^7 x_4^{29} x_5^{46}$   
 418.  $x_1^7 x_2^3 x_3^7 x_4^{61} x_5^{14}$     419.  $x_1^7 x_2^3 x_3^{13} x_4^7 x_5^{62}$     420.  $x_1^7 x_2^3 x_3^{13} x_4^{14} x_5^{55}$   
 421.  $x_1^7 x_2^3 x_3^{13} x_4^{15} x_5^{54}$     422.  $x_1^7 x_2^3 x_3^{13} x_4^{22} x_5^{47}$     423.  $x_1^7 x_2^3 x_3^{13} x_4^{23} x_5^{46}$   
 424.  $x_1^7 x_2^3 x_3^{13} x_4^{30} x_5^{39}$     425.  $x_1^7 x_2^3 x_3^{13} x_4^{31} x_5^{38}$     426.  $x_1^7 x_2^3 x_3^{13} x_4^{54} x_5^{15}$   
 427.  $x_1^7 x_2^3 x_3^{13} x_4^{55} x_5^{14}$     428.  $x_1^7 x_2^3 x_3^{13} x_4^{62} x_5^7$     429.  $x_1^7 x_2^3 x_3^{15} x_4^5 x_5^{62}$   
 430.  $x_1^7 x_2^3 x_3^{15} x_4^{13} x_5^{54}$     431.  $x_1^7 x_2^3 x_3^{15} x_4^{21} x_5^{46}$     432.  $x_1^7 x_2^3 x_3^{15} x_4^{29} x_5^{38}$   
 433.  $x_1^7 x_2^3 x_3^{15} x_4^{53} x_5^{14}$     434.  $x_1^7 x_2^3 x_3^{15} x_4^{61} x_5^6$     435.  $x_1^7 x_2^3 x_3^{29} x_4^6 x_5^{47}$   
 436.  $x_1^7 x_2^3 x_3^{29} x_4^7 x_5^{46}$     437.  $x_1^7 x_2^3 x_3^{29} x_4^{14} x_5^{39}$     438.  $x_1^7 x_2^3 x_3^{29} x_4^{15} x_5^{38}$   
 439.  $x_1^7 x_2^3 x_3^{29} x_4^{38} x_5^{15}$     440.  $x_1^7 x_2^3 x_3^{29} x_4^{39} x_5^{14}$     441.  $x_1^7 x_2^3 x_3^{29} x_4^{46} x_5^7$   
 442.  $x_1^7 x_2^3 x_3^{29} x_4^{47} x_5^6$     443.  $x_1^7 x_2^3 x_3^{31} x_4^5 x_5^{46}$     444.  $x_1^7 x_2^3 x_3^{31} x_4^{13} x_5^{38}$   
 445.  $x_1^7 x_2^3 x_3^{31} x_4^{37} x_5^{14}$     446.  $x_1^7 x_2^3 x_3^{31} x_4^{45} x_5^6$     447.  $x_1^7 x_2^3 x_3^{61} x_4^6 x_5^{15}$   
 448.  $x_1^7 x_2^3 x_3^{61} x_4^7 x_5^{14}$     449.  $x_1^7 x_2^3 x_3^{61} x_4^{14} x_5^7$     450.  $x_1^7 x_2^3 x_3^{61} x_4^{15} x_5^6$   
 451.  $x_1^7 x_2^3 x_3^4 x_4^{15} x_5^{62}$     452.  $x_1^7 x_2^3 x_3^4 x_4^{30} x_5^{47}$     453.  $x_1^7 x_2^3 x_3^4 x_4^{31} x_5^{46}$   
 454.  $x_1^7 x_2^3 x_3^4 x_4^{62} x_5^{15}$     455.  $x_1^7 x_2^3 x_3^4 x_4^{13} x_5^{62}$     456.  $x_1^7 x_2^3 x_3^4 x_4^{29} x_5^{46}$   
 457.  $x_1^7 x_2^3 x_3^4 x_4^{61} x_5^{14}$     458.  $x_1^7 x_2^3 x_3^4 x_4^9 x_5^{62}$     459.  $x_1^7 x_2^3 x_3^4 x_4^{15} x_5^{56}$   
 460.  $x_1^7 x_2^3 x_3^4 x_4^{24} x_5^{47}$     461.  $x_1^7 x_2^3 x_3^4 x_4^{25} x_5^{46}$     462.  $x_1^7 x_2^3 x_3^4 x_4^{31} x_5^{40}$   
 463.  $x_1^7 x_2^3 x_3^4 x_4^{56} x_5^{15}$     464.  $x_1^7 x_2^3 x_3^4 x_4^{57} x_5^{14}$     465.  $x_1^7 x_2^3 x_3^4 x_4^{62} x_5^7$   
 466.  $x_1^7 x_2^3 x_3^4 x_4^{14} x_5^{55}$     467.  $x_1^7 x_2^3 x_3^4 x_4^{15} x_5^{54}$     468.  $x_1^7 x_2^3 x_3^4 x_4^{22} x_5^{47}$   
 469.  $x_1^7 x_2^3 x_3^4 x_4^{23} x_5^{46}$     470.  $x_1^7 x_2^3 x_3^4 x_4^{30} x_5^{39}$     471.  $x_1^7 x_2^3 x_3^4 x_4^{31} x_5^{38}$   
 472.  $x_1^7 x_2^3 x_3^4 x_4^{54} x_5^{15}$     473.  $x_1^7 x_2^3 x_3^4 x_4^{55} x_5^{14}$     474.  $x_1^7 x_2^3 x_3^4 x_4^{62} x_5^7$   
 475.  $x_1^7 x_2^3 x_3^{11} x_4^5 x_5^{62}$     476.  $x_1^7 x_2^3 x_3^{11} x_4^{13} x_5^{54}$     477.  $x_1^7 x_2^3 x_3^{11} x_4^{21} x_5^{46}$   
 478.  $x_1^7 x_2^3 x_3^{11} x_4^{29} x_5^{38}$     479.  $x_1^7 x_2^3 x_3^{11} x_4^{53} x_5^{14}$     480.  $x_1^7 x_2^3 x_3^{11} x_4^{61} x_5^6$   
 481.  $x_1^7 x_2^3 x_3^{15} x_4^5 x_5^{62}$     482.  $x_1^7 x_2^3 x_3^{15} x_4^7 x_5^{56}$     483.  $x_1^7 x_2^3 x_3^{15} x_4^9 x_5^{54}$   
 484.  $x_1^7 x_2^3 x_3^{15} x_4^{15} x_5^{48}$     485.  $x_1^7 x_2^3 x_3^{15} x_4^{17} x_5^{46}$     486.  $x_1^7 x_2^3 x_3^{15} x_4^{23} x_5^{40}$   
 487.  $x_1^7 x_2^3 x_3^{15} x_4^{25} x_5^{38}$     488.  $x_1^7 x_2^3 x_3^{15} x_4^{49} x_5^{14}$     489.  $x_1^7 x_2^3 x_3^{15} x_4^{55} x_5^8$   
 490.  $x_1^7 x_2^3 x_3^{15} x_4^{57} x_5^6$     491.  $x_1^7 x_2^3 x_3^{25} x_4^6 x_5^{47}$     492.  $x_1^7 x_2^3 x_3^{25} x_4^7 x_5^{46}$   
 493.  $x_1^7 x_2^3 x_3^{25} x_4^{14} x_5^{39}$     494.  $x_1^7 x_2^3 x_3^{25} x_4^{15} x_5^{38}$     495.  $x_1^7 x_2^3 x_3^{25} x_4^{38} x_5^{15}$   
 496.  $x_1^7 x_2^3 x_3^{25} x_4^{39} x_5^{14}$     497.  $x_1^7 x_2^3 x_3^{25} x_4^{46} x_5^7$     498.  $x_1^7 x_2^3 x_3^{25} x_4^{47} x_5^6$   
 499.  $x_1^7 x_2^3 x_3^{27} x_4^5 x_5^{46}$     500.  $x_1^7 x_2^3 x_3^{27} x_4^{13} x_5^{38}$     501.  $x_1^7 x_2^3 x_3^{27} x_4^{37} x_5^{14}$   
 502.  $x_1^7 x_2^3 x_3^{27} x_4^{45} x_5^6$     503.  $x_1^7 x_2^3 x_3^{31} x_4^4 x_5^{46}$     504.  $x_1^7 x_2^3 x_3^{31} x_4^7 x_5^{40}$   
 505.  $x_1^7 x_2^3 x_3^{31} x_4^9 x_5^{38}$     506.  $x_1^7 x_2^3 x_3^{31} x_4^{33} x_5^{14}$     507.  $x_1^7 x_2^3 x_3^{31} x_4^{41} x_5^6$   
 508.  $x_1^7 x_2^3 x_3^{37} x_4^6 x_5^{15}$     509.  $x_1^7 x_2^3 x_3^{37} x_4^7 x_5^{14}$     510.  $x_1^7 x_2^3 x_3^{37} x_4^{14} x_5^7$   
 511.  $x_1^7 x_2^3 x_3^{37} x_4^{15} x_5^6$     512.  $x_1^7 x_2^3 x_3^{39} x_4^5 x_5^{14}$     513.  $x_1^7 x_2^3 x_3^{39} x_4^{13} x_5^6$   
 514.  $x_1^7 x_2^{11} x_3^5 x_4^6 x_5^{62}$     515.  $x_1^7 x_2^{11} x_3^5 x_4^{14} x_5^{55}$     516.  $x_1^7 x_2^{11} x_3^5 x_4^{15} x_5^{54}$   
 517.  $x_1^7 x_2^{11} x_3^5 x_4^{22} x_5^{47}$     518.  $x_1^7 x_2^{11} x_3^5 x_4^{23} x_5^{46}$     519.  $x_1^7 x_2^{11} x_3^5 x_4^{30} x_5^{39}$   
 520.  $x_1^7 x_2^{11} x_3^5 x_4^{31} x_5^{38}$     521.  $x_1^7 x_2^{11} x_3^5 x_4^{54} x_5^{15}$     522.  $x_1^7 x_2^{11} x_3^5 x_4^{55} x_5^{14}$   
 523.  $x_1^7 x_2^{11} x_3^5 x_4^{62} x_5^7$     524.  $x_1^7 x_2^{11} x_3^5 x_4^5 x_5^{62}$     525.  $x_1^7 x_2^{11} x_3^5 x_4^{13} x_5^{54}$   
 526.  $x_1^7 x_2^{11} x_3^5 x_4^{21} x_5^{46}$     527.  $x_1^7 x_2^{11} x_3^5 x_4^{29} x_5^{38}$     528.  $x_1^7 x_2^{11} x_3^5 x_4^{53} x_5^{14}$   
 529.  $x_1^7 x_2^{11} x_3^5 x_4^{61} x_5^6$     530.  $x_1^7 x_2^{11} x_3^5 x_4^3 x_5^{55}$     531.  $x_1^7 x_2^{11} x_3^5 x_4^3 x_5^{54}$

532.  $x_1^7 x_2^{11} x_3^{13} x_4^{22} x_5^{39}$  533.  $x_1^7 x_2^{11} x_3^{13} x_4^{23} x_5^{38}$  534.  $x_1^7 x_2^{11} x_3^{13} x_4^{54} x_5^7$   
 535.  $x_1^7 x_2^{11} x_3^{13} x_4^{55} x_5^6$  536.  $x_1^7 x_2^{11} x_3^{15} x_4^5 x_5^{54}$  537.  $x_1^7 x_2^{11} x_3^{15} x_4^{21} x_5^{38}$   
 538.  $x_1^7 x_2^{11} x_3^{15} x_4^5 x_5^6$  539.  $x_1^7 x_2^{11} x_3^{21} x_4^6 x_5^{47}$  540.  $x_1^7 x_2^{11} x_3^{21} x_4^7 x_5^{46}$   
 541.  $x_1^7 x_2^{11} x_3^{21} x_4^{14} x_5^{39}$  542.  $x_1^7 x_2^{11} x_3^{21} x_4^{15} x_5^{38}$  543.  $x_1^7 x_2^{11} x_3^{21} x_4^7 x_5^{15}$   
 544.  $x_1^7 x_2^{11} x_3^{21} x_4^{39} x_5^{14}$  545.  $x_1^7 x_2^{11} x_3^{21} x_4^{46} x_5^7$  546.  $x_1^7 x_2^{11} x_3^{21} x_4^{47} x_5^6$   
 547.  $x_1^7 x_2^{11} x_3^{23} x_4^5 x_5^{46}$  548.  $x_1^7 x_2^{11} x_3^{23} x_4^{13} x_5^{38}$  549.  $x_1^7 x_2^{11} x_3^{23} x_4^{37} x_5^{14}$   
 550.  $x_1^7 x_2^{11} x_3^{23} x_4^{45} x_5^6$  551.  $x_1^7 x_2^{11} x_3^{29} x_4^6 x_5^{39}$  552.  $x_1^7 x_2^{11} x_3^{29} x_4^7 x_5^{38}$   
 553.  $x_1^7 x_2^{11} x_3^{29} x_4^8 x_5^7$  554.  $x_1^7 x_2^{11} x_3^{29} x_4^{39} x_5^6$  555.  $x_1^7 x_2^{11} x_3^{31} x_4^5 x_5^{38}$   
 556.  $x_1^7 x_2^{11} x_3^{31} x_4^7 x_5^6$  557.  $x_1^7 x_2^{11} x_3^{53} x_4^6 x_5^{15}$  558.  $x_1^7 x_2^{11} x_3^{53} x_4^7 x_5^{14}$   
 559.  $x_1^7 x_2^{11} x_3^{53} x_4^{14} x_5^7$  560.  $x_1^7 x_2^{11} x_3^{53} x_4^{15} x_5^6$  561.  $x_1^7 x_2^{11} x_3^{55} x_4^5 x_5^{14}$   
 562.  $x_1^7 x_2^{11} x_3^{55} x_4^{13} x_5^6$  563.  $x_1^7 x_2^{11} x_3^{61} x_4^6 x_5^7$  564.  $x_1^7 x_2^{11} x_3^{61} x_4^7 x_5^6$   
 565.  $x_1^7 x_2^{15} x_3 x_4^7 x_5^{62}$  566.  $x_1^7 x_2^{15} x_3 x_4^{14} x_5^{55}$  567.  $x_1^7 x_2^{15} x_3 x_4^{15} x_5^{54}$   
 568.  $x_1^7 x_2^{15} x_3 x_4^{22} x_5^{47}$  569.  $x_1^7 x_2^{15} x_3 x_4^{23} x_5^{46}$  570.  $x_1^7 x_2^{15} x_3 x_4^{30} x_5^{39}$   
 571.  $x_1^7 x_2^{15} x_3 x_4^{31} x_5^{38}$  572.  $x_1^7 x_2^{15} x_3 x_4^{54} x_5^{15}$  573.  $x_1^7 x_2^{15} x_3 x_4^{55} x_5^{14}$   
 574.  $x_1^7 x_2^{15} x_3 x_4^{62} x_5^7$  575.  $x_1^7 x_2^{15} x_3 x_4^{62} x_5^6$  576.  $x_1^7 x_2^{15} x_3 x_4^{13} x_5^{54}$   
 577.  $x_1^7 x_2^{15} x_3^2 x_4^{21} x_5^{46}$  578.  $x_1^7 x_2^{15} x_3^2 x_4^{29} x_5^{38}$  579.  $x_1^7 x_2^{15} x_3^2 x_4^{53} x_5^{14}$   
 580.  $x_1^7 x_2^{15} x_3^2 x_4^{61} x_5^6$  581.  $x_1^7 x_2^{15} x_3^2 x_4^{62} x_5^5$  582.  $x_1^7 x_2^{15} x_3^2 x_4^7 x_5^{56}$   
 583.  $x_1^7 x_2^{15} x_3^2 x_4^9 x_5^{54}$  584.  $x_1^7 x_2^{15} x_3^2 x_4^{15} x_5^{48}$  585.  $x_1^7 x_2^{15} x_3^2 x_4^{17} x_5^{46}$   
 586.  $x_1^7 x_2^{15} x_3^2 x_4^{23} x_5^{40}$  587.  $x_1^7 x_2^{15} x_3^2 x_4^{25} x_5^{38}$  588.  $x_1^7 x_2^{15} x_3^2 x_4^{49} x_5^{14}$   
 589.  $x_1^7 x_2^{15} x_3^2 x_4^{55} x_5^8$  590.  $x_1^7 x_2^{15} x_3^2 x_4^{57} x_5^6$  591.  $x_1^7 x_2^{15} x_3^2 x_4^{58} x_5^{54}$   
 592.  $x_1^7 x_2^{15} x_3^{11} x_4^{21} x_5^{38}$  593.  $x_1^7 x_2^{15} x_3^{11} x_4^{53} x_5^6$  594.  $x_1^7 x_2^{15} x_3^{15} x_4^5 x_5^{54}$   
 595.  $x_1^7 x_2^{15} x_3^{15} x_4^7 x_5^{48}$  596.  $x_1^7 x_2^{15} x_3^{15} x_4^{16} x_5^{39}$  597.  $x_1^7 x_2^{15} x_3^{15} x_4^{17} x_5^{38}$   
 598.  $x_1^7 x_2^{15} x_3^{15} x_4^{23} x_5^{32}$  599.  $x_1^7 x_2^{15} x_3^{15} x_4^{48} x_5^7$  600.  $x_1^7 x_2^{15} x_3^{15} x_4^{49} x_5^6$   
 601.  $x_1^7 x_2^{15} x_3^{19} x_4^5 x_5^{46}$  602.  $x_1^7 x_2^{15} x_3^{19} x_4^{13} x_5^{38}$  603.  $x_1^7 x_2^{15} x_3^{19} x_4^{37} x_5^{14}$   
 604.  $x_1^7 x_2^{15} x_3^{19} x_4^{45} x_5^6$  605.  $x_1^7 x_2^{15} x_3^{23} x_4^5 x_5^{46}$  606.  $x_1^7 x_2^{15} x_3^{23} x_4^7 x_5^{40}$   
 607.  $x_1^7 x_2^{15} x_3^{23} x_4^9 x_5^{38}$  608.  $x_1^7 x_2^{15} x_3^{23} x_4^{33} x_5^{14}$  609.  $x_1^7 x_2^{15} x_3^{23} x_4^{41} x_5^6$   
 610.  $x_1^7 x_2^{15} x_3^{27} x_4^5 x_5^{38}$  611.  $x_1^7 x_2^{15} x_3^{27} x_4^{37} x_5^6$  612.  $x_1^7 x_2^{15} x_3^{31} x_4^5 x_5^{38}$   
 613.  $x_1^7 x_2^{15} x_3^{31} x_4^5 x_5^{14}$  614.  $x_1^7 x_2^{15} x_3^{31} x_4^{13} x_5^6$  615.  $x_1^7 x_2^{15} x_3^{31} x_4^5 x_5^{14}$   
 616.  $x_1^7 x_2^{15} x_3^{35} x_4^5 x_5^8$  617.  $x_1^7 x_2^{15} x_3^{35} x_4^9 x_5^6$  618.  $x_1^7 x_2^{15} x_3^{39} x_4^5 x_5^6$   
 619.  $x_1^7 x_2^{27} x_3^2 x_4^6 x_5^{47}$  620.  $x_1^7 x_2^{27} x_3^2 x_4^7 x_5^{46}$  621.  $x_1^7 x_2^{27} x_3^2 x_4^{14} x_5^{39}$   
 622.  $x_1^7 x_2^{27} x_3^2 x_4^{15} x_5^{38}$  623.  $x_1^7 x_2^{27} x_3^2 x_4^{38} x_5^{15}$  624.  $x_1^7 x_2^{27} x_3^2 x_4^{39} x_5^{14}$   
 625.  $x_1^7 x_2^{27} x_3^2 x_4^{46} x_5^7$  626.  $x_1^7 x_2^{27} x_3^2 x_4^{47} x_5^6$  627.  $x_1^7 x_2^{27} x_3^2 x_4^{54} x_5^{46}$   
 628.  $x_1^7 x_2^{27} x_3^2 x_4^{13} x_5^{38}$  629.  $x_1^7 x_2^{27} x_3^2 x_4^{37} x_5^{14}$  630.  $x_1^7 x_2^{27} x_3^2 x_4^{45} x_5^6$   
 631.  $x_1^7 x_2^{27} x_3^2 x_4^6 x_5^{39}$  632.  $x_1^7 x_2^{27} x_3^2 x_4^7 x_5^{38}$  633.  $x_1^7 x_2^{27} x_3^2 x_4^{38} x_5^7$   
 634.  $x_1^7 x_2^{27} x_3^2 x_4^{39} x_5^6$  635.  $x_1^7 x_2^{27} x_3^2 x_4^{15} x_5^{38}$  636.  $x_1^7 x_2^{27} x_3^2 x_4^{37} x_5^6$   
 637.  $x_1^7 x_2^{27} x_3^2 x_4^6 x_5^{15}$  638.  $x_1^7 x_2^{27} x_3^2 x_4^7 x_5^{14}$  639.  $x_1^7 x_2^{27} x_3^2 x_4^{14} x_5^7$   
 640.  $x_1^7 x_2^{27} x_3^2 x_4^{15} x_5^6$  641.  $x_1^7 x_2^{27} x_3^2 x_4^{39} x_5^{14}$  642.  $x_1^7 x_2^{27} x_3^2 x_4^{13} x_5^6$   
 643.  $x_1^7 x_2^{27} x_3^2 x_4^5 x_5^6$  644.  $x_1^7 x_2^{27} x_3^2 x_4^5 x_5^6$  645.  $x_1^7 x_2^{27} x_3^2 x_4^7 x_5^6$   
 646.  $x_1^7 x_2^{31} x_3 x_4^6 x_5^{47}$  647.  $x_1^7 x_2^{31} x_3 x_4^7 x_5^{46}$  648.  $x_1^7 x_2^{31} x_3 x_4^{14} x_5^{39}$   
 649.  $x_1^7 x_2^{31} x_3 x_4^{15} x_5^{38}$  650.  $x_1^7 x_2^{31} x_3 x_4^{38} x_5^{15}$  651.  $x_1^7 x_2^{31} x_3 x_4^{39} x_5^{14}$   
 652.  $x_1^7 x_2^{31} x_3 x_4^{46} x_5^7$  653.  $x_1^7 x_2^{31} x_3 x_4^{47} x_5^6$  654.  $x_1^7 x_2^{31} x_3 x_4^5 x_5^{46}$   
 655.  $x_1^7 x_2^{31} x_3 x_4^{13} x_5^{38}$  656.  $x_1^7 x_2^{31} x_3 x_4^{37} x_5^{14}$  657.  $x_1^7 x_2^{31} x_3 x_4^{45} x_5^6$   
 658.  $x_1^7 x_2^{31} x_3 x_4^{46} x_5^5$  659.  $x_1^7 x_2^{31} x_3 x_4^{40} x_5^5$  660.  $x_1^7 x_2^{31} x_3 x_4^9 x_5^{38}$

661.  $x_1^7 x_2^{31} x_3^7 x_4^{33} x_5^{14}$       662.  $x_1^7 x_2^{31} x_3^7 x_4^{41} x_5^6$       663.  $x_1^7 x_2^{31} x_3^{11} x_4^5 x_5^{38}$   
 664.  $x_1^7 x_2^{31} x_3^{11} x_4^{37} x_5^6$       665.  $x_1^7 x_2^{31} x_3^{15} x_4^5 x_5^{38}$       666.  $x_1^7 x_2^{31} x_3^{35} x_4^5 x_5^{14}$   
 667.  $x_1^7 x_2^{31} x_3^{35} x_4^{13} x_5^6$       668.  $x_1^7 x_2^{31} x_3^{39} x_4^5 x_5^{14}$       669.  $x_1^7 x_2^{31} x_3^{43} x_4^5 x_5^6$   
 670.  $x_1^7 x_2^{31} x_3^{47} x_4^5 x_5^6$       671.  $x_1^7 x_2^{59} x_3^5 x_4^6 x_5^{15}$       672.  $x_1^7 x_2^{59} x_3^5 x_4^7 x_5^{14}$   
 673.  $x_1^7 x_2^{59} x_3^5 x_4^{14} x_5^7$       674.  $x_1^7 x_2^{59} x_3^5 x_4^{15} x_5^6$       675.  $x_1^7 x_2^{59} x_3^5 x_4^5 x_5^{14}$   
 676.  $x_1^7 x_2^{59} x_3^5 x_4^{13} x_5^6$       677.  $x_1^7 x_2^{59} x_3^{13} x_4^6 x_5^7$       678.  $x_1^7 x_2^{59} x_3^{13} x_4^7 x_5^6$   
 679.  $x_1^7 x_2^{59} x_3^{15} x_4^5 x_5^6$       680.  $x_1^{15} x_2 x_3^6 x_4^{15} x_5^{55}$       681.  $x_1^{15} x_2 x_3^6 x_4^{23} x_5^{47}$   
 682.  $x_1^{15} x_2 x_3^6 x_4^{31} x_5^{39}$       683.  $x_1^{15} x_2 x_3^6 x_4^{55} x_5^{15}$       684.  $x_1^{15} x_2 x_3^7 x_4^7 x_5^{62}$   
 685.  $x_1^{15} x_2 x_3^7 x_4^{14} x_5^{55}$       686.  $x_1^{15} x_2 x_3^7 x_4^{15} x_5^{54}$       687.  $x_1^{15} x_2 x_3^7 x_4^{22} x_5^{47}$   
 688.  $x_1^{15} x_2 x_3^7 x_4^{23} x_5^{46}$       689.  $x_1^{15} x_2 x_3^7 x_4^{30} x_5^{39}$       690.  $x_1^{15} x_2 x_3^7 x_4^{31} x_5^{38}$   
 691.  $x_1^{15} x_2 x_3^7 x_4^{54} x_5^{15}$       692.  $x_1^{15} x_2 x_3^7 x_4^{55} x_5^{14}$       693.  $x_1^{15} x_2 x_3^7 x_4^{62} x_5^7$   
 694.  $x_1^{15} x_2 x_3^{14} x_4^7 x_5^{55}$       695.  $x_1^{15} x_2 x_3^{14} x_4^{23} x_5^{39}$       696.  $x_1^{15} x_2 x_3^{14} x_4^{55} x_5^7$   
 697.  $x_1^{15} x_2 x_3^{15} x_4^6 x_5^{55}$       698.  $x_1^{15} x_2 x_3^{15} x_4^5 x_5^{54}$       699.  $x_1^{15} x_2 x_3^{15} x_4^{22} x_5^{39}$   
 700.  $x_1^{15} x_2 x_3^{15} x_4^{23} x_5^{38}$       701.  $x_1^{15} x_2 x_3^{15} x_4^5 x_5^7$       702.  $x_1^{15} x_2 x_3^{15} x_4^{55} x_5^6$   
 703.  $x_1^{15} x_2 x_3^{22} x_4^7 x_5^{47}$       704.  $x_1^{15} x_2 x_3^{22} x_4^{15} x_5^{39}$       705.  $x_1^{15} x_2 x_3^{22} x_4^3 x_5^{15}$   
 706.  $x_1^{15} x_2 x_3^{22} x_4^7 x_5^6$       707.  $x_1^{15} x_2 x_3^{23} x_4^6 x_5^{47}$       708.  $x_1^{15} x_2 x_3^{23} x_4^7 x_5^{46}$   
 709.  $x_1^{15} x_2 x_3^{23} x_4^{14} x_5^{39}$       710.  $x_1^{15} x_2 x_3^{23} x_4^{15} x_5^{38}$       711.  $x_1^{15} x_2 x_3^{23} x_4^{38} x_5^{15}$   
 712.  $x_1^{15} x_2 x_3^{23} x_4^{39} x_5^{14}$       713.  $x_1^{15} x_2 x_3^{23} x_4^{46} x_5^7$       714.  $x_1^{15} x_2 x_3^{23} x_4^{47} x_5^6$   
 715.  $x_1^{15} x_2 x_3^{30} x_4^7 x_5^{39}$       716.  $x_1^{15} x_2 x_3^{30} x_4^{39} x_5^7$       717.  $x_1^{15} x_2 x_3^{31} x_4^6 x_5^{39}$   
 718.  $x_1^{15} x_2 x_3^{31} x_4^7 x_5^{38}$       719.  $x_1^{15} x_2 x_3^{31} x_4^{38} x_5^7$       720.  $x_1^{15} x_2 x_3^{31} x_4^{39} x_5^6$   
 721.  $x_1^{15} x_2 x_3^{34} x_4^7 x_5^{15}$       722.  $x_1^{15} x_2 x_3^{34} x_4^{15} x_5^7$       723.  $x_1^{15} x_2 x_3^{35} x_4^6 x_5^{15}$   
 724.  $x_1^{15} x_2 x_3^{35} x_4^7 x_5^{14}$       725.  $x_1^{15} x_2 x_3^{35} x_4^{14} x_5^7$       726.  $x_1^{15} x_2 x_3^{35} x_4^{15} x_5^6$   
 727.  $x_1^{15} x_2 x_3^{62} x_4^7 x_5^{14}$       728.  $x_1^{15} x_2^3 x_3^5 x_4^{62} x_5^6$       729.  $x_1^{15} x_2^3 x_3^5 x_4^{14} x_5^{55}$   
 730.  $x_1^{15} x_2^3 x_3^5 x_4^{15} x_5^{54}$       731.  $x_1^{15} x_2^3 x_3^5 x_4^{22} x_5^{47}$       732.  $x_1^{15} x_2^3 x_3^5 x_4^{23} x_5^{46}$   
 733.  $x_1^{15} x_2^3 x_3^5 x_4^{30} x_5^{39}$       734.  $x_1^{15} x_2^3 x_3^5 x_4^{31} x_5^{38}$       735.  $x_1^{15} x_2^3 x_3^5 x_4^{54} x_5^{15}$   
 736.  $x_1^{15} x_2^3 x_3^5 x_4^{55} x_5^{14}$       737.  $x_1^{15} x_2^3 x_3^5 x_4^{62} x_5^7$       738.  $x_1^{15} x_2^3 x_3^5 x_4^{62} x_5^6$   
 739.  $x_1^{15} x_2^3 x_3^7 x_4^{13} x_5^{54}$       740.  $x_1^{15} x_2^3 x_3^7 x_4^{21} x_5^{46}$       741.  $x_1^{15} x_2^3 x_3^7 x_4^{29} x_5^{38}$   
 742.  $x_1^{15} x_2^3 x_3^7 x_4^{53} x_5^{14}$       743.  $x_1^{15} x_2^3 x_3^7 x_4^{61} x_5^6$       744.  $x_1^{15} x_2^3 x_3^{13} x_4^6 x_5^{55}$   
 745.  $x_1^{15} x_2^3 x_3^{13} x_4^7 x_5^{54}$       746.  $x_1^{15} x_2^3 x_3^{13} x_4^{22} x_5^{39}$       747.  $x_1^{15} x_2^3 x_3^{13} x_4^{23} x_5^{38}$   
 748.  $x_1^{15} x_2^3 x_3^{13} x_4^{54} x_5^7$       749.  $x_1^{15} x_2^3 x_3^{13} x_4^{55} x_5^6$       750.  $x_1^{15} x_2^3 x_3^{15} x_4^5 x_5^{54}$   
 751.  $x_1^{15} x_2^3 x_3^{15} x_4^{21} x_5^{38}$       752.  $x_1^{15} x_2^3 x_3^{15} x_4^{53} x_5^6$       753.  $x_1^{15} x_2^3 x_3^{21} x_4^6 x_5^{47}$   
 754.  $x_1^{15} x_2^3 x_3^{21} x_4^7 x_5^{46}$       755.  $x_1^{15} x_2^3 x_3^{21} x_4^{14} x_5^{39}$       756.  $x_1^{15} x_2^3 x_3^{21} x_4^{15} x_5^{38}$   
 757.  $x_1^{15} x_2^3 x_3^{21} x_4^{38} x_5^{15}$       758.  $x_1^{15} x_2^3 x_3^{21} x_4^{39} x_5^{14}$       759.  $x_1^{15} x_2^3 x_3^{21} x_4^{46} x_5^7$   
 760.  $x_1^{15} x_2^3 x_3^{21} x_4^7 x_5^6$       761.  $x_1^{15} x_2^3 x_3^{23} x_4^5 x_5^{46}$       762.  $x_1^{15} x_2^3 x_3^{23} x_4^{13} x_5^{38}$   
 763.  $x_1^{15} x_2^3 x_3^{23} x_4^{37} x_5^{14}$       764.  $x_1^{15} x_2^3 x_3^{23} x_4^{45} x_5^6$       765.  $x_1^{15} x_2^3 x_3^{29} x_4^6 x_5^{39}$   
 766.  $x_1^{15} x_2^3 x_3^{29} x_4^7 x_5^{38}$       767.  $x_1^{15} x_2^3 x_3^{29} x_4^{38} x_5^7$       768.  $x_1^{15} x_2^3 x_3^{29} x_4^{39} x_5^6$   
 769.  $x_1^{15} x_2^3 x_3^{31} x_4^5 x_5^{38}$       770.  $x_1^{15} x_2^3 x_3^{31} x_4^{37} x_5^6$       771.  $x_1^{15} x_2^3 x_3^{33} x_4^6 x_5^{15}$   
 772.  $x_1^{15} x_2^3 x_3^{33} x_4^7 x_5^{14}$       773.  $x_1^{15} x_2^3 x_3^{33} x_4^{14} x_5^7$       774.  $x_1^{15} x_2^3 x_3^{33} x_4^{15} x_5^6$   
 775.  $x_1^{15} x_2^3 x_3^{35} x_4^5 x_5^{14}$       776.  $x_1^{15} x_2^3 x_3^{35} x_4^{13} x_5^6$       777.  $x_1^{15} x_2^3 x_3^{61} x_4^6 x_5^7$   
 778.  $x_1^{15} x_2^3 x_3^{61} x_4^7 x_5^6$       779.  $x_1^{15} x_2^7 x_3 x_4^7 x_5^{62}$       780.  $x_1^{15} x_2^7 x_3 x_4^{14} x_5^{55}$   
 781.  $x_1^{15} x_2^7 x_3 x_4^{15} x_5^{54}$       782.  $x_1^{15} x_2^7 x_3 x_4^{22} x_5^{47}$       783.  $x_1^{15} x_2^7 x_3 x_4^{23} x_5^{46}$   
 784.  $x_1^{15} x_2^7 x_3 x_4^{30} x_5^{39}$       785.  $x_1^{15} x_2^7 x_3 x_4^{31} x_5^{38}$       786.  $x_1^{15} x_2^7 x_3 x_4^{34} x_5^{15}$   
 787.  $x_1^{15} x_2^7 x_3 x_4^{55} x_5^{14}$       788.  $x_1^{15} x_2^7 x_3 x_4^{62} x_5^7$       789.  $x_1^{15} x_2^7 x_3 x_4^{62} x_5^6$

- 790.  $x_1^{15} x_2^7 x_3^3 x_4^{13} x_5^{54}$
- 793.  $x_1^{15} x_2^7 x_3^3 x_4^{53} x_5^{14}$
- 796.  $x_1^{15} x_2^7 x_3^7 x_4^5 x_5^{56}$
- 799.  $x_1^{15} x_2^7 x_3^7 x_4^{17} x_5^{46}$
- 802.  $x_1^{15} x_2^7 x_3^7 x_4^{49} x_5^{14}$
- 805.  $x_1^{15} x_2^7 x_3^{11} x_4^5 x_5^{54}$
- 808.  $x_1^{15} x_2^7 x_3^{15} x_4^5 x_5^{54}$
- 811.  $x_1^{15} x_2^7 x_3^{15} x_4^{17} x_5^{38}$
- 814.  $x_1^{15} x_2^7 x_3^{15} x_4^{49} x_5^6$
- 817.  $x_1^{15} x_2^7 x_3^{19} x_4^3 x_5^{14}$
- 820.  $x_1^{15} x_2^7 x_3^{23} x_4^5 x_5^{40}$
- 823.  $x_1^{15} x_2^7 x_3^{23} x_4^{41} x_5^6$
- 826.  $x_1^{15} x_2^7 x_3^{31} x_4^5 x_5^{38}$
- 829.  $x_1^{15} x_2^7 x_3^{55} x_4^5 x_5^{14}$
- 832.  $x_1^{15} x_2^7 x_3^{59} x_4^5 x_5^6$
- 835.  $x_1^{15} x_2^{15} x_3 x_4^{22} x_5^{39}$
- 838.  $x_1^{15} x_2^{15} x_3 x_4^{55} x_5^6$
- 841.  $x_1^{15} x_2^{15} x_3^3 x_4^{53} x_5^6$
- 844.  $x_1^{15} x_2^{15} x_3^7 x_4^{16} x_5^{39}$
- 847.  $x_1^{15} x_2^{15} x_3^7 x_4^{48} x_5^5$
- 850.  $x_1^{15} x_2^{15} x_3^{17} x_4^5 x_5^{38}$
- 853.  $x_1^{15} x_2^{15} x_3^{19} x_4^5 x_5^{38}$
- 856.  $x_1^{15} x_2^{15} x_3^{23} x_4^5 x_5^{32}$
- 859.  $x_1^{15} x_2^{15} x_3^{49} x_4^5 x_5^6$
- 862.  $x_1^{15} x_2^{19} x_3^7 x_4^{13} x_5^{38}$
- 865.  $x_1^{15} x_2^{23} x_3 x_4^{14} x_5^{39}$
- 868.  $x_1^{15} x_2^{23} x_3 x_4^{39} x_5^{14}$
- 871.  $x_1^{15} x_2^{23} x_3^3 x_4^5 x_5^{46}$
- 874.  $x_1^{15} x_2^{23} x_3^3 x_4^5 x_5^6$
- 877.  $x_1^{15} x_2^{23} x_3^7 x_4^5 x_5^{38}$
- 880.  $x_1^{15} x_2^{23} x_3^{11} x_4^5 x_5^{38}$
- 883.  $x_1^{15} x_2^{23} x_3^{35} x_4^5 x_5^{14}$
- 886.  $x_1^{15} x_2^{23} x_3^{43} x_4^5 x_5^6$
- 889.  $x_1^{15} x_2^{31} x_3^7 x_4^5 x_5^{38}$
- 892.  $x_1^{15} x_2^{31} x_3^3 x_4^5 x_5^{38}$
- 895.  $x_1^{15} x_2^{31} x_3^9 x_4^5 x_5^6$
- 898.  $x_1^{15} x_2^{55} x_3 x_4^{14} x_5^7$
- 901.  $x_1^{15} x_2^{55} x_3^3 x_4^{13} x_5^6$
- 904.  $x_1^{15} x_2^{55} x_3^9 x_4^5 x_5^6$
- 907.  $x_1^{31} x_2 x_3^6 x_4^7 x_5^{47}$
- 910.  $x_1^{31} x_2 x_3^9 x_4^7 x_5^7$
- 913.  $x_1^{31} x_2 x_3^5 x_4^{14} x_5^{39}$
- 916.  $x_1^{31} x_2 x_3^5 x_4^{39} x_5^{14}$
- 791.  $x_1^{15} x_2^7 x_3^3 x_4^{21} x_5^{46}$
- 794.  $x_1^{15} x_2^7 x_3^3 x_4^{61} x_5^6$
- 797.  $x_1^{15} x_2^7 x_3^7 x_4^5 x_5^{54}$
- 800.  $x_1^{15} x_2^7 x_3^7 x_4^{23} x_5^{40}$
- 803.  $x_1^{15} x_2^7 x_3^7 x_4^{55} x_5^8$
- 806.  $x_1^{15} x_2^7 x_3^{11} x_4^{21} x_5^{38}$
- 809.  $x_1^{15} x_2^7 x_3^{15} x_4^5 x_5^{48}$
- 812.  $x_1^{15} x_2^7 x_3^{15} x_4^{23} x_5^{32}$
- 815.  $x_1^{15} x_2^7 x_3^{19} x_4^5 x_5^{46}$
- 818.  $x_1^{15} x_2^7 x_3^{19} x_4^5 x_5^6$
- 821.  $x_1^{15} x_2^7 x_3^{23} x_4^5 x_5^{38}$
- 824.  $x_1^{15} x_2^7 x_3^{27} x_4^5 x_5^{38}$
- 827.  $x_1^{15} x_2^7 x_3^{51} x_4^5 x_5^{14}$
- 830.  $x_1^{15} x_2^7 x_3^{55} x_4^5 x_5^8$
- 833.  $x_1^{15} x_2^{15} x_3 x_4^5 x_5^{55}$
- 836.  $x_1^{15} x_2^{15} x_3 x_4^{23} x_5^{38}$
- 839.  $x_1^{15} x_2^{15} x_3^3 x_4^5 x_5^{54}$
- 842.  $x_1^{15} x_2^{15} x_3^7 x_4^5 x_5^{54}$
- 845.  $x_1^{15} x_2^{15} x_3^7 x_4^{17} x_5^{38}$
- 848.  $x_1^{15} x_2^{15} x_3^9 x_4^5 x_5^6$
- 851.  $x_1^{15} x_2^{15} x_3^{17} x_4^5 x_5^7$
- 854.  $x_1^{15} x_2^{15} x_3^{19} x_4^5 x_5^6$
- 857.  $x_1^{15} x_2^{15} x_3^{23} x_4^5 x_5^6$
- 860.  $x_1^{15} x_2^{15} x_3^{51} x_4^5 x_5^6$
- 863.  $x_1^{15} x_2^{23} x_3 x_4^5 x_5^{47}$
- 866.  $x_1^{15} x_2^{23} x_3 x_4^5 x_5^{38}$
- 869.  $x_1^{15} x_2^{23} x_3 x_4^6 x_5^7$
- 872.  $x_1^{15} x_2^{23} x_3^3 x_4^{13} x_5^{38}$
- 875.  $x_1^{15} x_2^{23} x_3^5 x_4^5 x_5^{46}$
- 878.  $x_1^{15} x_2^{23} x_3^7 x_4^5 x_5^{14}$
- 881.  $x_1^{15} x_2^{23} x_3^{11} x_4^5 x_5^6$
- 884.  $x_1^{15} x_2^{23} x_3^{35} x_4^5 x_5^6$
- 887.  $x_1^{15} x_2^{23} x_3^{47} x_4^5 x_5^6$
- 890.  $x_1^{15} x_2^{31} x_3 x_4^5 x_5^7$
- 893.  $x_1^{15} x_2^{31} x_3^3 x_4^5 x_5^6$
- 896.  $x_1^{15} x_2^{55} x_3 x_4^5 x_5^{15}$
- 899.  $x_1^{15} x_2^{55} x_3 x_4^5 x_5^6$
- 902.  $x_1^{15} x_2^{55} x_3^7 x_4^5 x_5^{14}$
- 905.  $x_1^{15} x_2^{55} x_3^{11} x_4^5 x_5^6$
- 908.  $x_1^{31} x_2 x_3^6 x_4^{15} x_5^{39}$
- 911.  $x_1^{31} x_2 x_3^7 x_4^7 x_5^{47}$
- 914.  $x_1^{31} x_2 x_3^5 x_4^{15} x_5^{38}$
- 917.  $x_1^{31} x_2 x_3^5 x_4^{46} x_5^7$
- 792.  $x_1^{15} x_2^7 x_3^3 x_4^{29} x_5^{38}$
- 795.  $x_1^{15} x_2^7 x_3^3 x_4^5 x_5^{62}$
- 798.  $x_1^{15} x_2^7 x_3^7 x_4^{15} x_5^{48}$
- 801.  $x_1^{15} x_2^7 x_3^7 x_4^{25} x_5^{38}$
- 804.  $x_1^{15} x_2^7 x_3^7 x_4^{57} x_5^6$
- 807.  $x_1^{15} x_2^7 x_3^{11} x_4^{53} x_5^6$
- 810.  $x_1^{15} x_2^7 x_3^{15} x_4^{16} x_5^{39}$
- 813.  $x_1^{15} x_2^7 x_3^{15} x_4^{48} x_5^7$
- 816.  $x_1^{15} x_2^7 x_3^{19} x_4^{13} x_5^{38}$
- 819.  $x_1^{15} x_2^7 x_3^{23} x_4^5 x_5^{46}$
- 822.  $x_1^{15} x_2^7 x_3^{23} x_4^5 x_5^{14}$
- 825.  $x_1^{15} x_2^7 x_3^{27} x_4^5 x_5^6$
- 828.  $x_1^{15} x_2^7 x_3^{51} x_4^5 x_5^6$
- 831.  $x_1^{15} x_2^7 x_3^{55} x_4^5 x_5^6$
- 834.  $x_1^{15} x_2^{15} x_3 x_4^5 x_5^{54}$
- 837.  $x_1^{15} x_2^{15} x_3 x_4^5 x_5^7$
- 840.  $x_1^{15} x_2^{15} x_3^3 x_4^{21} x_5^{38}$
- 843.  $x_1^{15} x_2^{15} x_3^7 x_4^5 x_5^{48}$
- 846.  $x_1^{15} x_2^{15} x_3^7 x_4^5 x_5^{32}$
- 849.  $x_1^{15} x_2^{15} x_3^{17} x_4^5 x_5^{39}$
- 852.  $x_1^{15} x_2^{15} x_3^{17} x_4^5 x_5^6$
- 855.  $x_1^{15} x_2^{15} x_3^{23} x_4^5 x_5^{38}$
- 858.  $x_1^{15} x_2^{15} x_3^{49} x_4^5 x_5^7$
- 861.  $x_1^{15} x_2^{15} x_3^{55} x_4^5 x_5^6$
- 864.  $x_1^{15} x_2^{23} x_3 x_4^5 x_5^{46}$
- 867.  $x_1^{15} x_2^{23} x_3 x_4^5 x_5^{15}$
- 870.  $x_1^{15} x_2^{23} x_3 x_4^6 x_5^6$
- 873.  $x_1^{15} x_2^{23} x_3^3 x_4^5 x_5^{14}$
- 876.  $x_1^{15} x_2^{23} x_3^3 x_4^5 x_5^{40}$
- 879.  $x_1^{15} x_2^{23} x_3^7 x_4^5 x_5^6$
- 882.  $x_1^{15} x_2^{23} x_3^{15} x_4^5 x_5^{38}$
- 885.  $x_1^{15} x_2^{23} x_3^{39} x_4^5 x_5^{14}$
- 888.  $x_1^{15} x_2^{31} x_3 x_4^5 x_5^{39}$
- 891.  $x_1^{15} x_2^{31} x_3 x_4^5 x_5^6$
- 894.  $x_1^{15} x_2^{31} x_3^7 x_4^5 x_5^{38}$
- 897.  $x_1^{15} x_2^{55} x_3 x_4^5 x_5^{14}$
- 900.  $x_1^{15} x_2^{55} x_3^3 x_4^5 x_5^{14}$
- 903.  $x_1^{15} x_2^{55} x_3^7 x_4^5 x_5^8$
- 906.  $x_1^{15} x_2^{55} x_3^{15} x_4^5 x_5^6$
- 909.  $x_1^{31} x_2 x_3^6 x_4^{39} x_5^{15}$
- 912.  $x_1^{31} x_2 x_3^7 x_4^7 x_5^{46}$
- 915.  $x_1^{31} x_2 x_3^5 x_4^{38} x_5^{15}$
- 918.  $x_1^{31} x_2 x_3^5 x_4^{47} x_5^6$

919.  $x_1^{31}x_2x_3^{14}x_4x_5^{39}$  920.  $x_1^{31}x_2x_3^{14}x_4x_5^{39}$  921.  $x_1^{31}x_2x_3^{15}x_4x_5^{39}$   
922.  $x_1^{31}x_2x_3^{15}x_4x_5^{38}$  923.  $x_1^{31}x_2x_3^{15}x_4x_5^{38}$  924.  $x_1^{31}x_2x_3^{15}x_4x_5^{39}$   
925.  $x_1^{31}x_2x_3^{38}x_4x_5^{15}$  926.  $x_1^{31}x_2x_3^{38}x_4x_5^{15}$  927.  $x_1^{31}x_2x_3^{39}x_4x_5^{15}$   
928.  $x_1^{31}x_2x_3^{39}x_4x_5^{14}$  929.  $x_1^{31}x_2x_3^{39}x_4x_5^{14}$  930.  $x_1^{31}x_2x_3^{39}x_4x_5^{15}$   
931.  $x_1^{31}x_2x_3^{46}x_4x_5^{7}$  932.  $x_1^{31}x_2x_3^{47}x_4x_5^{7}$  933.  $x_1^{31}x_2x_3^{47}x_4x_5^{6}$   
934.  $x_1^{31}x_2^3x_3^6x_4x_5^{47}$  935.  $x_1^{31}x_2^3x_3^5x_4x_5^{46}$  936.  $x_1^{31}x_2^3x_3^5x_4x_5^{39}$   
937.  $x_1^{31}x_2^3x_3^4x_4x_5^{38}$  938.  $x_1^{31}x_2^3x_3^5x_4x_5^{15}$  939.  $x_1^{31}x_2^3x_3^5x_4x_5^{14}$   
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943.  $x_1^{31}x_2^3x_3^4x_4x_5^{38}$  944.  $x_1^{31}x_2^3x_3^7x_4x_5^{14}$  945.  $x_1^{31}x_2^3x_3^7x_4x_5^{6}$   
946.  $x_1^{31}x_2^3x_3^{13}x_4x_5^{39}$  947.  $x_1^{31}x_2^3x_3^{13}x_4x_5^{38}$  948.  $x_1^{31}x_2^3x_3^{13}x_4x_5^{7}$   
949.  $x_1^{31}x_2^3x_3^{13}x_4x_5^{6}$  950.  $x_1^{31}x_2^3x_3^{15}x_4x_5^{38}$  951.  $x_1^{31}x_2^3x_3^{15}x_4x_5^{6}$   
952.  $x_1^{31}x_2^3x_3^{37}x_4x_5^{15}$  953.  $x_1^{31}x_2^3x_3^{37}x_4x_5^{14}$  954.  $x_1^{31}x_2^3x_3^{37}x_4x_5^{14}$   
955.  $x_1^{31}x_2^3x_3^{37}x_4x_5^{6}$  956.  $x_1^{31}x_2^3x_3^{39}x_4x_5^{14}$  957.  $x_1^{31}x_2^3x_3^{39}x_4x_5^{6}$   
958.  $x_1^{31}x_2^3x_3^{45}x_4x_5^{7}$  959.  $x_1^{31}x_2^3x_3^{45}x_4x_5^{6}$  960.  $x_1^{31}x_2^3x_3^{47}x_4x_5^{6}$   
961.  $x_1^{31}x_2^7x_3x_4x_5^{47}$  962.  $x_1^{31}x_2^7x_3x_4x_5^{46}$  963.  $x_1^{31}x_2^7x_3x_4x_5^{39}$   
964.  $x_1^{31}x_2^7x_3x_4x_5^{38}$  965.  $x_1^{31}x_2^7x_3x_4x_5^{15}$  966.  $x_1^{31}x_2^7x_3x_4x_5^{14}$   
967.  $x_1^{31}x_2^7x_3x_4x_5^{46}$  968.  $x_1^{31}x_2^7x_3x_4x_5^{6}$  969.  $x_1^{31}x_2^7x_3x_4x_5^{46}$   
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973.  $x_1^{31}x_2^7x_3x_4x_5^{46}$  974.  $x_1^{31}x_2^7x_3x_4x_5^{40}$  975.  $x_1^{31}x_2^7x_3x_4x_5^{38}$   
976.  $x_1^{31}x_2^7x_3x_4x_5^{14}$  977.  $x_1^{31}x_2^7x_3x_4x_5^{6}$  978.  $x_1^{31}x_2^7x_3x_4x_5^{38}$   
979.  $x_1^{31}x_2^7x_3x_4x_5^{6}$  980.  $x_1^{31}x_2^7x_3x_4x_5^{38}$  981.  $x_1^{31}x_2^7x_3x_4x_5^{14}$   
982.  $x_1^{31}x_2^7x_3x_4x_5^{6}$  983.  $x_1^{31}x_2^7x_3x_4x_5^{14}$  984.  $x_1^{31}x_2^7x_3x_4x_5^{6}$   
985.  $x_1^{31}x_2^7x_3x_4x_5^{6}$  986.  $x_1^{31}x_2^7x_3x_4x_5^{39}$  987.  $x_1^{31}x_2^7x_3x_4x_5^{38}$   
988.  $x_1^{31}x_2^7x_3x_4x_5^{7}$  989.  $x_1^{31}x_2^7x_3x_4x_5^{6}$  990.  $x_1^{31}x_2^7x_3x_4x_5^{38}$   
991.  $x_1^{31}x_2^7x_3x_4x_5^{6}$  992.  $x_1^{31}x_2^7x_3x_4x_5^{38}$  993.  $x_1^{31}x_2^7x_3x_4x_5^{6}$   
994.  $x_1^{31}x_2^7x_3x_4x_5^{15}$  995.  $x_1^{31}x_2^7x_3x_4x_5^{14}$  996.  $x_1^{31}x_2^7x_3x_4x_5^{14}$   
997.  $x_1^{31}x_2^7x_3x_4x_5^{6}$  998.  $x_1^{31}x_2^7x_3x_4x_5^{14}$  999.  $x_1^{31}x_2^7x_3x_4x_5^{6}$   
1000.  $x_1^{31}x_2^7x_3x_4x_5^{14}$  1001.  $x_1^{31}x_2^7x_3x_4x_5^{6}$  1002.  $x_1^{31}x_2^7x_3x_4x_5^{7}$   
1003.  $x_1^{31}x_2^7x_3x_4x_5^{6}$  1004.  $x_1^{31}x_2^7x_3x_4x_5^{6}$  1005.  $x_1^{31}x_2^7x_3x_4x_5^{6}$

$B_5^+((4)^3|(2)|(3)) = \{\bar{c}_t : 1 \leq t \leq 104\}$ , where

1.  $x_1x_2^{14}x_3^{23}x_4^{23}x_5^{31}$
2.  $x_1x_2^{14}x_3^{23}x_4^{31}x_5^{23}$
3.  $x_1x_2^{14}x_3^{31}x_4^{23}x_5^{23}$
4.  $x_1x_2^{15}x_3^{22}x_4^{23}x_5^{31}$
5.  $x_1x_2^{15}x_3^{22}x_4^{31}x_5^{23}$
6.  $x_1x_2^{15}x_3^{23}x_4^{22}x_5^{31}$
7.  $x_1x_2^{15}x_3^{23}x_4^{23}x_5^{30}$
8.  $x_1x_2^{15}x_3^{23}x_4^{30}x_5^{23}$
9.  $x_1x_2^{15}x_3^{31}x_4^{22}x_5^{22}$
10.  $x_1x_2^{15}x_3^{30}x_4^{23}x_5^{23}$
11.  $x_1x_2^{15}x_3^{31}x_4^{22}x_5^{23}$
12.  $x_1x_2^{15}x_3^{31}x_4^{23}x_5^{22}$
13.  $x_1x_2^{30}x_3^{15}x_4^{23}x_5^{23}$
14.  $x_1x_2^{31}x_3^{14}x_4^{23}x_5^{23}$
15.  $x_1x_2^{31}x_3^{15}x_4^{22}x_5^{23}$
16.  $x_1x_2^{31}x_3^{15}x_4^{23}x_5^{22}$
17.  $x_1^3x_2^{13}x_3^{22}x_4^{23}x_5^{31}$
18.  $x_1^3x_2^{13}x_3^{22}x_4^{31}x_5^{23}$
19.  $x_1^3x_2^{13}x_3^{23}x_4^{22}x_5^{31}$
20.  $x_1^3x_2^{13}x_3^{23}x_4^{23}x_5^{30}$
21.  $x_1^3x_2^{13}x_3^{23}x_4^{30}x_5^{23}$
22.  $x_1^3x_2^{13}x_3^{23}x_4^{31}x_5^{22}$
23.  $x_1^3x_2^{13}x_3^{30}x_4^{23}x_5^{23}$
24.  $x_1^3x_2^{13}x_3^{31}x_4^{22}x_5^{23}$
25.  $x_1^3x_2^{13}x_3^{31}x_4^{23}x_5^{22}$
26.  $x_1^3x_2^{15}x_3^{21}x_4^{22}x_5^{31}$
27.  $x_1^3x_2^{15}x_3^{21}x_4^{23}x_5^{30}$
28.  $x_1^3x_2^{15}x_3^{21}x_4^{30}x_5^{23}$
29.  $x_1^3x_2^{15}x_3^{21}x_4^{31}x_5^{22}$
30.  $x_1^3x_2^{15}x_3^{23}x_4^{21}x_5^{30}$
31.  $x_1^3x_2^{15}x_3^{23}x_4^{29}x_5^{22}$
32.  $x_1^3x_2^{15}x_3^{29}x_4^{22}x_5^{23}$
33.  $x_1^3x_2^{15}x_3^{29}x_4^{23}x_5^{22}$
34.  $x_1^3x_2^{15}x_3^{21}x_4^{21}x_5^{22}$
35.  $x_1^3x_2^{29}x_3^{14}x_4^{23}x_5^{23}$
36.  $x_1^3x_2^{29}x_3^{15}x_4^{22}x_5^{23}$



- |   |   |   |
|---|---|---|
| 37. $x_1^3 x_2^{29} x_3^{15} x_4^{23} x_5^{22}$ | 38. $x_1^3 x_2^{31} x_3^{13} x_4^{22} x_5^{23}$ | 39. $x_1^3 x_2^{31} x_3^{13} x_4^{23} x_5^{22}$ |
| 40. $x_1^3 x_2^{31} x_3^{15} x_4^{21} x_5^{22}$ | 41. $x_1^7 x_2^{11} x_3^{21} x_4^{22} x_5^{31}$ | 42. $x_1^7 x_2^{11} x_3^{21} x_4^{23} x_5^{30}$ |
| 43. $x_1^7 x_2^{11} x_3^{21} x_4^{30} x_5^{23}$ | 44. $x_1^7 x_2^{11} x_3^{21} x_4^{31} x_5^{22}$ | 45. $x_1^7 x_2^{11} x_3^{23} x_4^{21} x_5^{30}$ |
| 46. $x_1^7 x_2^{11} x_3^{23} x_4^{29} x_5^{22}$ | 47. $x_1^7 x_2^{11} x_3^{29} x_4^{22} x_5^{23}$ | 48. $x_1^7 x_2^{11} x_3^{29} x_4^{23} x_5^{22}$ |
| 49. $x_1^7 x_2^{11} x_3^{31} x_4^{21} x_5^{22}$ | 50. $x_1^7 x_2^{15} x_3^{19} x_4^{21} x_5^{30}$ | 51. $x_1^7 x_2^{15} x_3^{19} x_4^{29} x_5^{22}$ |
| 52. $x_1^7 x_2^{15} x_3^{27} x_4^{21} x_5^{22}$ | 53. $x_1^7 x_2^{27} x_3^{13} x_4^{22} x_5^{23}$ | 54. $x_1^7 x_2^{27} x_3^{13} x_4^{23} x_5^{22}$ |
| 55. $x_1^7 x_2^{27} x_3^{15} x_4^{21} x_5^{22}$ | 56. $x_1^7 x_2^{31} x_3^{11} x_4^{21} x_5^{22}$ | 57. $x_1^{15} x_2 x_3^{22} x_4^{23} x_5^{31}$   |
| 58. $x_1^{15} x_2 x_3^{22} x_4^{31} x_5^{23}$   | 59. $x_1^{15} x_2 x_3^{23} x_4^{22} x_5^{31}$   | 60. $x_1^{15} x_2 x_3^{23} x_4^{23} x_5^{30}$   |
| 61. $x_1^{15} x_2 x_3^{23} x_4^{30} x_5^{23}$   | 62. $x_1^{15} x_2 x_3^{23} x_4^{31} x_5^{22}$   | 63. $x_1^{15} x_2 x_3^{30} x_4^{23} x_5^{23}$   |
| 64. $x_1^{15} x_2 x_3^{31} x_4^{22} x_5^{23}$   | 65. $x_1^{15} x_2 x_3^{31} x_4^{23} x_5^{22}$   | 66. $x_1^{15} x_2 x_3^{21} x_4^{22} x_5^{31}$   |
| 67. $x_1^{15} x_2 x_3^{21} x_4^{23} x_5^{30}$   | 68. $x_1^{15} x_2 x_3^{21} x_4^{30} x_5^{23}$   | 69. $x_1^{15} x_2 x_3^{21} x_4^{31} x_5^{22}$   |
| 70. $x_1^{15} x_2 x_3^{23} x_4^{21} x_5^{30}$   | 71. $x_1^{15} x_2 x_3^{23} x_4^{29} x_5^{22}$   | 72. $x_1^{15} x_2 x_3^{29} x_4^{22} x_5^{23}$   |
| 73. $x_1^{15} x_2 x_3^{29} x_4^{23} x_5^{22}$   | 74. $x_1^{15} x_2 x_3^{31} x_4^{21} x_5^{22}$   | 75. $x_1^{15} x_2 x_3^{19} x_4^{21} x_5^{30}$   |
| 76. $x_1^{15} x_2 x_3^{19} x_4^{29} x_5^{22}$   | 77. $x_1^{15} x_2 x_3^{27} x_4^{21} x_5^{22}$   | 78. $x_1^{15} x_2 x_3^{17} x_4^{22} x_5^{23}$   |
| 79. $x_1^{15} x_2 x_3^{17} x_4^{23} x_5^{22}$   | 80. $x_1^{15} x_2 x_3^{19} x_4^{21} x_5^{22}$   | 81. $x_1^{15} x_2 x_3 x_4^{22} x_5^{31}$        |
| 82. $x_1^{15} x_2 x_3 x_4^{23} x_5^{30}$        | 83. $x_1^{15} x_2 x_3 x_4^{30} x_5^{23}$        | 84. $x_1^{15} x_2 x_3 x_4^{31} x_5^{22}$        |
| 85. $x_1^{15} x_2 x_3 x_4^{23} x_5^{30}$        | 86. $x_1^{15} x_2 x_3 x_4^{29} x_5^{22}$        | 87. $x_1^{15} x_2 x_3 x_4^{21} x_5^{22}$        |
| 88. $x_1^{15} x_2 x_3 x_4^{23} x_5^{30}$        | 89. $x_1^{15} x_2 x_3 x_4^{22} x_5^{22}$        | 90. $x_1^{15} x_2 x_3 x_4^{22} x_5^{23}$        |
| 91. $x_1^{15} x_2 x_3 x_4^{23} x_5^{22}$        | 92. $x_1^{15} x_2 x_3 x_4^{21} x_5^{22}$        | 93. $x_1^{15} x_2 x_3 x_4^{23} x_5^{22}$        |
| 94. $x_1^{31} x_2 x_3 x_4^{23} x_5^{23}$        | 95. $x_1^{31} x_2 x_3 x_4^{22} x_5^{23}$        | 96. $x_1^{31} x_2 x_3 x_4^{23} x_5^{22}$        |
| 97. $x_1^{31} x_2 x_3 x_4^{22} x_5^{23}$        | 98. $x_1^{31} x_2 x_3 x_4^{23} x_5^{22}$        | 99. $x_1^{31} x_2 x_3 x_4^{21} x_5^{22}$        |
| 100. $x_1^{31} x_2 x_3 x_4^{21} x_5^{22}$       | 101. $x_1^{31} x_2 x_3 x_4^{22} x_5^{23}$       | 102. $x_1^{31} x_2 x_3 x_4^{23} x_5^{22}$       |
| 103. $x_1^{31} x_2 x_3 x_4^{22} x_5^{23}$       | 104. $x_1^{31} x_2 x_3 x_4^{23} x_5^{22}$       |   |

$B_5^+((4)|^4|(2)) = \{\tilde{c}_t : 1 \leq t \leq 156\}$ , where

- |   |   |   |
|---|---|---|
| 1. $x_1 x_2^{14} x_3^{15} x_4^{31} x_5^{31}$    | 2. $x_1 x_2^{14} x_3^{31} x_4^{15} x_5^{31}$    | 3. $x_1 x_2^{14} x_3^{31} x_4^{31} x_5^{15}$    |
| 4. $x_1 x_2^{15} x_3^{14} x_4^{31} x_5^{31}$    | 5. $x_1 x_2^{15} x_3^{15} x_4^{30} x_5^{31}$    | 6. $x_1 x_2^{15} x_3^{15} x_4^{31} x_5^{30}$    |
| 7. $x_1 x_2^{15} x_3^{30} x_4^{15} x_5^{31}$    | 8. $x_1 x_2^{15} x_3^{30} x_4^{31} x_5^{15}$    | 9. $x_1 x_2^{15} x_3^{31} x_4^{14} x_5^{31}$    |
| 10. $x_1 x_2^{15} x_3^{31} x_4^{15} x_5^{30}$   | 11. $x_1 x_2^{15} x_3^{31} x_4^{30} x_5^{15}$   | 12. $x_1 x_2^{15} x_3^{31} x_4^{31} x_5^{14}$   |
| 13. $x_1 x_2^{30} x_3^{15} x_4^{15} x_5^{31}$   | 14. $x_1 x_2^{30} x_3^{15} x_4^{31} x_5^{15}$   | 15. $x_1 x_2^{30} x_3^{31} x_4^{15} x_5^{15}$   |
| 16. $x_1 x_2^{31} x_3^{14} x_4^{15} x_5^{31}$   | 17. $x_1 x_2^{31} x_3^{14} x_4^{31} x_5^{15}$   | 18. $x_1 x_2^{31} x_3^{15} x_4^{14} x_5^{31}$   |
| 19. $x_1 x_2^{31} x_3^{15} x_4^{15} x_5^{30}$   | 20. $x_1 x_2^{31} x_3^{15} x_4^{30} x_5^{15}$   | 21. $x_1 x_2^{31} x_3^{15} x_4^{31} x_5^{14}$   |
| 22. $x_1 x_2^{31} x_3^{30} x_4^{15} x_5^{15}$   | 23. $x_1 x_2^{31} x_3^{31} x_4^{14} x_5^{15}$   | 24. $x_1 x_2^{31} x_3^{31} x_4^{15} x_5^{14}$   |
| 25. $x_1^3 x_2^{13} x_3^{14} x_4^{31} x_5^{31}$ | 26. $x_1^3 x_2^{13} x_3^{15} x_4^{30} x_5^{31}$ | 27. $x_1^3 x_2^{13} x_3^{15} x_4^{31} x_5^{30}$ |
| 28. $x_1^3 x_2^{13} x_3^{30} x_4^{15} x_5^{31}$ | 29. $x_1^3 x_2^{13} x_3^{30} x_4^{31} x_5^{15}$ | 30. $x_1^3 x_2^{13} x_3^{31} x_4^{14} x_5^{31}$ |
| 31. $x_1^3 x_2^{13} x_3^{31} x_4^{15} x_5^{30}$ | 32. $x_1^3 x_2^{13} x_3^{31} x_4^{30} x_5^{15}$ | 33. $x_1^3 x_2^{13} x_3^{31} x_4^{31} x_5^{14}$ |
| 34. $x_1^3 x_2^{15} x_3^{13} x_4^{30} x_5^{31}$ | 35. $x_1^3 x_2^{15} x_3^{13} x_4^{31} x_5^{30}$ | 36. $x_1^3 x_2^{15} x_3^{15} x_4^{29} x_5^{30}$ |
| 37. $x_1^3 x_2^{15} x_3^{29} x_4^{14} x_5^{31}$ | 38. $x_1^3 x_2^{15} x_3^{29} x_4^{15} x_5^{30}$ | 39. $x_1^3 x_2^{15} x_3^{29} x_4^{30} x_5^{15}$ |
| 40. $x_1^3 x_2^{15} x_3^{29} x_4^{31} x_5^{14}$ | 41. $x_1^3 x_2^{15} x_3^{31} x_4^{13} x_5^{30}$ | 42. $x_1^3 x_2^{15} x_3^{31} x_4^{29} x_5^{14}$ |
| 43. $x_1^3 x_2^{29} x_3^{14} x_4^{15} x_5^{31}$ | 44. $x_1^3 x_2^{29} x_3^{14} x_4^{31} x_5^{15}$ | 45. $x_1^3 x_2^{29} x_3^{15} x_4^{14} x_5^{31}$ |
| 46. $x_1^3 x_2^{29} x_3^{15} x_4^{15} x_5^{30}$ | 47. $x_1^3 x_2^{29} x_3^{15} x_4^{30} x_5^{15}$ | 48. $x_1^3 x_2^{29} x_3^{15} x_4^{31} x_5^{14}$ |
| 49. $x_1^3 x_2^{29} x_3^{30} x_4^{15} x_5^{15}$ | 50. $x_1^3 x_2^{29} x_3^{31} x_4^{14} x_5^{15}$ | 51. $x_1^3 x_2^{29} x_3^{31} x_4^{15} x_5^{14}$ |
| 52. $x_1^3 x_2^{31} x_3^{13} x_4^{14} x_5^{31}$ | 53. $x_1^3 x_2^{31} x_3^{13} x_4^{15} x_5^{30}$ | 54. $x_1^3 x_2^{31} x_3^{13} x_4^{30} x_5^{15}$ |

55.  $x_1^3 x_2^{31} x_3^{13} x_4^{31} x_5^{14}$   
58.  $x_1^3 x_2^{31} x_3^{29} x_4^{14} x_5^{15}$   
61.  $x_1^7 x_2^{11} x_3^{13} x_4^{30} x_5^{31}$   
64.  $x_1^7 x_2^{11} x_3^{29} x_4^{14} x_5^{31}$   
67.  $x_1^7 x_2^{11} x_3^{29} x_4^{31} x_5^{14}$   
70.  $x_1^7 x_2^{15} x_3^{11} x_4^{29} x_5^{30}$   
73.  $x_1^7 x_2^{27} x_3^{13} x_4^{14} x_5^{31}$   
76.  $x_1^7 x_2^{27} x_3^{13} x_4^{31} x_5^{14}$   
79.  $x_1^7 x_2^{27} x_3^{29} x_4^{14} x_5^{15}$   
82.  $x_1^7 x_2^{31} x_3^{11} x_4^{13} x_5^{30}$   
85.  $x_1^{15} x_2 x_3^{14} x_4^{31} x_5^{31}$   
88.  $x_1^{15} x_2 x_3^{30} x_4^{19} x_5^{31}$   
91.  $x_1^{15} x_2 x_3^{31} x_4^{19} x_5^{30}$   
94.  $x_1^{15} x_2^3 x_3^{13} x_4^{30} x_5^{31}$   
97.  $x_1^{15} x_2^3 x_3^{29} x_4^{14} x_5^{31}$   
100.  $x_1^{15} x_2^3 x_3^{29} x_4^{31} x_5^{14}$   
103.  $x_1^{15} x_2^7 x_3^{11} x_4^{29} x_5^{30}$   
106.  $x_1^{15} x_2^{15} x_3 x_4^{30} x_5^{31}$   
109.  $x_1^{15} x_2^{15} x_3^{15} x_4^{17} x_5^{30}$   
112.  $x_1^{15} x_2^{15} x_3^3 x_4^{30}$   
115.  $x_1^{15} x_2^3 x_3^{27} x_4^{13} x_5^{14}$   
118.  $x_1^{15} x_2^{31} x_3 x_4^{30} x_5^{15}$   
121.  $x_1^{15} x_2^{31} x_3^3 x_4^{29} x_5^{14}$   
124.  $x_1^{31} x_2 x_3^{14} x_4^{15} x_5^{31}$   
127.  $x_1^{31} x_2 x_3^{15} x_4^{15} x_5^{30}$   
130.  $x_1^{31} x_2 x_3^{30} x_4^{15} x_5^{15}$   
133.  $x_1^{31} x_2^3 x_3^{13} x_4^{14} x_5^{31}$   
136.  $x_1^{31} x_2^3 x_3^{13} x_4^{31} x_5^{14}$   
139.  $x_1^{31} x_2^3 x_3^{29} x_4^{15} x_5^{15}$   
142.  $x_1^{31} x_2^3 x_3^{11} x_4^{13} x_5^{30}$   
145.  $x_1^{31} x_2^{15} x_3 x_4^{31} x_5^{31}$   
148.  $x_1^{31} x_2^{15} x_3 x_4^{31} x_5^{14}$   
151.  $x_1^{31} x_2^{15} x_3^{15} x_4 x_5^{30}$   
154.  $x_1^{31} x_2^{31} x_3 x_4^{15} x_5^{14}$   
56.  $x_1^3 x_2^{31} x_3^{15} x_4^{13} x_5^{30}$   
59.  $x_1^3 x_2^{31} x_3^{29} x_4^{15} x_5^{14}$   
62.  $x_1^7 x_2^{11} x_3^{13} x_4^{31} x_5^{30}$   
65.  $x_1^7 x_2^{11} x_3^{29} x_4^{15} x_5^{30}$   
68.  $x_1^7 x_2^{11} x_3^{31} x_4^{13} x_5^{30}$   
71.  $x_1^7 x_2^{15} x_3^{27} x_4^{13} x_5^{30}$   
74.  $x_1^7 x_2^{27} x_3^{13} x_4^{15} x_5^{30}$   
77.  $x_1^7 x_2^{27} x_3^{15} x_4^{13} x_5^{30}$   
80.  $x_1^7 x_2^{27} x_3^{29} x_4^{15} x_5^{14}$   
83.  $x_1^7 x_2^{31} x_3^{11} x_4^{29} x_5^{14}$   
86.  $x_1^{15} x_2 x_3^{15} x_4^{30} x_5^{31}$   
89.  $x_1^{15} x_2 x_3^{30} x_4^{31} x_5^{15}$   
92.  $x_1^{15} x_2 x_3^{31} x_4^{30} x_5^{15}$   
95.  $x_1^{15} x_2^3 x_3^{13} x_4^{31} x_5^{30}$   
98.  $x_1^{15} x_2^3 x_3^{29} x_4^{15} x_5^{30}$   
101.  $x_1^{15} x_2^3 x_3^{31} x_4^{13} x_5^{30}$   
104.  $x_1^{15} x_2^7 x_3^{27} x_4^{13} x_5^{30}$   
107.  $x_1^{15} x_2^{15} x_3 x_4^{31} x_5^{30}$   
110.  $x_1^{15} x_2^{15} x_3^{19} x_4^{13} x_5^{30}$   
113.  $x_1^{15} x_2^{23} x_3^{11} x_4^{13} x_5^{30}$   
116.  $x_1^{15} x_2^{31} x_3 x_4^{14} x_5^{31}$   
119.  $x_1^{15} x_2^{31} x_3 x_4^{31} x_5^{14}$   
122.  $x_1^{15} x_2^{31} x_3^{15} x_4 x_5^{30}$   
125.  $x_1^{31} x_2 x_3^{14} x_4^{31} x_5^{15}$   
128.  $x_1^{31} x_2 x_3^{15} x_4^{30} x_5^{15}$   
131.  $x_1^{31} x_2 x_3^{31} x_4^{14} x_5^{15}$   
134.  $x_1^{31} x_2^3 x_3^{13} x_4^{15} x_5^{30}$   
137.  $x_1^{31} x_2^3 x_3^{15} x_4^{13} x_5^{30}$   
140.  $x_1^{31} x_2^3 x_3^{29} x_4^{15} x_5^{14}$   
143.  $x_1^{31} x_2^3 x_3^{11} x_4^{29} x_5^{14}$   
146.  $x_1^{31} x_2^{15} x_3 x_4^{15} x_5^{30}$   
149.  $x_1^{31} x_2^{15} x_3^3 x_4^{13} x_5^{30}$   
152.  $x_1^{31} x_2^{15} x_3^{31} x_4 x_5^{14}$   
155.  $x_1^{31} x_2^{31} x_3 x_4^{13} x_5^{14}$   
57.  $x_1^3 x_2^{31} x_3^{15} x_4^{29} x_5^{14}$   
60.  $x_1^3 x_2^{31} x_3^{31} x_4^{13} x_5^{14}$   
63.  $x_1^7 x_2^{11} x_3^{15} x_4^{29} x_5^{30}$   
66.  $x_1^7 x_2^{11} x_3^{29} x_4^{30} x_5^{15}$   
69.  $x_1^7 x_2^{11} x_3^{31} x_4^{29} x_5^{14}$   
72.  $x_1^7 x_2^{15} x_3^{27} x_4^{29} x_5^{14}$   
75.  $x_1^7 x_2^{27} x_3^{13} x_4^{30} x_5^{15}$   
78.  $x_1^7 x_2^{27} x_3^{15} x_4^{29} x_5^{14}$   
81.  $x_1^7 x_2^{27} x_3^{31} x_4^{13} x_5^{14}$   
84.  $x_1^7 x_2^{31} x_3^{27} x_4^{13} x_5^{14}$   
87.  $x_1^{15} x_2 x_3^{15} x_4^{31} x_5^{30}$   
90.  $x_1^{15} x_2 x_3^{31} x_4^{14} x_5^{31}$   
93.  $x_1^{15} x_2 x_3^{31} x_4^{14} x_5^{14}$   
96.  $x_1^{15} x_2^3 x_3^{15} x_4^{29} x_5^{30}$   
99.  $x_1^{15} x_2^3 x_3^{29} x_4^{30} x_5^{15}$   
102.  $x_1^{15} x_2^3 x_3^{31} x_4^{29} x_5^{14}$   
105.  $x_1^{15} x_2^7 x_3^{27} x_4^{29} x_5^{14}$   
108.  $x_1^{15} x_2^{15} x_3^3 x_4^{29} x_5^{30}$   
111.  $x_1^{15} x_2^{15} x_3^{19} x_4^{29} x_5^{14}$   
114.  $x_1^{15} x_2^{23} x_3^{11} x_4^{29} x_5^{14}$   
117.  $x_1^{15} x_2^{31} x_3 x_4^{15} x_5^{30}$   
120.  $x_1^{15} x_2^{31} x_3^3 x_4^{13} x_5^{30}$   
123.  $x_1^{15} x_2^{31} x_3^{31} x_4 x_5^{14}$   
126.  $x_1^{31} x_2 x_3^{15} x_4^{14} x_5^{31}$   
129.  $x_1^{31} x_2 x_3^{15} x_4^{31} x_5^{14}$   
132.  $x_1^{31} x_2 x_3^{31} x_4^{15} x_5^{14}$   
135.  $x_1^{31} x_2^3 x_3^{13} x_4^{30} x_5^{15}$   
138.  $x_1^{31} x_2^3 x_3^{15} x_4^{29} x_5^{14}$   
141.  $x_1^{31} x_2^3 x_3^{31} x_4^{13} x_5^{14}$   
144.  $x_1^{31} x_2^3 x_3^{27} x_4^{13} x_5^{14}$   
147.  $x_1^{31} x_2^{15} x_3^3 x_4^{30} x_5^{15}$   
150.  $x_1^{31} x_2^{15} x_3^3 x_4^{29} x_5^{14}$   
153.  $x_1^{31} x_2^{31} x_3^{14} x_4 x_5^{15}$   
156.  $x_1^{31} x_2^{31} x_3^{15} x_4 x_5^{14}$

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