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# THE HIT PROBLEM OF RANK FIVE IN A GENERIC DEGREE

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**Abstract**

Let $E^k$ be an elementary abelian 2-group of rank $k$ and let $BE^k$ be the classifying space of $E^k$. Then, $P_k := H^*(BE^k) \cong \mathbb{F}_2[x_1, x_2, \ldots, x_k]$, a polynomial algebra in $k$ generators $x_1, x_2, \ldots, x_k$, with the degree of each $x_i$ being 1. This algebra is regarded as a module over the mod-2 Steenrod algebra, $\mathcal{A}$.

We study the *Peterson hit problem* of finding a minimal set of generators for $\mathcal{A}$-module $P_k$. It is an open problem in Algebraic Topology. In this paper, we explicitly determine a minimal set of $\mathcal{A}$-generators for $P_5$ in terms of the admissible monomials for the case of the generic degree $m = 2^{d+2} + 2^{d+1} - 3$ with $d \geqslant 6$.

## 1 Introduction

Let $E^k$ be an elementary abelian 2-group of rank $k$ and let $BE^k$ be the classifying space of $E^k$. Then,

$$P_k := H^*(BE^k) \cong \mathbb{F}_2[x_1, x_2, \ldots, x_k],$$

a polynomial algebra in $k$ generators $x_1, x_2, \ldots, x_k$, each of degree 1. Here, the cohomology is taken with coefficients in the prime field $\mathbb{F}_2$ of two elements. The algebra $P_k$ is a module over the mod-2 Steenrod algebra, $\mathcal{A}$. The action

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of $\mathcal{A}$ on $P_k$ is determined by the elementary properties of the Steenrod squares $Sq^i$ and subject to the Cartan formula (see Steenrod and Epstein [18]).

An element $g$ in $P_k$ is called *hit* if it can be written as a finite sum $g = \sum_{u\geqslant 0} Sq^{2^u}(g_u)$ for suitable polynomials $g_u \in P_k$. That means $g$ belongs to $\mathcal{A}^+P_k$, where $\mathcal{A}^+$ is the augmentation ideal of $\mathcal{A}$.

We study the problem, set up by Frank Peterson, of determining a minimal set of generators for the polynomial algebra $P_k$ as a module over the Steenrod algebra. It is called the *Peterson hit problem of rank $k$*. In other words, we want to determine a basis of the $\mathbb{F}_2$-vector space

$$QP_k := P_k/\mathcal{A}^+P_k = \mathbb{F}_2 \otimes_{\mathcal{A}} P_k.$$

The hit problem was studied by Peterson [7], Wood [30], Singer [16], and Priddy [11], who showed its relation to many problems in the homotopy theory respectively in cobordism theory, modular representation theory, Adams spectral sequence for the stable homotopy of spheres, and stable homotopy type of classifying spaces of finite groups. Then, this problem was studied by Carlisle-Wood [1], Crabb-Hubbuck [2], Janfada-Wood [3], Nam [6], Kameko [4], Mothebe [5], Repka-Selick [12], Singer [17], Silverman [13, 14], Silverman-Singer [15], Sum-Tin [24], Tin [26], Walker and Wood [27], Wood [31], the present author [19, 20, 22] and others.

The space $QP_k$ was explicitly computed by Peterson [7] for $k = 1, 2$, by Kameko [4] for $k = 3$ and by the present author [20] for $k = 4$. The problem is still open for any $k > 4$. Recently, Walker and Wood presented the results on the hit problem and its applications to representations of general linear groups in the monographs [28, 29].

Denote by $(P_k)_m$ and $(QP_k)_m$ the subspaces of degree $m$ homogeneous polynomials in the spaces $P_k$ and $QP_k$ respectively, where $m$ is an any nonnegative integer. Set $\mu(m) = \min\{u \in \mathbb{Z} : \alpha(m+u) \leqslant u\}$, where $\alpha(a)$ denotes the number of one in dyadic expansion of a positive integer $a$.

An early result for the hit problem was established by Wood.

**Theorem 1.1** (See Wood [30]). *If $\mu(m) > k$, then $(QP_k)_m = 0$.*

Note that this theorem is originally Peterson's conjecture in [7].

Kameko's homomorphism $\widetilde{Sq}^0_* : QP_k \to QP_k$ is one of important tools in the study of the hit problem. This homomorphism is induced by the $\mathbb{F}_2$-linear map $\phi : P_k \to P_k$ given by

$$\phi(x) = \begin{cases} z, & \text{if } x = x_1x_2\ldots x_kz^2,\\ 0, & \text{otherwise,}\end{cases}$$

for any monomial $x \in P_k$. Note that $\phi$ is not an $\mathcal{A}$-homomorphism. However, for any non-negative integer $s$, $\phi Sq^{2s} = Sq^s\phi$, and $\phi Sq^{2s+1} = 0$.

**Theorem 1.2** (See Kameko [4]). *If $q$ is a positive integer such that $\mu(2q+k) = k$, then*

$$(\widetilde{Sq}^0_*)_{(k,q)} := \widetilde{Sq}^0_* : (QP_k)_{2q+k} \longrightarrow (QP_k)_q$$

*is an isomorphism of the $GL_k$-modules.*

From Theorems 1.1 and 1.2, the hit problem is reduced to the case of degree $m$ such that $\mu(m) < k$. For $\mu(m) = k-1$, the problem was studied by Crabb-Hubbuck [2], Nam [6], Repka-Selick [12], Walker-Wood [27] and the present author [19, 20]. A new tool for studying the Peterson hit problem has been presented in [22] when the author studies it for the case $\mu(m) = k-2$.

Recently, many authors study the hit problem for the case $k = 5$ and the problem has been explicitly determined for the case of degree $m$ such that $\alpha(m+s) = 1$ with $s = \mu(m) < 5$ (see e.g. [23]).

In this paper, we study the hit problem for a case of $k = 5$ and a family of degree $m$ such that $\alpha(m+s) = 2$. More precisely, we explicitly compute the space $(QP_k)_{2^{d+2}+2^{d+1}-3}$, with $d \geqslant 6$, in terms of the admissible monomials (see Section 2). To do this, we consider Kameko's homomorphism

$$(\widetilde{Sq}^0_*)_{(5,2^{d+1}+2^d-4)} : (QP_5)_{2^{d+2}+2^{d+1}-3} \longrightarrow (QP_5)_{2^{d+1}+2^d-4}.$$

The space $(QP_5)_{2^{d+1}+2^d-4}$ can easily determine by using the results in [20]. So, we need to compute the space $\operatorname{Ker}((\widetilde{Sq}^0_*)_{(5,2^{d+1}+2^d-4)})$. We prove the following.

**Theorem 1.3.** *For any integer $d \geqslant 4$, there exist exactly 1085 classes of degree $m = 2^{d+2} + 2^{d+1} - 3$ in $\operatorname{Ker}((\widetilde{Sq}^0_*)_{(5,2^{d+1}+2^d-4)})$ represented by the admissible monomials in $P_5$. Consequently $\dim \operatorname{Ker}((\widetilde{Sq}^0_*)_{(5,2^{d+1}+2^d-4)}) = 1085$.*

A simple computation using Theorems 1.3 and 1.4 in our work [20] we easily obtain $\dim(QP_5)_{(2^{d+1}+2^d-4)} = (2^5-1)\dim(QP_4)_8 = 31 \times 55 = 1705$ for any $d \geqslant 6$. Since $(\widetilde{Sq}^0_*)_{(5,2^{d+1}+2^d-4)}$ is an epimorphism, by combining this and Theorem 1.3 we get the following.

**Corollary 1.4.** *For any integer $d \geqslant 6$, we have*

$$\dim(QP_5)_{(2^{d+2}+2^{d+1}-3)} = 2790.$$

For $d \leqslant 2$, the computation is easy and the results have been published by other authors. We also study the problem for $d = 3, 4, 5$. We have $1690 \leqslant \dim(QP_5)_{45} \leqslant 1739$, $2466 \leqslant \dim(QP_5)_{93} \leqslant 2511$ and $2790 \leqslant \dim(QP_5)_{189} \leqslant 2796$. The exact dimension of $(QP_5)_{(2^{d+2}+2^{d+1}-3)}$ for $3 \leqslant d < 6$ can be checked by using a computer calculation (see Section 4).

The paper is organized as follows. In Section 2, we recall some needed results on the admissible monomials in $P_k$, the criteria of Singer and Silverman on the hit monomials. The detailed proof of Theorem 1.3 is presented in Section 3. In Section 4 we present some results of computation for $(QP_5)_{(2^{d+1}+2^d-4)}$ with $3 \leqslant d \leqslant 5$. In Sections 5 and 6, we list the needed admissible monomials in $P_5$ for the proof of main results.

# 2 Some preliminaries on the hit problem

In this section, we recall some needed definitions and results on the weight vector of a monomial, the admissible monomials, the criteria for hit monomials and the notion of strongly monomial from Kameko [4], Singer [17] and the present author [20, 22].

## 2.1 The weight vector of a monomial and the admissible monomials

**Definition 2.1.1.** Let $x = x_1^{s_1}x_2^{s_2}\ldots x_k^{s_k} \in P_k$. We denote $\nu_j(x) = s_j, 1 \leqslant j \leqslant k$, and $\nu(x) = \max\{\nu_j(x) : 1 \leqslant j \leqslant k\}$. We define the sequences

$$\omega(x) = (\omega_1(x), \omega_2(x), \ldots, \omega_i(x), \ldots), \quad \sigma(x) = (\nu_1(x), \nu_2(x), \ldots, \nu_k(x)),$$

where $\omega_i(x) = \sum_{1\leqslant j\leqslant k} \alpha_{i-1}(\nu_j(x)),\ i \geqslant 1$. Then, $\omega(x)$ and $\sigma(x)$ are called the weight vector and the exponent vector of $x$ respectively.

A weight vector $\omega$ is a sequence of non-negative integers $(\omega_1, \omega_2, \ldots, \omega_i, \ldots)$ such that $\omega_i = 0$ for $i \gg 0$.

The sets of weight vectors and exponent vectors respectively are ordered by the left lexicographical order.

**Definition 2.1.2.** Let $\omega$ be a weight vector and $f$, $h$ two polynomials of the same degree in $P_k$.

i) $f \equiv h$ if and only if $f + h \in \mathcal{A}^+P_k$.

ii) $g \equiv_\omega h$ if and only if $g + h \in \mathcal{A}^+P_k + P_k^-(\omega)$.

Note that the polynomial $f$ is hit if and only if $f \equiv 0$.

It is easy to see that the relations $\equiv$ and $\equiv_\omega$ are equivalence ones. We denote

$$QP_k(\omega) = P_k(\omega)/((\mathcal{A}^+P_k \cap P_k(\omega)) + P_k^-(\omega)).$$

It is easy to see that for any degree $n$, we have

$$(QP_k)_n \cong \bigoplus_{\deg \omega = n} QP_k(\omega), \quad \text{(see e.g. [21])}. \tag{2.1}$$

**Definition 2.1.3.** Let $v$, $w$ be monomials in $P_k$ with $\deg v = \deg w$. We define $v < w$ if and only if one of the following conditions holds:

i) $\omega(v) < \omega(w)$;

ii) $\omega(v) = \omega(w)$ and $\sigma(v) < \sigma(w)$.

**Definition 2.1.4.** A monomial $x$ in $P_k$ is said to be inadmissible if there are monomials $w_1, w_2, \ldots, w_s$ such that $w_j < x$ for $j = 1, 2, \ldots, s$ and $x + \sum_{j=1}^{s} w_j \in \mathcal{A}^+ P_k$. A monomial $x$ is said to be admissible if it is not inadmissible.

Obviously, the set of all the admissible monomials of degree $m$ in $P_k$ is a minimal set of $\mathcal{A}$-generators for $P_k$ in degree $m$.

Denote by $\mathcal{A}_u$ the sub-Hopf algebra of $\mathcal{A}$ generated by $Sq^i$ with $0 \leqslant i < 2^u$, and $\mathcal{A}_u^+ = \mathcal{A}^+ \cap \mathcal{A}_u$.

**Definition 2.1.5.** A monomial $x$ in $P_k$ is said to be strictly inadmissible if and only if there exist monomials $w_1, w_2, \ldots, w_s$ such that $w_j < x$, for $j = 1, 2, \ldots, s$ and $x + \sum_{j=1}^{s} v_j \in \mathcal{A}_u^+ P_k$ with $u = \max\{i : \omega_i(x) > 0\}$.

**Theorem 2.1.6** (See Kameko [4], Sum [19])**.** *Let $x, y, w$ be monomials in $P_k$ such that $\omega_i(x) = 0$ for $i > t > 0$, $\omega_u(w) \neq 0$ and $\omega_i(w) = 0$ for $i > u > 0$.*

i) *If $w$ is inadmissible, then so is $uw^{2^t}$.*

ii) *If $w$ is strictly inadmissible, then so is $wv^{2^u}$.*

## 2.2 Some criteria for hit monomials

We recall a result of Singer [17] on the hit monomials in $P_k$.

**Definition 2.2.1.** A monomial $z$ in $P_k$ is called a spike if $\nu_j(z) = 2^{s_j} - 1$ for $s_j$ a non-negative integer and $j = 1, 2, \ldots, k$. If $s_1 > s_2 > \ldots > s_{r-1} \geqslant s_r > 0$ and $s_j = 0$ for $j > r$, then $z$ is called the minimal spike.

The following is a criterion for hit monomials in $P_k$.

**Theorem 2.2.2** (See Singer [17, Theorem 1.2])**.** *Suppose $f \in P_k$ is a monomial of degree $m$ with $\mu(m) \leqslant k$. Let $z$ be the minimal spike of degree $m$. If $\omega(f) < \omega(z)$, then $f$ is hit.*

It can be shown that $x_1^{31}x_2^7x_3^7$ is the minimal spike of degree 45 and the monomial $x_1^5x_2^{13}x_3^9x_4^{13}x_5^5$ is hit, but

$$\omega(x_1^5x_2^{13}x_3^9x_4^{13}x_5^5) = (5,0,4,3,0) > (3,3,3,1,1) = \omega(x_1^{31}x_2^7x_3^7).$$

Thus, this criterion is not enough to determine all hit monomials. Hence, we need another criterion for hit polynomials in $P_k$.

**Theorem 2.2.3** (See Silverman [14, Theorem 1.2]). *Let $f$ be a polynomial of the form $ph^{2^m}$ for some homogeneous polynomials $p$ and $h$. If $\deg p < (2^m - 1)\mu(\deg h)$, then $f$ is hit.*

This result leads to a criterion in terms of the minimal spike which stronger version of Theorem 2.2.2.

**Theorem 2.2.4** (See Walker and Wood [27, Theorem 14.1.3]). *Let $f \in P_k$ be a monomial of degree $m$, where $\mu(m) \leqslant k$ and let $z$ be the minimal spike of degree $m$. If there is an index $r$ such that $\sum_{i=1}^r 2^{i-1}\omega_i(x) < \sum_{i=1}^r 2^{i-1}\omega_i(z)$, then $f$ is hit.*

## 2.3 On the kernel of Kameko's homomorphism

In this subsection, we recall some notions and results from [22] for the kernel of Kemeko's homomorphism which will be used in the proof of the main result.

**Definition 2.3.1.** Let $z$ be the minimal spike of degree $m$. Denote by $\mathcal{P}_{(k,m)}$ the subspace of $P_k$ spanned by all monomials $f$ of degree $m$ such that

$$\sum_{1\leqslant i\leqslant r} 2^{i-1}\omega_i(f) < \sum_{1\leqslant i\leqslant r} 2^{i-1}\omega_i(z),$$

for some index $r \geqslant 1$.

**Definition 2.3.2.** A monomial $f$ of degree $m$ in $P_k$ is said to be strongly inadmissible if there exist monomials $y_1, y_2, \ldots, y_u$ of the same weight vector $\omega(f)$ such that $y_t < f$, $1 \leqslant t \leqslant u$, and

$$x + y_1 + y_2 + \ldots + y_u \in P_k^-(\omega(f)) + \mathcal{A}_s^+ P_k + \mathcal{P}_{(k,m)},$$

where $s = \max\{i : \omega_i(f) > 0\}$.

**Notation 2.3.3.** Let $K$ be a finite sequence of positive integers. Then, there are positive integers $d_0, d_1, \ldots, d_s$ and $r_0, r_1, \ldots, r_s$ such that $r_{i+1} \neq r_i$ and $K = (r_0)|^{d_0}|(r_1)|^{d_1}|\ldots|(r_s)|^{d_r}$. We define the reduced length of $K$ by $\mathrm{rl}(K) = d_1 + d_2 + \ldots + d_s$. For example, with $K = (4,4,4,2,3,3) = (4)|^3|(2)|(3)|^2$, we have $d_0 = 3$, $d_1 = 1$, $d_2 = 2$, hence $\mathrm{rl}(K) = d_1 + d_2 = 3$.

Denote by $\mathsf{PSeq}_k^d$ the set of all pairs $(\mathcal{U},\mathcal{V})$ of sequences $\mathcal{U} = (u_1, u_2, \ldots, u_d)$, $\mathcal{V} = (v_1, v_2, \ldots, v_d)$, where $u_t$, $v_t$ are integers such that $1 \leqslant u_t < v_t \leqslant k$, for $1 \leqslant t \leqslant d$, and by $\mathsf{PInc}_k^d$ the set of all $(\mathcal{U},\mathcal{V}) \in \mathsf{PSeq}_k^d$ such that $u_1 \leqslant u_2 \leqslant \ldots \leqslant u_d$ and $v_1 \leqslant v_2 \leqslant \ldots \leqslant v_d$. By convention, $\mathsf{PSeq}_k^0 = \emptyset$. For $(\mathcal{U},\mathcal{V}) \in \mathsf{PSeq}_k^d$, we define

$$X_{(\mathcal{U},\mathcal{V})} = \prod_{1\leqslant t\leqslant d} X_{u_t,v_t}^{2^{d-t}} \in P_k((k-2)|^d).$$

Here, $X_{\mathbb{J}} = X_{j_1,j_2,\ldots,j_s} = \prod_{j\notin\mathbb{J}} x_j$ for $\mathbb{J} = \{j_1, j_2, \ldots, j_s\} \subset \{1, 2, \ldots k\}$.

**Definition 2.3.4.** Let $h_0$ be a positive integer, $h_0 > 2$, and $\mathcal{B}$ be a subset of $\mathsf{PInc}_k^{h_0}$. The set $\mathcal{B}$ is said to be compatible with $(k-2)|^{h_0}$ if the following conditions hold:

i) For any $(\mathcal{U},\mathcal{V}) \in \mathcal{B}$, $\mathrm{rl}(\mathcal{U}) \leqslant h_0 - 2$ and $\mathrm{rl}(\mathcal{V}) \leqslant h_0 - 2$,
ii) For any $(\mathcal{H},\mathcal{K}) \in \mathsf{PSeq}_k^{h_0}$, we have

$$X_{(\mathcal{H},\mathcal{K})} + \sum_{v=\min\mathcal{H}+1}^{\min\mathcal{K}} \sum_{(\mathcal{U},\mathcal{V})\in\mathcal{B}_v} X_{(\mathcal{U},\mathcal{V})} \in P_k^-((k-1)|^{h_0}) + \mathcal{A}_{h_0}^+ P_k + \mathcal{P}_{(k,n_{n_0})},$$

where $\mathcal{B}_v$ is a set of some pairs $(\mathcal{U},\mathcal{V}) \in \mathcal{B}$ such that $\min\mathcal{U} = \min\mathcal{H}$, $\min\mathcal{V} = v$ and $n_{h_0} = (k-2)(2^{h_0}-1)$.

**Theorem 2.3.5** (See [22]). *Let $h_0$ be a positive integer, $h_0 > 2$, and let $k \geqslant 4$, $m = \sum_{i=1}^{k-2}(2^{d_i}-1)$ with $d_i$ positive integers such that $d_1 > d_2 > \ldots > d_{k-3} \geqslant d_{k-2} = d \geqslant h_0$. Denote $q = \sum_{i=1}^{k-3}(2^{d_i-d}-1)$. Suppose the set $\mathcal{B} \subset \mathsf{PInc}_k^{h_0}$ is compatible with $(k-2)|^{h_0}$. Then,*

$$\overline{\mathcal{B}} := \bigcup_{(\mathcal{U},\mathcal{V})\in\mathcal{B}} \Big\{X_{(\mathcal{U},\mathcal{V})}(X_{i,j})^{2^d-2^{h_0}}(\theta_{J_{(i,j)}}(y))^{2^d} : y \in B_{k-2}(q)\Big\}$$

*is a set of generators for $\mathrm{Ker}(\widetilde{Sq}^0_*)_{(k,\frac{n}{2})}$, where $i = \min\mathcal{U} = i_1$, $j = \min\mathcal{V} = v_1$, $J_{(i,j)} = (1\ldots,\hat{i},\ldots,\hat{j},\ldots,k)$ and $B_{k-2}(q)$ is the set of all the admissible monomials of the degree $q$ in $P_{k-2}$. Consequently, $\dim\mathrm{Ker}(\widetilde{Sq}^0_*)_{(k,\frac{n}{2})} \leqslant |\mathcal{B}|\dim(QP_{k-2})_q$.*

We need the following for the proof of Theorem 1.3.

**Proposition 2.3.6** (See [22]). *The set*

$$\mathcal{B}_5 = \{(\mathcal{U},\mathcal{V}) \in \mathsf{PSeq}_5^6 : X_{(\mathcal{U},\mathcal{V})} \in B_5((3)|^6)\} \subset \mathsf{PInc}_5^6$$

*is compatible with $(3)|^6$.*

## 2.4 Some other needed results and notations

We set

$$P_k^0 = \langle\{x \in P_k : \nu_j(x) = 0 \text{ for some } j\}\rangle,$$
$$P_k^+ = \langle\{x \in P_k : \nu_j(x) > 0, \text{ for all } j\}\rangle.$$

Then, $P_k^0$ and $P_k^+$ are the $\mathcal{A}$-submodules of $P_k$ and $QP_k = QP_k^0 \oplus QP_k^+$, where $QP_k^0 = P_k^0/\mathcal{A}^+P_k^0$ and $QP_k^+ = P_k^+/\mathcal{A}^+P_k^+$.

We denote

$$\mathcal{N}_k = \{(i;I) : I = (t_1, t_2, \ldots, t_s), 1 \leqslant i < t_1 < \ldots < t_s \leqslant k,\ 0 \leqslant s < k\}.$$

For any $(i;I) \in \mathcal{N}_k$, we define the homomorphism $p_{(i;I)} : P_k \to P_{k-1}$ of $\mathcal{A}$-algebras by

$$p_{(i;I)}(x_u) = \begin{cases} x_u, & \text{if } 1 \leqslant u < i,\\ \sum_{t\in I} x_{t-1}, & \text{if } u = i,\\ x_{u-1}, & \text{if } i < u \leqslant k. \end{cases} \tag{2.2}$$

**Lemma 2.4.1** (See [9])**.** *For any monomial $u$ in $P_k$, we have*

$$p_{(i;I)}(u) \in P_{k-1}(\omega(u)).$$

It is easy to see that if $\omega$ is a weight vector of degree $m$ and $u \in P_k(\omega)$, then $p_{(i;I)}(u) \in P_{k-1}(\omega)$. Hence, $p_{(i;I)}$ passes to homomorphisms

$$p^{(\omega)}_{(i;I)} : QP_k(\omega) \longrightarrow QP_{k-1}(\omega),\ \ p^{(m)}_{(i;I)} : (QP_k)_m \longrightarrow (QP_{k-1})_m.$$

For $J = (s_1, s_2, \ldots, s_r) : 1 \leqslant s_1 < \ldots < s_r \leqslant k$, we define a monomorphism $\theta_J : P_r \to P_k$ of $\mathcal{A}$-algebras by substituting

$$\theta_J(x_v) = x_{s_v} \ \text{ for } \ 1 \leqslant v \leqslant r. \tag{2.3}$$

Clearly, for an arbitrary weight vector $\omega$ of degree $m$,

$$Q\theta_J(P_r^+)(\omega) \cong QP_r^+(\omega) \text{ and } (Q\theta_J(P_r^+))_m \cong (QP_r^+)_m$$

for $1 \leqslant r \leqslant k$. Here, $Q\theta_J(P_r^+) = \theta_J(P_r^+)/\mathcal{A}^+\theta_J(P_r^+)$. Then, by a simple calculation using the result in Wood [30] and (2.1), we obtain the following.

**Proposition 2.4.2** (See Walker and Wood [28])**.** *Suppose $\omega$ is a weight vector of degree $n$. Then, we have*

$$\dim QP_k(\omega) = \sum_{\mu(m)\leqslant u\leqslant k} \binom{k}{u} \dim QP_u^+(\omega),$$
$$\dim(QP_k)_n = \sum_{\mu(m)\leqslant u\leqslant k} \binom{k}{u} \dim(QP_u^+)_n.$$

Set $J_s = (1, \ldots, \hat{s}, \ldots, k)$ for $1 \leqslant s \leqslant k$.

**Proposition 2.4.3** (See Mothebe and Uys [5])**.** *Suppose $s$, $h$ are integers such that $1 \leqslant s \leqslant k$. If $f$ is an admissible monomial in $P_{k-1}$ then so is $x_s^{2^h-1}\theta_{J_s}(f)$ in $P_k$.*

**Notation 2.4.4.** For any nonnegative integer $m$, we denote $B_k(m)$ the set of all admissible monomials of degree $m$ in $P_k$. Set $B_k^0(m) = B_k(m) \cap P_k^0,\ B_k^+(m) = B_k(m) \cap P_k^+$. For any weight vector $\omega$ of degree $m$, we denote

$$B_k(\omega) = B_k(m) \cap P_k(\omega),\ B_k^+(\omega) = B_k^+(m) \cap P_k(\omega).$$

For a homogeneous polynomial $h \in P_k$, we denote $[h]$ the class in $QP_k$ represented by $h$. If $\omega$ is a weight vector and $h \in P_k(\omega)$, then we denote $[h]_\omega$ the class in $QP_k(\omega)$ represented by $h$. For a subset $H$ of $P_k$, we denote $[H] = \{[h] : h \in H\}$. For $H \subset P_k(\omega)$, we set $[H]_\omega = \{[h]_\omega : h \in H\}$. Then, $[B_k(\omega)]_\omega$ and $[B_k^+(\omega)]_\omega$, are respectively the basses of the spaces $QP_k(\omega)$ and $QP_k^+(\omega) := QP_k(\omega) \cap QP_k^+$. Denote by $|H|$ the cardinal of $H$.

# 3 The indecomposables of $P_5$ in the degree $2^{d+2}+2^{d+1}-3$

In this section, we consider Kameko's homomorphism

$$(\widetilde{Sq}^0_*)_{(5,q)} : (QP_5)_m \longrightarrow (QP_5)_q$$

where $q = 2^{d+1} + 2^d - 4$ and $m = 2q + 5 = 2^{d+2} + 2^{d+1} - 3$ with $d$ a positive integer.

## 3.1 The weight vector of admissible monomials in the space $\mathrm{Ker}\big((\widetilde{Sq}^0_*)_{(5,q)}$

First of all, we determine the weight vectors of the admissible monomials in $\mathrm{Ker}\big((\widetilde{Sq}^0_*)_{(5,q)}\big)$.

**Lemma 3.1.1.** *Let $x$ be an admissible monomial of degree $m = 2^{d+2}+2^{d+1}-3$ in $P_5$ with $d$ a positive integer. If $[x] \in \mathrm{Ker}\big((\widetilde{Sq}^0_*)_{(5,q)}\big)$, then either $\omega(x) = (3)|^d|(1)|^2$ or $\omega(x) = (3)|^{d+1}$.*

*Proof.* Since $x$ is admissible and $[x] \in \mathrm{Ker}\big((\widetilde{Sq}^0_*)_{(5,q)}\big)$, using Lemma 3.1.1 in [22] we get $\omega_i(x) = 3$ for $1 \leqslant i \leqslant d$. Then, $x = \bar{x}y^{2^d}$ with $\bar{x}$ a monomial of weight vector $(3)|^d$ and $y$ a monomial of degree 3 in $P_5$. Since $x$ is admissible, by Theorem 2.1.6, $y$ is also admissible. Hence, $\omega(y) = (1)|^2$ or $\omega(y) = (3)$. The lemma is proved. $\square$

By Lemma 3.1.1, we have

$$\mathrm{Ker}\big((\widetilde{Sq}^0_*)_{(5,q)}\big) \cong QP_5((3)|^{d+1}) \bigoplus QP_5((3)|^d|(1)|^2).$$

The space $QP_5((3)|^{d+1})$ has been determined in [22, Theorem 4.2].

**Proposition 3.1.2.** *If $d \geqslant 4$, then we have* $\dim QP_5((3)|^{d+1}) = 155$.

The admissible monomials of weight vector $(3)|^{d+1}$ are listed in the Appendix of the work [22].

## 3.2 Proof of Theorem 1.3

To prove Theorem 1.3, we need only to compute the space $QP_5((3)|^d|(1)|^2)$. We need some technical lemmas. The following follows from [20].

**Lemma 3.2.1.** *Let $\{i, j, t, u\}$ be a subset of $\{1, 2, 3, 4, 5\}$ such that $i < j < t < u$. The following monomials are strictly inadmissible:*

$$x_i^2x_jx_tx_u,\ x_i^3x_j^4x_t^3x_u^3,\ x_i^7x_j^8x_t^3x_u^3,\ x_i^7x_j^7x_t^8x_u^7,\ x_i^{15}x_j^{15}x_t^{15}x_u^{16}.$$

**Lemma 3.2.2.**

i) *If $v$ is one of the monomials:*

$$\begin{array}{llll} x_1^7x_2^7x_3^{15}x_4^{16} & x_1^7x_2^{15}x_3^7x_4^{16} & x_1^7x_2^{15}x_3^{16}x_4^7 & x_1^7x_2^{15}x_3^{17}x_4^6 \\ x_1^{15}x_2^7x_3^7x_4^{16} & x_1^{15}x_2^7x_3^{16}x_4^7 & x_1^{15}x_2^7x_3^{17}x_4^6 & x_1^{15}x_2^{19}x_3^5x_4^6, \end{array}$$

*then the monomial $\theta_{J_s}(x)$, $1 \leqslant s \leqslant 5$, is strictly inadmissible.*

ii) *Let $(i, j, t, u, v)$ be an arbitrary permutation of $(1, 2, 3, 4, 5)$. The following monomials are strictly inadmissible:*

$$\begin{array}{l} x_i^{15}x_j^{19}x_tx_u^4x_v^6;\ x_i^{15}x_j^{18}x_tx_u^4x_v^7;\ x_i^{15}x_j^{18}x_tx_u^5x_v^6;\ x_i^{15}x_j^{17}x_tx_u^6x_v^6;\ x_i^{15}x_j^{16}x_tx_u^6x_v^7; \\ x_i^{15}x_j^{19}x_t^5x_u^2x_v^4;\ x_i^{15}x_j^{17}x_t^2x_u^4x_v^7;\ x_i^{15}x_j^{19}x_t^3x_u^4x_v^4;\ x_i^{15}x_j^{20}x_t^3x_u^3x_v^4; \\ x_ix_j^{14}x_t^{16}x_u^7x_v^7;\ i < u;\ x_i^{15}x_j^{17}x_t^3x_u^6x_v^4,\ j < v;\ x_i^{15}x_j^{16}x_t^3x_u^5x_v^6,\ j < u < v. \end{array}$$

iii) *The following monomials are strictly inadmissible:*

$$\begin{array}{lllll} x_1x_2^{14}x_3^{19}x_4^5x_5^6 & x_1^3x_2^7x_3^{11}x_4^4x_5^{20} & x_1^3x_2^7x_3^{11}x_4^{20}x_5^4 & x_1^3x_2^7x_3^{24}x_4^5x_5^6 & x_1^3x_2^{13}x_3^6x_4^{16}x_5^7 \\ x_1^3x_2^{13}x_3^6x_4^{17}x_5^6 & x_1^3x_2^{13}x_3^7x_4^{16}x_5^6 & x_1^3x_2^{13}x_3^7x_4^{18}x_5^4 & x_1^3x_2^{13}x_3^{16}x_4^6x_5^7 & x_1^3x_2^{13}x_3^{16}x_4^7x_5^6 \\ x_1^3x_2^{13}x_3^{17}x_4^6x_5^6 & x_1^3x_2^{13}x_3^{18}x_4^4x_5^7 & x_1^3x_2^{13}x_3^{18}x_4^5x_5^6 & x_1^3x_2^{13}x_3^{18}x_4^7x_5^4 & x_1^3x_2^{13}x_3^{19}x_4^4x_5^6 \\ x_1^3x_2^{13}x_3^{19}x_4^6x_5^4 & x_1^7x_2^3x_3^{11}x_4^4x_5^{20} & x_1^7x_2^3x_3^{11}x_4^{20}x_5^4 & x_1^7x_2^3x_3^{24}x_4^5x_5^6 & x_1^7x_2^9x_3^2x_4^5x_5^{22} \\ x_1^7x_2^9x_3^2x_4^{21}x_5^6 & x_1^7x_2^9x_3^3x_4^4x_5^{22} & x_1^7x_2^9x_3^3x_4^{20}x_5^6 & x_1^7x_2^9x_3^3x_4^{22}x_5^4 & x_1^7x_2^9x_3^{18}x_4^5x_5^6 \\ x_1^7x_2^9x_3^{19}x_4^4x_5^6 & x_1^7x_2^9x_3^{19}x_4^6x_5^4 & x_1^7x_2^{11}x_3^3x_4^4x_5^{20} & x_1^7x_2^{11}x_3^3x_4^{20}x_5^4 & x_1^7x_2^{11}x_3^{16}x_4^5x_5^6 \\ x_1^7x_2^{11}x_3^{19}x_4^4x_5^4 & x_1^7x_2^{25}x_3^2x_4^5x_5^6 & x_1^7x_2^{25}x_3^3x_4^4x_5^6 & x_1^7x_2^{25}x_3^3x_4^6x_5^4 & \end{array}$$

*Proof.* Note that all monomials in the lemma are of weight vector $(3)|^3|(1)|^2$. Part i) has been prove in [20]. We prove Part ii) for $x = x_1x_2^{15}x_3^{19}x_4^4x_5^6$, $y = x_1^7x_2^{15}x_3x_4^{16}x_5^6$ and $z = x_1^{15}x_2x_3^6x_4^{16}x_5^7$. We have

$$x = x_1x_2^8x_3^{23}x_4^6x_5^7 + x_1x_2^9x_3^{23}x_4^6x_5^6 + x_1x_2^{10}x_3^{23}x_4^4x_5^7 + x_1x_2^{11}x_3^{23}x_4^4x_5^6$$

$$\begin{aligned}
&+ x_1x_2^{15}x_3^{16}x_4^6x_5^7 + x_1x_2^{15}x_3^{17}x_4^6x_5^6 + x_1x_2^{15}x_3^{18}x_4^4x_5^7\\
&+ Sq^1\big(x_1x_2^{15}x_3^{15}x_4^4x_5^9 + x_1x_2^{15}x_3^{15}x_4^5x_5^8 + x_1x_2^{15}x_3^{16}x_4^5x_5^7 + x_1x_2^{15}x_3^{17}x_4^4x_5^7\\
&+ x_1x_2^{16}x_3^{15}x_4^5x_5^7 + x_1x_2^{17}x_3^{15}x_4^4x_5^7 + x_1^4x_2^{15}x_3^{15}x_4^3x_5^7\big) + Sq^2\big(x_1x_2^{15}x_3^{15}x_4^6x_5^6\\
&+ x_1^2x_2^{15}x_3^{15}x_4^3x_5^8 + x_1^2x_2^{15}x_3^{16}x_4^3x_5^7 + x_1^2x_2^{16}x_3^{15}x_4^3x_5^7\big)\\
&+ Sq^4\big(x_1x_2^{15}x_3^{15}x_4^4x_5^6\big) + Sq^8\big(x_1x_2^8x_3^{15}x_4^6x_5^7 + x_1x_2^9x_3^{15}x_4^6x_5^6\\
&+ x_1x_2^{10}x_3^{15}x_4^4x_5^7 + x_1x_2^{11}x_3^{15}x_4^4x_5^6\big) \mod(P_5^-((3)|^3|(1)|^2)),\\
y &= x_1x_2^{15}x_3^4x_4^7x_5^{18} + x_1x_2^{15}x_3^4x_4^{18}x_5^7 + x_1x_2^{19}x_3^4x_4^7x_5^{14} + x_1x_2^{19}x_3^4x_4^{14}x_5^7\\
&+ x_1^4x_2^{19}x_3^4x_4^7x_5^{11} + x_1^4x_2^{19}x_3^4x_4^{11}x_5^7 + x_1^5x_2^{15}x_3x_4^6x_5^{18} + x_1^5x_2^{15}x_3x_4^{18}x_5^6\\
&+ x_1^5x_2^{15}x_3^2x_4^7x_5^{16} + x_1^5x_2^{15}x_3^2x_4^{16}x_5^7 + x_1^5x_2^{19}x_3x_4^6x_5^{14} + x_1^5x_2^{19}x_3x_4^{14}x_5^6\\
&+ x_1^5x_2^{19}x_3^2x_4^7x_5^{12} + x_1^5x_2^{19}x_3^2x_4^{12}x_5^7 + x_1^7x_2^8x_3^4x_4^7x_5^{19} + x_1^7x_2^8x_3^4x_4^{19}x_5^7\\
&+ x_1^7x_2^9x_3x_4^6x_5^{22} + x_1^7x_2^9x_3x_4^{22}x_5^6 + x_1^7x_2^9x_3^2x_4^7x_5^{20} + x_1^7x_2^9x_3^2x_4^{20}x_5^7\\
&+ x_1^7x_2^{15}x_3x_4^6x_5^{16} + Sq^1\big(x_1x_2^{15}x_3^2x_4^{13}x_5^{13} + x_1x_2^{15}x_3^8x_4^7x_5^{13} + x_1x_2^{15}x_3^8x_4^{13}x_5^7\\
&+ x_1^3x_2^{15}x_3x_4^{12}x_5^{13} + x_1^3x_2^{15}x_3x_4^{13}x_5^{12} + x_1^3x_2^{15}x_3^4x_4^{11}x_5^{11} + x_1^7x_2^{15}x_3x_4^8x_5^{13}\\
&+ x_1^7x_2^{15}x_3x_4^9x_5^{12} + x_1^7x_2^{15}x_3x_4^{12}x_5^9 + x_1^7x_2^{15}x_3x_4^{13}x_5^8 + x_1^7x_2^{15}x_3^4x_4^7x_5^{11}\\
&+ x_1^7x_2^{15}x_3^4x_4^{11}x_5^7\big) + Sq^2\big(x_1x_2^{15}x_3^2x_4^{11}x_5^{14} + x_1x_2^{15}x_3^2x_4^{14}x_5^{11} + x_1^2x_2^{15}x_3^4x_4^{11}x_5^{11}\\
&+ x_1^2x_2^{15}x_3^8x_4^7x_5^{11} + x_1^2x_2^{15}x_3^8x_4^{11}x_5^7 + x_1^3x_2^{15}x_3x_4^{10}x_5^{14} + x_1^3x_2^{15}x_3x_4^{14}x_5^{10}\\
&+ x_1^3x_2^{15}x_3^2x_4^{11}x_5^{12} + x_1^3x_2^{15}x_3^2x_4^{12}x_5^{11} + x_1^4x_2^{15}x_3^2x_4^{11}x_5^{11} + x_1^7x_2^{15}x_3x_4^6x_5^{14}\\
&+ x_1^7x_2^{15}x_3x_4^{10}x_5^{10} + x_1^7x_2^{15}x_3x_4^{14}x_5^6 + x_1^7x_2^{15}x_3^2x_4^7x_5^{12} + x_1^7x_2^{15}x_3^2x_4^8x_5^{11}\\
&+ x_1^7x_2^{15}x_3^2x_4^{11}x_5^8 + x_1^7x_2^{15}x_3^2x_4^{12}x_5^7\big) + Sq^4\big(x_1x_2^{15}x_3^4x_4^7x_5^{14}\\
&+ x_1x_2^{15}x_3^4x_4^{14}x_5^7 + x_1^2x_2^{15}x_3^2x_4^{11}x_5^{11} + x_1^4x_2^{15}x_3^4x_4^7x_5^{11} + x_1^4x_2^{15}x_3^4x_4^{11}x_5^7\\
&+ x_1^5x_2^{15}x_3x_4^6x_5^{14} + x_1^5x_2^{15}x_3x_4^{14}x_5^6 + x_1^5x_2^{15}x_3^2x_4^7x_5^{12} + x_1^5x_2^{15}x_3^2x_4^{12}x_5^7\big)\\
&+ Sq^8\big(x_1^7x_2^8x_3^4x_4^7x_5^{11} + x_1^7x_2^8x_3^4x_4^{11}x_5^7 + x_1^7x_2^9x_3x_4^6x_5^{14} + x_1^7x_2^9x_3x_4^{14}x_5^6\\
&+ x_1^7x_2^9x_3^2x_4^7x_5^{12} + x_1^7x_2^9x_3^2x_4^{12}x_5^7\big) \mod(P_5^-((3)|^3|(1)|^2)),
\end{aligned}$$

Hence, the monomials $x$ and $y$ are strictly inadmissible. We have

$$\begin{aligned}
z &= x_1^9x_2x_3^6x_4^7x_5^{22} + x_1^9x_2x_3^6x_4^{22}x_5^7 + x_1^{11}x_2x_3^4x_4^7x_5^{22} + x_1^{11}x_2x_3^4x_4^{22}x_5^7\\
&+ x_1^{15}x_2x_3^4x_4^7x_5^{18} + x_1^{15}x_2x_3^4x_4^{18}x_5^7 + x_1^{15}x_2x_3^6x_4^7x_5^{16} + Sq^1\big(x_1^{15}x_2x_3^2x_4^{13}x_5^{13}\\
&+ x_1^{15}x_2x_3^3x_4^{13}x_5^{12} + x_1^{15}x_2x_3^5x_4^{11}x_5^{12} + x_1^{15}x_2^2x_3^5x_4^9x_5^{13} + x_1^{15}x_2^2x_3^5x_4^{13}x_5^9\\
&+ x_1^{15}x_2^4x_3^3x_4^{11}x_5^{11} + x_1^{19}x_2x_3^2x_4^{11}x_5^{11} + x_1^{19}x_2x_3^3x_4^{10}x_5^{11}\big) + Sq^2\big(x_1^{15}x_2x_3^2x_4^{11}x_5^{14}\\
&+ x_1^{15}x_2x_3^2x_4^{14}x_5^{11} + x_1^{15}x_2x_3^3x_4^{14}x_5^{10} + x_1^{15}x_2x_3^5x_4^9x_5^{13} + x_1^{15}x_2x_3^5x_4^{13}x_5^9\\
&+ x_1^{15}x_2x_3^6x_4^7x_5^{14} + x_1^{15}x_2x_3^6x_4^{11}x_5^{10} + x_1^{15}x_2x_3^6x_4^{14}x_5^7 + x_1^{15}x_2^2x_3^3x_4^{11}x_5^{12}\\
&+ x_1^{15}x_2^2x_3^3x_4^{12}x_5^{11} + x_1^{15}x_2^2x_3^4x_4^{11}x_5^{11} + x_1^{15}x_2^4x_3^2x_4^{11}x_5^{11} + x_1^{15}x_2^4x_3^3x_4^{10}x_5^{11}\\
&+ x_1^{17}x_2x_3^3x_4^9x_5^{13} + x_1^{17}x_2x_3^3x_4^{13}x_5^9\big) + Sq^4\big(x_1^{15}x_2x_3^4x_4^7x_5^{14} + x_1^{15}x_2x_3^4x_4^{14}x_5^7
\end{aligned}$$

$$+ x_1^{15}x_2^{2}x_3^{2}x_4^{11}x_5^{11} + x_1^{15}x_2^{2}x_3^{3}x_4^{10}x_5^{11}\big) + Sq^8\big(x_1^{9}x_2x_3^{6}x_4^{7}x_5^{14} + x_1^{9}x_2x_3^{6}x_4^{14}x_5^{7}$$
$$+ x_1^{11}x_2x_3^{4}x_4^{7}x_5^{14} + x_1^{11}x_2x_3^{4}x_4^{14}x_5^{7}\big) \mod(P_5^-((3)|^3|(1)|^2)),$$

Hence, the monomial $z$ is strictly inadmissible.

We now prove Part iii) for $u = x_1x_2^{14}x_3^{19}x_4^{5}x_5^{6}$. We have

$$\begin{aligned}
u = {} & x_1x_2^{5}x_3^{15}x_4^{6}x_5^{18} + x_1x_2^{5}x_3^{19}x_4^{6}x_5^{14} + x_1x_2^{6}x_3^{15}x_4^{7}x_5^{16} + x_1x_2^{6}x_3^{15}x_4^{16}x_5^{7} \\
& + x_1x_2^{6}x_3^{17}x_4^{7}x_5^{14} + x_1x_2^{6}x_3^{19}x_4^{12}x_5^{7} + x_1x_2^{7}x_3^{16}x_4^{7}x_5^{14} + x_1x_2^{7}x_3^{17}x_4^{6}x_5^{14} \\
& + x_1x_2^{7}x_3^{18}x_4^{7}x_5^{12} + x_1x_2^{7}x_3^{19}x_4^{6}x_5^{12} + x_1x_2^{8}x_3^{23}x_4^{6}x_5^{7} + x_1x_2^{8}x_3^{23}x_4^{7}x_5^{6} \\
& + x_1x_2^{10}x_3^{23}x_4^{5}x_5^{6} + x_1x_2^{12}x_3^{18}x_4^{7}x_5^{7} + x_1x_2^{14}x_3^{16}x_4^{7}x_5^{7} + x_1x_2^{14}x_3^{17}x_4^{6}x_5^{7} \\
& + x_1x_2^{14}x_3^{17}x_4^{7}x_5^{6} + Sq^1\big(x_1x_2^{3}x_3^{15}x_4^{12}x_5^{13} + x_1x_2^{3}x_3^{19}x_4^{10}x_5^{11} + x_1x_2^{5}x_3^{15}x_4^{7}x_5^{16} \\
& + x_1x_2^{5}x_3^{15}x_4^{10}x_5^{13} + x_1x_2^{5}x_3^{15}x_4^{11}x_5^{12} + x_1x_2^{5}x_3^{15}x_4^{12}x_5^{11} + x_1x_2^{5}x_3^{19}x_4^{7}x_5^{12} \\
& + x_1x_2^{7}x_3^{15}x_4^{8}x_5^{13} + x_1x_2^{7}x_3^{15}x_4^{9}x_5^{12} + x_1x_2^{7}x_3^{16}x_4^{7}x_5^{13} + x_1x_2^{7}x_3^{17}x_4^{7}x_5^{12} \\
& + x_1x_2^{8}x_3^{15}x_4^{7}x_5^{13} + x_1x_2^{12}x_3^{15}x_4^{7}x_5^{9} + x_1x_2^{12}x_3^{15}x_4^{9}x_5^{7} + x_1x_2^{12}x_3^{17}x_4^{7}x_5^{7} \\
& + x_1x_2^{13}x_3^{15}x_4^{7}x_5^{8} + x_1x_2^{13}x_3^{15}x_4^{8}x_5^{7} + x_1x_2^{13}x_3^{16}x_4^{7}x_5^{7} + x_1^{2}x_2^{5}x_3^{15}x_4^{9}x_5^{13} \\
& + x_1^{2}x_2^{13}x_3^{15}x_4^{5}x_5^{9} + x_1^{4}x_2^{7}x_3^{15}x_4^{7}x_5^{11} + x_1^{4}x_2^{11}x_3^{15}x_4^{7}x_5^{7}\big) + Sq^2\big(x_1x_2^{3}x_3^{15}x_4^{10}x_5^{14} \\
& + x_1x_2^{5}x_3^{15}x_4^{9}x_5^{13} + x_1x_2^{6}x_3^{15}x_4^{7}x_5^{14} + x_1x_2^{7}x_3^{15}x_4^{6}x_5^{14} + x_1x_2^{10}x_3^{15}x_4^{7}x_5^{10} \\
& + x_1x_2^{10}x_3^{15}x_4^{10}x_5^{7} + x_1x_2^{11}x_3^{15}x_4^{6}x_5^{10} + x_1x_2^{13}x_3^{15}x_4^{5}x_5^{9} + x_1x_2^{14}x_3^{15}x_4^{6}x_5^{7} \\
& + x_1x_2^{14}x_3^{15}x_4^{7}x_5^{6} + x_1^{2}x_2^{7}x_3^{15}x_4^{7}x_5^{12} + x_1^{2}x_2^{7}x_3^{15}x_4^{8}x_5^{11} + x_1^{2}x_2^{7}x_3^{16}x_4^{7}x_5^{11} \\
& + x_1^{2}x_2^{8}x_3^{15}x_4^{7}x_5^{11} + x_1^{2}x_2^{11}x_3^{15}x_4^{7}x_5^{8} + x_1^{2}x_2^{11}x_3^{15}x_4^{8}x_5^{7} + x_1^{2}x_2^{11}x_3^{16}x_4^{7}x_5^{7} \\
& + x_1^{2}x_2^{12}x_3^{15}x_4^{7}x_5^{7} + x_1^{4}x_2^{3}x_3^{15}x_4^{10}x_5^{11} + x_1x_2^{3}x_3^{17}x_4^{9}x_5^{13} + x_1x_2^{11}x_3^{17}x_4^{5}x_5^{9}\big) \\
& + Sq^4\big(x_1x_2^{5}x_3^{15}x_4^{6}x_5^{14} + x_1x_2^{6}x_3^{15}x_4^{7}x_5^{12} + x_1x_2^{6}x_3^{15}x_4^{12}x_5^{7} + x_1x_2^{7}x_3^{15}x_4^{6}x_5^{12} \\
& + x_1x_2^{14}x_3^{15}x_4^{5}x_5^{6} + x_1^{2}x_2^{3}x_3^{15}x_4^{10}x_5^{11} + x_1^{2}x_2^{5}x_3^{15}x_4^{7}x_5^{12}\big) + Sq^8\big(x_1x_2^{8}x_3^{15}x_4^{6}x_5^{7} \\
& + x_1x_2^{8}x_3^{15}x_4^{7}x_5^{6} + x_1x_2^{10}x_3^{15}x_4^{5}x_5^{6}\big) \mod(P_5^-((3)|^3|(1)|^2)),
\end{aligned}$$

The above equality shows that the monomial $u$ is strictly inadmissible. $\square$

**Lemma 3.2.3.** *Let $(i,j,t,u,v)$ be an arbitrary permutation of $(1,2,3,4,5)$. The following monomials are strictly inadmissible:*

$$x_i^{3}x_j^{13}x_t^{14}x_u^{31}x_v^{32},\ x_i^{3}x_j^{13}x_t^{30}x_u^{15}x_v^{32},\ x_i^{3}x_j^{29}x_t^{14}x_u^{15}x_v^{32},\ i<j<t, u<v$$

*and*

| | | | |
|---|---|---|---|
| $x_1^{3}x_2^{13}x_3^{30}x_4^{33}x_5^{14}$ | $x_1^{3}x_2^{13}x_3^{31}x_4^{34}x_5^{12}$ | $x_1^{3}x_2^{15}x_3^{29}x_4^{34}x_5^{12}$ | $x_1^{3}x_2^{29}x_3^{14}x_4^{33}x_5^{14}$ |
| $x_1^{3}x_2^{29}x_3^{15}x_4^{34}x_5^{12}$ | $x_1^{3}x_2^{31}x_3^{13}x_4^{34}x_5^{12}$ | $x_1^{3}x_2^{31}x_3^{37}x_4^{10}x_5^{12}$ | $x_1^{7}x_2^{7}x_3^{9}x_4^{10}x_5^{60}$ |
| $x_1^{7}x_2^{11}x_3^{13}x_4^{30}x_5^{32}$ | $x_1^{7}x_2^{11}x_3^{29}x_4^{14}x_5^{32}$ | $x_1^{7}x_2^{11}x_3^{29}x_4^{32}x_5^{14}$ | $x_1^{7}x_2^{11}x_3^{29}x_4^{34}x_5^{12}$ |
| $x_1^{7}x_2^{27}x_3^{13}x_4^{14}x_5^{32}$ | $x_1^{7}x_2^{27}x_3^{13}x_4^{32}x_5^{14}$ | $x_1^{7}x_2^{27}x_3^{13}x_4^{34}x_5^{12}$ | $x_1^{7}x_2^{27}x_3^{37}x_4^{10}x_5^{12}$ |
| $x_1^{15}x_2^{3}x_3^{29}x_4^{34}x_5^{12}$ | $x_1^{15}x_2^{31}x_3^{33}x_4^{2}x_5^{12}$ | $x_1^{31}x_2^{3}x_3^{13}x_4^{34}x_5^{12}$ | $x_1^{31}x_2^{3}x_3^{37}x_4^{10}x_5^{12}$ |
| $x_1^{31}x_2^{15}x_3^{33}x_4^{2}x_5^{12}$ | $x_1^{31}x_2^{35}x_3^{5}x_4^{10}x_5^{12}$ | | |

*Proof.* The monomials in the lemma are of weight vector $(3)|^4|(1)|^2$. We prove the lemma for $v = x_1^3x_2^{31}x_3^{37}x_4^{10}x_5^{12}$ and $w = x_1^{15}x_2^3x_3^{29}x_4^{32}x_5^{14}$. By a direct computation using the Cartan formula we have

$$\begin{aligned}
v &= x_1x_2^{31}x_3^{15}x_4^{32}x_5^{14} + x_1x_2^{31}x_3^{38}x_4^{9}x_5^{14} + x_1x_2^{39}x_3^{15}x_4^{24}x_5^{14} + x_1x_2^{39}x_3^{30}x_4^{9}x_5^{14}\\
&\quad + x_1^2x_2^{35}x_3^{29}x_4^{13}x_5^{14} + x_1^2x_2^{37}x_3^{27}x_4^{13}x_5^{14} + x_1^3x_2^{17}x_3^{45}x_4^{14}x_5^{14} + x_1^3x_2^{19}x_3^{45}x_4^{12}x_5^{14}\\
&\quad + x_1^3x_2^{19}x_3^{45}x_4^{14}x_5^{12} + x_1^3x_2^{20}x_3^{43}x_4^{13}x_5^{14} + x_1^3x_2^{23}x_3^{15}x_4^{12}x_5^{40} + x_1^3x_2^{23}x_3^{45}x_4^{10}x_5^{12}\\
&\quad + x_1^3x_2^{31}x_3^{15}x_4^{12}x_5^{32} + x_1^3x_2^{31}x_3^{32}x_4^{13}x_5^{14} + x_1^3x_2^{31}x_3^{33}x_4^{12}x_5^{14} + x_1^3x_2^{31}x_3^{33}x_4^{14}x_5^{12}\\
&\quad + Sq^1\big(x_1x_2^{31}x_3^{15}x_4^{21}x_5^{24} + x_1x_2^{31}x_3^{29}x_4^{11}x_5^{20} + x_1x_2^{31}x_3^{29}x_4^{17}x_5^{14}\\
&\quad + x_1x_2^{35}x_3^{27}x_4^{11}x_5^{18} + x_1^3x_2^{31}x_3^{15}x_4^{19}x_5^{24} + x_1^3x_2^{31}x_3^{27}x_4^{13}x_5^{18} + x_1^3x_2^{31}x_3^{27}x_4^{17}x_5^{14}\\
&\quad + x_1^3x_2^{35}x_3^{27}x_4^{9}x_5^{18} + x_1^3x_2^{35}x_3^{27}x_4^{13}x_5^{14}\big) + Sq^2\big(x_1x_2^{31}x_3^{15}x_4^{22}x_5^{22}\\
&\quad + x_1x_2^{31}x_3^{27}x_4^{18}x_5^{14} + x_1x_2^{31}x_3^{30}x_4^{7}x_5^{22} + x_1^2x_2^{31}x_3^{15}x_4^{19}x_5^{24} + x_1^2x_2^{31}x_3^{27}x_4^{13}x_5^{18}\\
&\quad + x_1^2x_2^{31}x_3^{27}x_4^{17}x_5^{14} + x_1^2x_2^{35}x_3^{27}x_4^{13}x_5^{14} + x_1^4x_2^{31}x_3^{27}x_4^{11}x_5^{18} + x_1^5x_2^{31}x_3^{27}x_4^{10}x_5^{18}\\
&\quad + x_1^5x_2^{31}x_3^{27}x_4^{14}x_5^{14}\big) + Sq^4\big(x_1x_2^{31}x_3^{23}x_4^{20}x_5^{14} + x_1x_2^{31}x_3^{30}x_4^{7}x_5^{20}\\
&\quad + x_1^2x_2^{31}x_3^{27}x_4^{11}x_5^{18} + x_1^3x_2^{31}x_3^{23}x_4^{12}x_5^{20} + x_1^3x_2^{31}x_3^{27}x_4^{10}x_5^{18} + x_1^3x_2^{31}x_3^{27}x_4^{14}x_5^{14}\\
&\quad + x_1^3x_2^{31}x_3^{28}x_4^{13}x_5^{14} + x_1^3x_2^{31}x_3^{29}x_4^{12}x_5^{14} + x_1^3x_2^{31}x_3^{29}x_4^{14}x_5^{12}\big)\\
&\quad + Sq^8\big(x_1x_2^{31}x_3^{15}x_4^{24}x_5^{14} + x_1x_2^{31}x_3^{30}x_4^{9}x_5^{14} + x_1^3x_2^{31}x_3^{15}x_4^{12}x_5^{24}\\
&\quad + x_1^3x_2^{31}x_3^{29}x_4^{10}x_5^{12}\big) + Sq^{16}\big(x_1^3x_2^{17}x_3^{29}x_4^{14}x_5^{14} + x_1^3x_2^{19}x_3^{29}x_4^{12}x_5^{14}\\
&\quad + x_1^3x_2^{19}x_3^{29}x_4^{14}x_5^{12} + x_1^3x_2^{20}x_3^{27}x_4^{13}x_5^{14} + x_1^3x_2^{23}x_3^{15}x_4^{12}x_5^{24}\\
&\quad + x_1^3x_2^{23}x_3^{29}x_4^{10}x_5^{12}\big) \mod(P_5^-((3)|^4|(1)|^2)),\\
w &= x_1^{11}x_2^3x_3^{15}x_4^{28}x_5^{36} + x_1^{11}x_2^3x_3^{15}x_4^{36}x_5^{28} + x_1^{11}x_2^3x_3^{29}x_4^{14}x_5^{36} + x_1^{11}x_2^3x_3^{29}x_4^{36}x_5^{14}\\
&\quad + x_1^{11}x_2^3x_3^{37}x_4^{14}x_5^{28} + x_1^{11}x_2^3x_3^{37}x_4^{28}x_5^{14} + x_1^{15}x_2^3x_3^{15}x_4^{28}x_5^{32} + x_1^{15}x_2^3x_3^{15}x_4^{32}x_5^{28}\\
&\quad + x_1^{15}x_2^3x_3^{17}x_4^{14}x_5^{44} + x_1^{15}x_2^3x_3^{17}x_4^{44}x_5^{14} + x_1^{15}x_2^3x_3^{29}x_4^{14}x_5^{32}\\
&\quad + Sq^1\big(x_1^{11}x_2^3x_3^{27}x_4^{25}x_5^{26} + x_1^{19}x_2^3x_3^{19}x_4^{25}x_5^{26} + x_1^{19}x_2^3x_3^{23}x_4^{21}x_5^{26}\\
&\quad + x_1^{19}x_2^3x_3^{23}x_4^{25}x_5^{22} + x_1^{19}x_2^3x_3^{27}x_4^{17}x_5^{26} + x_1^{19}x_2^3x_3^{27}x_4^{21}x_5^{22} + x_1^{19}x_2^3x_3^{27}x_4^{25}x_5^{18}\big)\\
&\quad + Sq^2\big(x_1^{15}x_2^5x_3^{27}x_4^{26}x_5^{18} + x_1^{15}x_2^5x_3^{27}x_4^{22}x_5^{22} + x_1^{15}x_2^5x_3^{23}x_4^{26}x_5^{22}\\
&\quad + x_1^{15}x_2^5x_3^{19}x_4^{26}x_5^{26} + x_1^{15}x_2^5x_3^{27}x_4^{18}x_5^{26} + x_1^7x_2^5x_3^{27}x_4^{26}x_5^{26}\big)\\
&\quad + Sq^4\big(x_1^7x_2^3x_3^{23}x_4^{28}x_5^{28} + x_1^7x_2^3x_3^{27}x_4^{26}x_5^{26} + x_1^7x_2^3x_3^{29}x_4^{22}x_5^{28} + x_1^7x_2^3x_3^{29}x_4^{28}x_5^{22}\\
&\quad + x_1^{15}x_2^3x_3^{15}x_4^{28}x_5^{28} + x_1^{15}x_2^3x_3^{19}x_4^{26}x_5^{26} + x_1^{15}x_2^3x_3^{21}x_4^{22}x_5^{28} + x_1^{15}x_2^3x_3^{21}x_4^{28}x_5^{22}\\
&\quad + x_1^{15}x_2^3x_3^{23}x_4^{20}x_5^{28} + x_1^{15}x_2^3x_3^{23}x_4^{26}x_5^{22} + x_1^{15}x_2^3x_3^{23}x_4^{28}x_5^{20} + x_1^{15}x_2^3x_3^{27}x_4^{18}x_5^{26}\\
&\quad + x_1^{15}x_2^3x_3^{27}x_4^{22}x_5^{22} + x_1^{15}x_2^3x_3^{27}x_4^{26}x_5^{18} + x_1^{15}x_2^3x_3^{29}x_4^{14}x_5^{28} + x_1^{15}x_2^3x_3^{29}x_4^{20}x_5^{22}\\
&\quad + x_1^{15}x_2^3x_3^{29}x_4^{22}x_5^{20} + x_1^{15}x_2^3x_3^{29}x_4^{28}x_5^{14}\big) + Sq^8\big(x_1^{11}x_2^3x_3^{15}x_4^{28}x_5^{28}\\
&\quad + x_1^{11}x_2^3x_3^{29}x_4^{14}x_5^{28} + x_1^{11}x_2^3x_3^{29}x_4^{28}x_5^{14}\big) + Sq^{16}\big(x_1^{15}x_2^3x_3^{17}x_4^{14}x_5^{28}
\end{aligned}$$

$$+ x_1^{15}x_2^{3}x_3^{17}x_4^{28}x_5^{14}\big) \mod(P_5^-((3)|^4|(1)|^2)).$$

Thus, the monomials $v$ and $w$ are strictly inadmissible. $\square$

**Lemma 3.2.4.** *The following monomials are strictly inadmissible:*

$$\begin{array}{llll} x_1^{3}x_2^{29}x_3^{30}x_4^{31}x_5^{32} & x_1^{3}x_2^{29}x_3^{31}x_4^{30}x_5^{32} & x_1^{3}x_2^{31}x_3^{29}x_4^{30}x_5^{32} & x_1^{7}x_2^{27}x_3^{29}x_4^{30}x_5^{32} \\ x_1^{7}x_2^{27}x_3^{29}x_4^{30}x_5^{32} & x_1^{7}x_2^{27}x_3^{29}x_4^{32}x_5^{30} & x_1^{7}x_2^{27}x_3^{29}x_4^{34}x_5^{28} & x_1^{31}x_2^{3}x_3^{29}x_4^{30}x_5^{32} \end{array}$$

*Proof.* The monomials in the lemma are of weight vector $(3)|^5|(1)$. We prove the lemma for $t = x_1^3x_2^{29}x_3^{30}x_4^{31}x_5^{32}$. A direct computation shows that

$$\begin{aligned} t &= x_1^{2}x_2^{27}x_3^{29}x_4^{31}x_5^{36} + x_1^{2}x_2^{27}x_3^{29}x_4^{37}x_5^{30} + x_1^{2}x_2^{29}x_3^{29}x_4^{31}x_5^{34} + x_1^{2}x_2^{29}x_3^{29}x_4^{35}x_5^{30} \\ &+ x_1^{2}x_2^{29}x_3^{33}x_4^{31}x_5^{30} + x_1^{3}x_2^{17}x_3^{30}x_4^{29}x_5^{46} + x_1^{3}x_2^{17}x_3^{30}x_4^{45}x_5^{30} + x_1^{3}x_2^{17}x_3^{46}x_4^{29}x_5^{30} \\ &+ x_1^{3}x_2^{21}x_3^{24}x_4^{31}x_5^{46} + x_1^{3}x_2^{21}x_3^{24}x_4^{47}x_5^{30} + x_1^{3}x_2^{21}x_3^{26}x_4^{29}x_5^{46} + x_1^{3}x_2^{21}x_3^{26}x_4^{45}x_5^{30} \\ &+ x_1^{3}x_2^{21}x_3^{40}x_4^{31}x_5^{30} + x_1^{3}x_2^{21}x_3^{42}x_4^{29}x_5^{30} + x_1^{3}x_2^{27}x_3^{29}x_4^{36}x_5^{30} + x_1^{3}x_2^{28}x_3^{29}x_4^{31}x_5^{34} \\ &+ x_1^{3}x_2^{28}x_3^{29}x_4^{35}x_5^{30} + x_1^{3}x_2^{28}x_3^{33}x_4^{31}x_5^{30} + x_1^{3}x_2^{29}x_3^{24}x_4^{31}x_5^{38} + x_1^{3}x_2^{29}x_3^{24}x_4^{39}x_5^{30} \\ &+ x_1^{3}x_2^{29}x_3^{26}x_4^{29}x_5^{38} + x_1^{3}x_2^{29}x_3^{26}x_4^{37}x_5^{30} + x_1^{3}x_2^{29}x_3^{30}x_4^{29}x_5^{34} \\ &+ Sq^1\big(x_1^{3}x_2^{27}x_3^{29}x_4^{31}x_5^{34} + x_1^{3}x_2^{27}x_3^{29}x_4^{35}x_5^{30} + x_1^{3}x_2^{27}x_3^{33}x_4^{31}x_5^{30}\big) \\ &+ Sq^2\big(x_1^{2}x_2^{27}x_3^{29}x_4^{31}x_5^{34} + x_1^{2}x_2^{27}x_3^{29}x_4^{35}x_5^{30} + x_1^{2}x_2^{27}x_3^{33}x_4^{31}x_5^{30} \\ &+ x_1^{5}x_2^{27}x_3^{30}x_4^{31}x_5^{30}\big) + Sq^4\big(x_1^{3}x_2^{27}x_3^{30}x_4^{31}x_5^{30} + x_1^{3}x_2^{29}x_3^{30}x_4^{29}x_5^{30}\big) \\ &+ Sq^8\big(x_1^{3}x_2^{29}x_3^{24}x_4^{31}x_5^{30} + x_1^{3}x_2^{29}x_3^{26}x_4^{29}x_5^{30}\big) + Sq^{16}\big(x_1^{3}x_2^{17}x_3^{30}x_4^{29}x_5^{30} \\ &+ x_1^{3}x_2^{21}x_3^{24}x_4^{31}x_5^{30} + x_1^{3}x_2^{21}x_3^{26}x_4^{29}x_5^{30}\big) \mod(P_5^-((3)|^4|(1)|^2)). \end{aligned}$$

This equality shows that the monomials $t$ is strictly inadmissible. The others can be proved by a similar computation. $\square$

We now prove Theorem 1.3.

*Proof of Theorem 1.3.* We observe that for any $x \in B_5((3)|^d)$, with $d \geqslant 5$, there exists uniquely a sequence $J_x = (j_1^x, j_2^x, j_3^x)$ such that $j_u^x > 16$ for $u = 1, 2, 3$. We set

$$A(d) = \left\{x\theta_{J_x}(y) : x \in B_5((3)|^d),\, y \in B_3((1)|^2)\right\},$$
$$B(d) = \left\{x_i^{2^{d+2}-1}\theta_{J_i}(z) : z \in B_4^+((2)|^d),\, 1 \leqslant i \leqslant 5\right\}.$$

Since $|B_5((3)|^d)| = 155$, $|B_3((1)|^2)| = 6$ and $|B_4^+((2)|^d)| = 13$, we get $|A(d)| = 155 \times 6 = 930$ and $|B(d)| = 65$.

By a simple computation we have $A(d) = B_5^0((3)|^d|(1)|^2) \cup B(d) \cup C(d)$, where $C(d) = \{a_j = a_{d,j} : 1 \leqslant j \leqslant 475\}$ with $a_j$, $1 \leqslant j \leqslant 475$, as listed in the Section 5. We prove Theorem 1.3 by proving $B_5((3)|^d|(1)|^2) = A(d)$.

Let $X$ be an admissible monomial of weight vector $(3)|^d|(1)|^2$. Then we have $X = xy^{2^d}$ with $x$ a monomial of weight vector $(3)|^d$ and $y$ a monomial of weight vector $(1)|^2$ in $P_5$. Since $X$ is admissible, $x$ and $y$ are also admissible. If $d \geqslant 6$, then $A(d) = \overline{\mathcal{B}_5}$ as determined in Theorem 2.3.5 with $k = 5$ and $\mathcal{B} = \mathcal{B}_5$. By combining Proposition 2.3.6 and Theorem 2.3.5 we obtain $B_5((3)|^d|(1)|^2) \subset A(d)$.

For $d < 6$, let $x \in B_5((3)|^d)$ and $y \in B_5((1)|^2)$. By a direct computation we see that if $xy^{2^d} \notin A(d)$, then there is a monomial $w$ as given in one of Lemmas from 3.2.1 to 3.2.4 such that $xy^{2^d} = \bar{x}w^{2^r}z^{2^{r+s}}$, where $\bar{x}$ and $z$ are suitable monomials and $2 \leqslant r \leqslant 6$, $s = 0, 1$. By Theorem 2.1.6, $xy^{2^d}$ is inadmissible and we have a contradiction. Hence, we get $B_5((3)|^d|(1)|^2) \subset A(d)$.

We prove $B_5((3)|^d|(1)|^2) = A(d)$ by proving the set $[A(d)]_{\omega_d}$ is linearly independent in $QP_5(\omega_d)$ with $\omega_d = (3)|^d|(1)|^2$. We prove this for $d \geqslant 5$. The case $d = 4$ is prove by a similar computation with some minor changes.

Consider the subspaces $\langle [B(d)]_{\omega_d}\rangle \subset QP_5(\omega_d)$ and $\langle [C(d)]_{\omega_d}\rangle \subset QP_5(\omega_d)$. It is easy to see that for any $x \in B(d)$, we have $x = x_i^{2^{d+2}-1}\theta_{J_i}(y)$ with $y$ an admissible monomial of weight vector $(2)|^d$ in $P_4$. By Proposition 2.4.3, $x$ is admissible. Hence $\dim\langle [B(d)]_{\omega_d}\rangle = 65$. Since $\nu(x) = 2^{d+2} - 1$ for all $x \in B(d)$ and $\nu(x) < 2^{d+2} - 1$ for all $x \in C(d)$, we obtain $\langle [B(d)]_{\omega_d}\rangle \cap \langle [C(d)]_{\omega_d}\rangle = \{0\}$. Hence, we need only to prove the set $[C(d)]_{\omega_d} = \{[a_{d,t}]_{\omega_d} : 1 \leqslant t \leqslant 475\}$ is linearly independent in $QP_5(\omega_d)$, where the monomials $a_t = a_{d,t}$, $1 \leqslant t \leqslant 475$, are determined as in Section 5.

Suppose there is a linear relation

$$\mathcal{U} := \sum_{1 \leqslant t \leqslant 475} \gamma_t a_{d,t} \equiv_{\omega_d} 0, \tag{3.1}$$

where $\gamma_t \in \mathbb{F}_2$. We denote $\gamma_{\mathbb{J}} = \sum_{t \in \mathbb{J}} \gamma_t$ for any $\mathbb{J} \subset \{t \in \mathbb{N} : 1 \leqslant t \leqslant 475\}$.

Let $w_u = w_{d,u}$, $1 \leqslant u \leqslant 66$, be as in Section 5 and the homomorphism $p_{(i;I)} : P_5 \to P_4$ which is defined by (2.2) for $k = 5$. From Lemma 2.4.1, we see that $p_{(i;I)}$ passes to a homomorphism from $QP_5(\omega_d)$ to $QP_4(\omega_d)$. By applying $p_{(1;2)}$, $p_{(1;3)}$, and $p_{(4;5)}$ to (3.1), we obtain

$$\begin{aligned}
p_{(1;2)}(\mathcal{U}) \equiv_{\omega_d} {}& \gamma_{16}w_2 + \gamma_{22}w_5 + \gamma_{23}w_6 + \gamma_{25}w_7 + \gamma_{26}w_8 + \gamma_{28}w_{10} + \gamma_{29}w_{11} \\
&+ \gamma_{34}w_{12} + \gamma_{36}w_{13} + \gamma_{37}w_{14} + \gamma_{39}w_{15} + \gamma_{40}w_{16} + \gamma_{17}w_{20} + \gamma_{18}w_{21} \\
&+ \gamma_{19}w_{22} + \gamma_{20}w_{23} + \gamma_{21}w_{25} + \gamma_{24}w_{26} + \gamma_{27}w_{27} + \gamma_{30}w_{28} + \gamma_{31}w_{29} \\
&+ \gamma_{32}w_{30} + \gamma_{33}w_{31} + \gamma_{35}w_{32} + \gamma_{38}w_{33} + \gamma_{41}w_{34} + \gamma_{42}w_{35} + \gamma_{\{85,227\}}w_{37} \\
&+ \gamma_{\{86,228\}}w_{38} + \gamma_{\{87,229\}}w_{39} + \gamma_{\{88,230\}}w_{40} + \gamma_{\{89,231\}}w_{41} + \gamma_{\{90,232\}}w_{42} \\
&+ \gamma_{109}w_{44} + \gamma_{110}w_{45} + \gamma_{111}w_{46} + \gamma_{112}w_{47} + \gamma_{113}w_{49} + \gamma_{114}w_{50} + \gamma_{115}w_{51} \\
&+ \gamma_{116}w_{52} + \gamma_{117}w_{53} + \gamma_{139}w_{55} + \gamma_{140}w_{56} + \gamma_{141}w_{57} + \gamma_{142}w_{58} \\
&+ \gamma_{143}w_{59} + \gamma_{144}w_{60} + \gamma_{145}w_{61} + \gamma_{146}w_{62} + \gamma_{163}w_{63} + \gamma_{164}w_{64} \\
&+ \gamma_{165}w_{65} + \gamma_{166}w_{66} \equiv_{\omega_d} 0,\\
p_{(1;3)}(\mathcal{U}) \equiv_{\omega_d} {}& \gamma_{16}w_2 + \gamma_{\{1,51,167\}}w_5 + \gamma_{\{2,52,168\}}w_6 + \gamma_{\{3,54,169\}}w_7
\end{aligned}$$

$$
\begin{aligned}
&+ \gamma_{\{4,55,170\}}w_8 + \gamma_{28}w_{10} + \gamma_{29}w_{11} + \gamma_{\{8,69,187\}}w_{12} + \gamma_{\{9,71,188\}}w_{13}\\
&+ \gamma_{\{10,72,189\}}w_{14} + \gamma_{\{11,74,190\}}w_{15} + \gamma_{40}w_{16} + \gamma_{\{14,82,203\}}w_{17}\\
&+ \gamma_{\{15,84,204\}}w_{18} + \gamma_{43}w_{20} + \gamma_{44}w_{21} + \gamma_{45}w_{22} + \gamma_{46}w_{23} + \gamma_{\{50,209\}}w_{25}\\
&+ \gamma_{\{53,210\}}w_{26} + \gamma_{\{56,211\}}w_{27} + \gamma_{63}w_{28} + \gamma_{64}w_{29} + \gamma_{65}w_{30} + \gamma_{66}w_{31}\\
&+ \gamma_{\{70,218\}}w_{32} + \gamma_{\{73,219\}}w_{33} + \gamma_{79}w_{34} + \gamma_{80}w_{35} + \gamma_{\{83,224\}}w_{36} + \gamma_{91}w_{37}\\
&+ \gamma_{92}w_{38} + \gamma_{93}w_{39} + \gamma_{100}w_{40} + \gamma_{101}w_{41} + \gamma_{106}w_{42} + \gamma_{122}w_{44} + \gamma_{123}w_{45}\\
&+ \gamma_{125}w_{46} + \gamma_{126}w_{47} + \gamma_{121}w_{49} + \gamma_{124}w_{50} + \gamma_{127}w_{51} + \gamma_{134}w_{52} + \gamma_{136}w_{53}\\
&+ \gamma_{138}w_{54} + \gamma_{149}w_{55} + \gamma_{151}w_{56} + \gamma_{152}w_{57} + \gamma_{154}w_{58} + \gamma_{150}w_{59} + \gamma_{153}w_{60}\\
&+ \gamma_{159}w_{61} + \gamma_{161}w_{62} \equiv_{\omega_d} 0,\\
p_{(4;5)}(\mathcal{U}) &\equiv_{\omega_d} \gamma_{116}w_1 + \gamma_{117}w_2 + \gamma_{134}w_4 + \gamma_{135}w_5 + \gamma_{136}w_6 + \gamma_{137}w_7 + \gamma_{138}w_8\\
&+ \gamma_{145}w_{10} + \gamma_{146}w_{11} + \gamma_{159}w_{12} + \gamma_{160}w_{13} + \gamma_{161}w_{14} + \gamma_{162}w_{15} + \gamma_{166}w_{16}\\
&+ \gamma_{300}w_{19} + \gamma_{301}w_{20} + \gamma_{302}w_{21} + \gamma_{303}w_{22} + \gamma_{304}w_{23} + \gamma_{314}w_{25} + \gamma_{315}w_{26}\\
&+ \gamma_{316}w_{27} + \gamma_{329}w_{28} + \gamma_{330}w_{29} + \gamma_{331}w_{30} + \gamma_{332}w_{31} + \gamma_{339}w_{32} + \gamma_{340}w_{33}\\
&+ \gamma_{347}w_{34} + \gamma_{348}w_{35} + \gamma_{397}w_{37} + \gamma_{398}w_{38} + \gamma_{399}w_{39} + \gamma_{406}w_{40} + \gamma_{407}w_{41}\\
&+ \gamma_{411}w_{42} + \gamma_{428}w_{43} + \gamma_{429}w_{44} + \gamma_{430}w_{45} + \gamma_{431}w_{46} + \gamma_{432}w_{47} + \gamma_{442}w_{49}\\
&+ \gamma_{443}w_{50} + \gamma_{444}w_{51} + \gamma_{445}w_{52} + \gamma_{446}w_{53} + \gamma_{459}w_{55} + \gamma_{460}w_{56} + \gamma_{461}w_{57}\\
&+ \gamma_{462}w_{58} + \gamma_{469}w_{59} + \gamma_{470}w_{60} + \gamma_{471}w_{61} + \gamma_{472}w_{62} \equiv_{\omega_d} 0.
\end{aligned}
$$

From the above equalities, we get

$$\gamma_j = 0 \text{ for } j \in \mathbb{J}_1,\ \gamma_i = \gamma_j \text{ for } (i,j) \in \mathbb{K}_1, \tag{3.2}$$

where $\mathbb{J}_1 = \{16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 63, 64, 65, 66, 79, 80, 91, 92, 93, 100, 101, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 121, 122, 123, 124, 125, 126, 127, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 149, 150, 151, 152, 153, 154, 159, 160, 161, 162, 163, 164, 165, 166, 300, 301, 302, 303, 304, 314, 315, 316, 329, 330, 331, 332, 339, 340, 347, 348, 397, 398, 399, 406, 407, 411, 428, 429, 430, 431, 432, 442, 443, 444, 445, 446, 459, 460, 461, 462, 469, 470, 471, 472\}$ and $\mathbb{K}_1 = \{(50,209), (53,210), (56,211), (70,218), (73,219), (83,224), (85,227), (86,228), (87,229), (88,230), (89,231), (90,232)\}$.

Apply the homomorphisms $p_{(2;3)}$, $p_{(2;4)}$, $p_{(3;4)}$ to (3.1) and using (3.2), to obtain

$$
\begin{aligned}
p_{(2;3)}(\mathcal{U}) &\equiv_{\omega_d} \gamma_{\{1,175,234,288\}}w_5 + \gamma_{\{2,176,235,289\}}w_6 + \gamma_{\{3,178,237,291\}}w_7\\
&+ \gamma_{\{4,179,238,292\}}w_8 + \gamma_{\{8,193,249,319\}}w_{12} + \gamma_{\{9,195,251,321\}}w_{13}\\
&+ \gamma_{\{10,196,252,322\}}w_{14} + \gamma_{\{11,198,254,324\}}w_{15} + \gamma_{\{14,206,261,342\}}w_{17}\\
&+ \gamma_{\{15,208,263,344\}}w_{18} + \gamma_{167}w_{20} + \gamma_{168}w_{21} + \gamma_{169}w_{22} + \gamma_{170}w_{23}
\end{aligned}
$$

$$
\begin{aligned}
&+ \gamma_{\{50,85,174,233,287\}}w_{25} + \gamma_{\{53,86,177,236,290\}}w_{26} + \gamma_{\{56,87,180,239,293\}}w_{27}\\
&+ \gamma_{187}w_{28} + \gamma_{188}w_{29} + \gamma_{189}w_{30} + \gamma_{190}w_{31} + \gamma_{\{70,88,194,250,320\}}w_{32}\\
&+ \gamma_{\{73,89,197,253,323\}}w_{33} + \gamma_{203}w_{34} + \gamma_{204}w_{35} + \gamma_{\{83,90,207,262,343\}}w_{36}\\
&+ \gamma_{349}w_{37} + \gamma_{350}w_{38} + \gamma_{351}w_{39} + \gamma_{358}w_{40} + \gamma_{359}w_{41} + \gamma_{364}w_{42}\\
&+ \gamma_{416}w_{44} + \gamma_{417}w_{45} + \gamma_{419}w_{46} + \gamma_{420}w_{47} + \gamma_{415}w_{49} + \gamma_{418}w_{50} + \gamma_{421}w_{51}\\
&+ \gamma_{449}w_{55} + \gamma_{451}w_{56} + \gamma_{452}w_{57} + \gamma_{454}w_{58} + \gamma_{450}w_{59} + \gamma_{453}w_{60} \equiv_{\omega_d} 0,
\end{aligned}
$$

$$
\begin{aligned}
p_{(2;4)}(\mathcal{U}) &\equiv_{\omega_d} \gamma_{\{2,52,168\}}w_2 + \gamma_{\{4,55,170\}}w_3 + \gamma_{\{5,57,182,246\}}w_5 + \gamma_{\{6,59,184,247\}}w_7\\
&+ \gamma_{\{7,61,186,248\}}w_9 + \gamma_{\{8,69,187\}}w_{10} + \gamma_{\{10,72,189\}}w_{11} + \gamma_{\{12,75,200,259\}}w_{13}\\
&+ \gamma_{\{13,77,202,260\}}w_{15} + \gamma_{\{14,82,203\}}w_{16} + \gamma_{\{171,212,353,367\}}w_{20} + \gamma_{176}w_{21}\\
&+ \gamma_{\{172,214,355,369\}}w_{22} + \gamma_{179}w_{23} + \gamma_{\{173,216,357,371\}}w_{24} + \gamma_{181}w_{25}\\
&+ \gamma_{183}w_{26} + \gamma_{185}w_{27} + \gamma_{193}w_{28} + \gamma_{\{191,220,361,376\}}w_{29} + \gamma_{196}w_{30}\\
&+ \gamma_{\{192,222,363,378\}}w_{31} + \gamma_{199}w_{32} + \gamma_{201}w_{33} + \gamma_{\{205,225,366,382\}}w_{35}\\
&+ \gamma_{206}w_{34} + \gamma_{352}w_{37} + \gamma_{354}w_{38} + \gamma_{356}w_{39} + \gamma_{360}w_{40} + \gamma_{362}w_{41} + \gamma_{365}w_{42}\\
&+ \gamma_{\{412,423,433\}}w_{44} + \gamma_{417}w_{45} + \gamma_{\{413,425,435\}}w_{46} + \gamma_{420}w_{47}\\
&+ \gamma_{\{414,427,437\}}w_{48} + \gamma_{422}w_{49} + \gamma_{424}w_{50} + \gamma_{426}w_{51} + \gamma_{449}w_{55} + \gamma_{452}w_{57}\\
&+ \gamma_{\{447,456,463\}}w_{56} + \gamma_{\{448,458,465\}}w_{58} + \gamma_{455}w_{59} + \gamma_{457}w_{60} \equiv_{\omega_d} 0,
\end{aligned}
$$

$$
\begin{aligned}
p_{(3;4)}(\mathcal{U}) &\equiv_{\omega_d} \gamma_{\{118,128,306,311\}}w_5 + \gamma_{\{119,130,308,312\}}w_7 + \gamma_{\{120,132,310,313\}}w_9\\
&+ \gamma_{\{147,155,334,337\}}w_{13} + \gamma_{\{148,157,336,338\}}w_{15} + \gamma_{\{284,287,294,392,473\}}w_{20}\\
&+ \gamma_{289}w_{21} + \gamma_{\{285,290,296,394,474\}}w_{22} + \gamma_{292}w_{23} + \gamma_{\{286,293,298,396,475\}}w_{24}\\
&+ \gamma_{305}w_{25} + \gamma_{307}w_{26} + \gamma_{309}w_{27} + \gamma_{319}w_{28} + \gamma_{\{317,320,325,401,404\}}w_{29}\\
&+ \gamma_{322}w_{30} + \gamma_{\{318,323,327,403,405\}}w_{31} + \gamma_{333}w_{32} + \gamma_{335}w_{33} + \gamma_{342}w_{34}\\
&+ \gamma_{\{341,343,345,409,410\}}w_{35} + \gamma_{391}w_{37} + \gamma_{393}w_{38} + \gamma_{395}w_{39} + \gamma_{400}w_{40}\\
&+ \gamma_{402}w_{41} + \gamma_{408}w_{42} + \gamma_{\{412,415,422,434,439\}}w_{44} + \gamma_{417}w_{45}\\
&+ \gamma_{\{413,418,424,436,440\}}w_{46} + \gamma_{420}w_{47} + \gamma_{\{414,421,426,438,441\}}w_{48} + \gamma_{433}w_{49}\\
&+ \gamma_{435}w_{50} + \gamma_{437}w_{51} + \gamma_{449}w_{55} + \gamma_{\{447,450,455,464,467\}}w_{56} + \gamma_{452}w_{57}\\
&+ \gamma_{\{448,453,457,466,468\}}w_{58} + \gamma_{463}w_{59} + \gamma_{465}w_{60} \equiv_{\omega_d} 0.
\end{aligned}
$$

Computing from these equalities gives

$$\gamma_j = 0 \text{ for } j \in \mathbb{J}_2,\ \gamma_i = \gamma_j \text{ for } (i,j) \in \mathbb{K}_2, \tag{3.3}$$

where $\mathbb{J}_2 = \{167, 168, 169, 170, 176, 179, 181, 183, 185, 187, 188, 189, 190, 193, 196, 199, 201, 203, 204, 206, 289, 292, 305, 307, 309, 319, 322, 333, 335, 342, 349, 350, 351, 352, 354, 356, 358, 359, 360, 362, 364, 365, 391, 393, 395, 400, 402, 408, 415, 416, 417, 418, 419, 420, 421, 422, 424, 426, 433, 435, 437, 449, 450, 451, 452, 453, 454, 455, 457, 463, 465\}$ and $\mathbb{K}_2 = \{(1,51), (2,52), (2,235), (3,54), (4,55), (4,238), (8,69), (8,249), (9,71),$

(10,72), (10,252), (11,74), (14,82), (14,261), (15,84), (412,423), (413,425), (414,427), (447,456), (448,458)}.

Apply the homomorphism $p_{(1;4)}, p_{(3;5)}$ to (3.1) and using (3.2), (3.3), to get

$$\begin{aligned}
p_{(1;4)}(\mathcal{U}) \equiv_{\omega_d}\ & \gamma_{\{5,58\}}w_5 + \gamma_{\{6,60\}}w_7 + \gamma_{\{7,62\}}w_9 + \gamma_{\{12,76\}}w_{13} + \gamma_{\{13,78\}}w_{15}\\
&+ \gamma_{\{47,95,212,240,266,294,325,345\}}w_{20} + \gamma_2 w_{21} + \gamma_{\{48,97,214,242,268,296,327\}}w_{22}\\
&+ \gamma_4 w_{23} + \gamma_{\{49,99,216,244,270,298\}}w_{24} + \gamma_{57}w_{25} + \gamma_{59}w_{26} + \gamma_{61}w_{27} + \gamma_8 w_{28}\\
&+ \gamma_{\{67,103,220,255,275,325\}}w_{29} + \gamma_{10}w_{30} + \gamma_{\{68,105,222,257,277,327\}}w_{31}\\
&+ \gamma_{75}w_{32} + \gamma_{77}w_{33} + \gamma_{14}w_{34} + \gamma_{\{81,108,225,264,281,345\}}w_{35} + \gamma_{94}w_{37}\\
&+ \gamma_{\{96,325\}}w_{38} + \gamma_{\{98,327,345\}}w_{39} + \gamma_{102}w_{40} + \gamma_{104}w_{41} + \gamma_{107}w_{42}\\
&+ \gamma_{\{118,129\}}w_{44} + \gamma_{\{119,131\}}w_{46} + \gamma_{\{120,133\}}w_{48} + \gamma_{128}w_{49} + \gamma_{130}w_{50}\\
&+ \gamma_{132}w_{51} + \gamma_{\{147,156\}}w_{56} + \gamma_{\{148,158\}}w_{58} + \gamma_{155}w_{59} + \gamma_{157}w_{60} \equiv_{\omega_d} 0,\\
p_{(3;5)}(\mathcal{U}) \equiv_{\omega_d}\ & \gamma_{\{118,129\}}w_4 + \gamma_{\{119,131\}}w_6 + \gamma_{\{120,133\}}w_8 + \gamma_{\{147,156\}}w_{12}\\
&+ \gamma_{\{148,158\}}w_{14} + \gamma_{\{284,295\}}w_{19} + \gamma_{288}w_{20} + \gamma_{\{285,297\}}w_{21} + \gamma_{291}w_{22}\\
&+ \gamma_{\{286,299\}}w_{23} + \gamma_{306}w_{25} + \gamma_{308}w_{26} + \gamma_{310}w_{27} + \gamma_{\{317,326\}}w_{28} + \gamma_{321}w_{29}\\
&+ \gamma_{\{318,328\}}w_{30} + \gamma_{324}w_{31} + \gamma_{334}w_{32} + \gamma_{336}w_{33} + \gamma_{\{341,346\}}w_{34} + \gamma_{344}w_{35}\\
&+ \gamma_{392}w_{37} + \gamma_{394}w_{38} + \gamma_{396}w_{39} + \gamma_{401}w_{40} + \gamma_{403}w_{41} + \gamma_{409}w_{42} + \gamma_{434}w_{49}\\
&+ \gamma_{436}w_{50} + \gamma_{438}w_{51} + \gamma_{464}w_{59} + \gamma_{466}w_{60} \equiv_{\omega_d} 0.
\end{aligned}$$

From these equalities it implies

$$\gamma_j = 0 \text{ for } j \in \mathbb{J}_3,\ \gamma_i = \gamma_j \text{ for } (i,j) \in \mathbb{K}_3, \tag{3.4}$$

where $\mathbb{J}_3 = \{2,\ 4,\ 8,\ 10,\ 14,\ 52,\ 55,\ 57,\ 59,\ 61,\ 69,\ 72,\ 75,\ 77,\ 82,\ 94,\ 102,\ 104,\ 107,\ 128,\ 130,\ 132,\ 155,\ 157,\ 235,\ 238,\ 249,\ 252,\ 261,\ 288,\ 291,\ 306,\ 308,\ 310,\ 321,\ 324,\ 334,\ 336,\ 344,\ 392,\ 394,\ 396,\ 401,\ 403,\ 409,\ 434,\ 436,\ 438,\ 464,\ 466\}$ and $\mathbb{K}_3 = \{$(5,58), (6,60), (7,62), (12,76), (13,78), (96,325), (118,129), (118,311), (119,131), (119,312), (120,133), (120,313), (147,156), (147,337), (148,158), (148,338), (284,295), (285,297), (286,299), (317,326)}.

By applying the homomorphisms $p_{(1;5)}, p_{(2;5)}$ to (3.1) and using (3.2), (3.3), (3.4), we get

$$\begin{aligned}
p_{(1;5)}(\mathcal{U}) \equiv_{\omega_d}\ & \gamma_{\{1,175,234\}}w_1 + \gamma_{\{3,178,237\}}w_2 + \gamma_{\{5,182,246\}}w_4 + \gamma_{\{6,184,247\}}w_6\\
&+ \gamma_{\{7,186,248\}}w_8 + \gamma_{\{9,195,251\}}w_{10} + \gamma_{\{11,198,254\}}w_{11} + \gamma_{\{12,200,259\}}w_{12}\\
&+ \gamma_{\{13,202,260\}}w_{14} + \gamma_{\{15,208,263\}}w_{16} + \gamma_{\{47,50,85,213,241,267,272,284\}}w_{19}\\
&+ \gamma_1 w_{20} + \gamma_{\{48,53,86,96,215,243,269,273,285,317\}}w_{21} + \gamma_3 w_{22}\\
&+ \gamma_{\{49,56,87,98,217,245,271,274,286,318,341\}}w_{23} + \gamma_5 w_{25} + \gamma_6 w_{26} + \gamma_7 w_{27}\\
&+ \gamma_{\{67,70,88,221,256,276,279,317\}}w_{28} + \gamma_9 w_{29} + \gamma_{\{68,73,89,223,258,278,280,318\}}w_{30}\\
&+ \gamma_{11}w_{31} + \gamma_{12}w_{32} + \gamma_{13}w_{33} + \gamma_{\{81,83,90,226,265,282,283,341\}}w_{34} + \gamma_{15}w_{35}
\end{aligned}$$

$$
\begin{aligned}
&+ \gamma_{\{95,317,341\}}w_{37} + \gamma_{\{97,318\}}w_{38} + \gamma_{99}w_{39} + \gamma_{103}w_{40} + \gamma_{105}w_{41} + \gamma_{108}w_{42}\\
&+ \gamma_{118}w_{49} + \gamma_{119}w_{50} + \gamma_{120}w_{51} + \gamma_{147}w_{59} + \gamma_{148}w_{60} \equiv_{\omega_d} 0,\\
p_{(2;5)}(\mathcal{U}) &\equiv_{\omega_d} \gamma_{\{171,174,213,368,373\}}w_{19} + \gamma_{175}w_{20} + \gamma_{\{172,177,215,370,374\}}w_{21}\\
&+ \gamma_{178}w_{22} + \gamma_{\{173,180,217,372,375\}}w_{23} + \gamma_{182}w_{25} + \gamma_{184}w_{26} + \gamma_{186}w_{27}\\
&+ \gamma_{\{191,194,221,377,380\}}w_{28} + \gamma_{195}w_{29} + \gamma_{\{192,197,223,379,381\}}w_{30} + \gamma_{198}w_{31}\\
&+ \gamma_{200}w_{32} + \gamma_{202}w_{33} + \gamma_{\{205,207,226,383,384\}}w_{34} + \gamma_{208}w_{35} + \gamma_{353}w_{37}\\
&+ \gamma_{355}w_{38} + \gamma_{357}w_{39} + \gamma_{361}w_{40} + \gamma_{363}w_{41} + \gamma_{366}w_{42} + \gamma_{412}w_{49} + \gamma_{413}w_{50}\\
&+ \gamma_{414}w_{51} + \gamma_{447}w_{59} + \gamma_{448}w_{60} \equiv_{\omega_d} 0.
\end{aligned}
$$

By a direct calculation using the above equalities we have

$$\gamma_j = 0 \text{ for } j \in \mathbb{J}_4,\ \gamma_{97} = \gamma_{318}, \tag{3.5}$$

where $\mathbb{J}_4 = \{1,\ 3,\ 5,\ 6,\ 7,\ 9,\ 11,\ 12,\ 13,\ 15,\ 51,\ 54,\ 58,\ 60,\ 62,\ 71,\ 74,\ 76,\ 78,\ 84,\ 99,\ 103,\ 105,\ 108,\ 118,\ 119,\ 120,\ 129,\ 131,\ 133,\ 147,\ 148,\ 156,\ 158,\ 175,\ 178,\ 182,\ 184,\ 186,\ 195,\ 198,\ 200,\ 202,\ 208,\ 234,\ 237,\ 246,\ 247,\ 248,\ 251,\ 254,\ 259,\ 260,\ 263,\ 311,\ 312,\ 313,\ 337,\ 338,\ 353,\ 355,\ 357,\ 361,\ 363,\ 366,\ 412,\ 413,\ 414,\ 423,\ 425,\ 427,\ 439,\ 440,\ 441,\ 447,\ 448,\ 456,\ 458,\ 467,\ 468\}$.

Apply the homomorphisms $p_{(1;(2,3))}, p_{(1;(2,4))}$ to (3.1) and using (3.2), (3.3), (3.4), (3.5) to obtain

$$
\begin{aligned}
p_{(1;(2,3))}(\mathcal{U}) &\equiv_{\omega_d} \gamma_{174}w_{25} + \gamma_{177}w_{26} + \gamma_{180}w_{27} + \gamma_{194}w_{32} + \gamma_{197}w_{33} + \gamma_{207}w_{36}\\
&+ \gamma_{233}w_{37} + \gamma_{236}w_{38} + \gamma_{239}w_{39} + \gamma_{250}w_{40} + \gamma_{253}w_{41} + \gamma_{262}w_{42} + \gamma_{287}w_{49}\\
&+ \gamma_{290}w_{50} + \gamma_{293}w_{51} + \gamma_{320}w_{59} + \gamma_{323}w_{60} + \gamma_{343}w_{65} \equiv_{\omega_d} 0,\\
p_{(1;(2,4))}(\mathcal{U}) &\equiv_{\omega_d} \gamma_{\{47,85,95,96,171,212,240,241,266,294,345,367\}}w_{20}\\
&+ \gamma_{\{48,86,97,172,214,242,243,268,296,327,369\}}w_{22}\\
&+ \gamma_{\{49,87,173,216,244,245,270,298,371\}}w_{24} + \gamma_{\{67,88,96,191,220,255,256,275,376\}}w_{29}\\
&+ \gamma_{\{68,89,192,222,257,258,277,327,378\}}w_{31} + \gamma_{\{81,90,205,225,264,265,281,345,382\}}w_{35}\\
&+ \gamma_{240}w_{37} + \gamma_{242}w_{38} + \gamma_{\{98,244,327,345\}}w_{39} + \gamma_{255}w_{40} + \gamma_{257}w_{41} + \gamma_{264}w_{42}\\
&+ \gamma_{294}w_{49} + \gamma_{296}w_{50} + \gamma_{298}w_{51} + \gamma_{96}w_{59} + \gamma_{327}w_{60} + \gamma_{345}w_{65} \equiv_{\omega_d} 0.
\end{aligned}
$$

From the above relations we obtain

$$\gamma_j = 0 \text{ for } j \in \mathbb{J}_5,\ \gamma_i = \gamma_j \text{ for } (i,j) \in \mathbb{K}_5, \tag{3.6}$$

where $\mathbb{J}_5 = \{96,\ 98,\ 174,\ 177,\ 180,\ 194,\ 197,\ 207,\ 233,\ 236,\ 239,\ 240,\ 242,\ 244,\ 250,\ 253,\ 255,\ 257,\ 262,\ 264,\ 287,\ 290,\ 293,\ 294,\ 296,\ 298,\ 320,\ 323,\ 325,\ 327,\ 343,\ 345\}$ and $\mathbb{K}_5 = \{(50,85),\ (53,86),\ (56,87),\ (70,88),\ (73,89),\ (83,90),\ (97,405),\ (284,473),\ (285,474),\ (286,475),\ (317,404),\ (341,410)\}$.

Applying the homomorphisms $p_{(1;(3,4))}$, $p_{(1;(2,5))}$ to (3.1) and using (3.2), (3.3), (3.4), (3.5), (3.6) gives

$$\begin{aligned}
p_{(1;(3,4))}(\mathcal{U}) &\equiv_{\omega_d} \gamma_{\{171,213,368,373\}}w_5 + \gamma_{\{172,215,370,374\}}w_7 + \gamma_{\{173,217,372,375\}}w_9\\
&+ \gamma_{\{191,221,377,380\}}w_{13} + \gamma_{\{192,223,379,381\}}w_{15} + \gamma_{\{205,226,383,384\}}w_{18}\\
&+ \gamma_{\{47,50,95,212,266,267,367,385\}}w_{20} + \gamma_{\{48,53,97,214,268,269,369,386\}}w_{22}\\
&+ \gamma_{\{49,56,216,270,271,371,387\}}w_{24} + \gamma_{\{212,272,367\}}w_{25} + \gamma_{\{214,273,369\}}w_{26}\\
&+ \gamma_{\{216,274,371\}}w_{27} + \gamma_{\{67,70,220,275,276,376,388\}}w_{29}\\
&+ \gamma_{\{68,73,222,277,278,378,389\}}w_{31} + \gamma_{\{220,279,376\}}w_{32} + \gamma_{\{222,280,378\}}w_{33}\\
&+ \gamma_{\{81,83,225,281,282,382,390\}}w_{35} + \gamma_{\{225,283,382\}}w_{36} + \gamma_{266}w_{37} + \gamma_{268}w_{38}\\
&+ \gamma_{270}w_{39} + \gamma_{275}w_{40} + \gamma_{277}w_{41} + \gamma_{281}w_{42} \equiv_{\omega_d} 0,\\
p_{(1;(2,5))}(\mathcal{U}) &\equiv_{\omega_d} \gamma_{\{47,50,171,213,241,267,272,284,368,373,385\}}w_{19}\\
&+ \gamma_{\{48,53,172,215,243,269,273,285,317,370,374,386\}}w_{21}\\
&+ \gamma_{\{49,56,97,173,217,245,271,274,286,341,372,375,387\}}w_{23}\\
&+ \gamma_{\{67,70,191,221,256,276,279,317,377,380,388\}}w_{28}\\
&+ \gamma_{\{68,73,97,192,223,258,278,280,379,381,389\}}w_{30}\\
&+ \gamma_{\{81,83,205,226,265,282,283,341,383,384,390\}}w_{34} + \gamma_{\{95,241,317,341\}}w_{37}\\
&+ \gamma_{243}w_{38} + \gamma_{245}w_{39} + \gamma_{256}w_{40} + \gamma_{258}w_{41} + \gamma_{265}w_{42} + \gamma_{284}w_{49}\\
&+ \gamma_{285}w_{50} + \gamma_{286}w_{51} + \gamma_{317}w_{59} + \gamma_{97}w_{60} + \gamma_{341}w_{65} \equiv_{\omega_d} 0.
\end{aligned}$$

By computing from these relations we obtain

$$\gamma_j = 0 \text{ for } j \in \mathbb{J}_6,\ \gamma_i = \gamma_j \text{ for } (i,j) \in \mathbb{K}_6, \tag{3.7}$$

where $\mathbb{J}_6 = \{$ 95, 97, 241, 243, 245, 256, 258, 265, 266, 268, 270, 275, 277, 281, 284, 285, 286, 295, 297, 299, 317, 318, 326, 328, 341, 346, 404, 405, 410, 473, 474, 475$\}$ and $\mathbb{K}_6 = \{$(47,212), (48,214), (49,216), (67,220), (68,222), (81,225)$\}$.

Apply the homomorphisms $p_{(1;(3,5))}$, $p_{(1;(4,5))}$ to (3.1) and using (3.2), (3.3), (3.4), (3.5), (3.6), (3.6) to get

$$\begin{aligned}
p_{(1;(3,5))}(\mathcal{U}) &\equiv_{\omega_d} \gamma_{\{47,171,367\}}w_4 + \gamma_{\{48,172,369\}}w_6 + \gamma_{\{49,173,371\}}w_8\\
&+ \gamma_{\{67,191,376\}}w_{12} + \gamma_{\{68,192,378\}}w_{14} + \gamma_{\{81,205,382\}}w_{17}\\
&+ \gamma_{\{47,50,213,267,272,368\}}w_{19} + \gamma_{\{48,53,215,269,273,370\}}w_{21}\\
&+ \gamma_{\{49,56,217,271,274,372\}}w_{23} + \gamma_{\{213,272,368\}}w_{25} + \gamma_{\{215,273,370\}}w_{26}\\
&+ \gamma_{\{217,274,372\}}w_{27} + \gamma_{\{67,70,221,276,279,377\}}w_{28} + \gamma_{\{68,73,223,278,280,379\}}w_{30}\\
&+ \gamma_{\{221,279,377\}}w_{32} + \gamma_{\{223,280,379\}}w_{33} + \gamma_{\{81,83,226,282,283,383\}}w_{34}\\
&+ \gamma_{\{226,283,383\}}w_{36} + \gamma_{267}w_{37} + \gamma_{269}w_{38} + \gamma_{271}w_{39} + \gamma_{276}w_{40}\\
&+ \gamma_{278}w_{41} + \gamma_{282}w_{42} \equiv_{\omega_d} 0,
\end{aligned}$$

$$\begin{aligned}
p_{(1;(4,5))}(\mathcal{U}) &\equiv_{\omega_d} \gamma_{\{213,267,272,373,385\}}w_{19} + \gamma_{\{213,267,373,385\}}w_{20}\\
&+ \gamma_{\{215,269,273,374,386\}}w_{21} + \gamma_{\{215,269,374,386\}}w_{22} + \gamma_{\{217,271,274,375,387\}}w_{23}\\
&+ \gamma_{\{217,271,375,387\}}w_{24} + \gamma_{\{221,276,279,380,388\}}w_{28} + \gamma_{\{221,276,380,388\}}w_{29}\\
&+ \gamma_{\{223,278,280,381,389\}}w_{30} + \gamma_{\{223,278,381,389\}}w_{31} + \gamma_{\{226,282,283,384,390\}}w_{34}\\
&+ \gamma_{\{226,282,384,390\}}w_{35} + \gamma_{272}w_{37} + \gamma_{273}w_{38} + \gamma_{274}w_{39} + \gamma_{279}w_{40}\\
&+ \gamma_{280}w_{41} + \gamma_{283}w_{42} \equiv_{\omega_d} 0.
\end{aligned}$$

By a direct computation from the above equalities we obtain $\gamma_j = 0$ for $1 \leqslant j \leqslant 475$. The theorem is proved. $\square$

### 3.3 On a structure of of the space $\mathrm{Ker}\big((\widetilde{Sq}^0_*)_{(5,20)}\big)$

In this section we present some results on a structure of $\mathrm{Ker}\big((\widetilde{Sq}^0_*)_{(5,20)}\big)$. Consider the homomorphism $p_{(i;I)} : P_k \to P_{k-1}$ as defined by (2.2). We set

$$\widetilde{SF}_k(\omega) = \bigcap_{(i;I)\in\mathcal{N}_k} \mathrm{Ker}(p^{(\omega)}_{(i;I)}) \text{ and } \widetilde{QP}_k(\omega) = QP_k(\omega)/\widetilde{SF}_k(\omega),$$
$$(\widetilde{SF}_k)_m = \bigcap_{(i;I)\in\mathcal{N}_k} \mathrm{Ker}(p^{(m)}_{(i;I)}) \text{ and } (\widetilde{QP}_k)_m = (QP_k)_m/(\widetilde{SF}_k)_m,$$

where $m = \deg\omega$. Note that $\widetilde{SF}_k(\omega) \subset QP^+_k(\omega)$ and $(\widetilde{SF}_k)_m \subset (QP_k)_m$.

We see that $\widetilde{SF}_k(\omega)$ and $(\widetilde{SF}_k)_m$ are respectively $\mathbb{F}_2$-subspaces of $QP_k(\omega)$ and $(\widetilde{QP}_k)_m$. Then we have

$$QP_k(\omega) \cong \widetilde{QP}_k(\omega) \bigoplus \widetilde{SF}_k(\omega),\ (QP_k)_m \cong (\widetilde{QP}_k)_m \bigoplus (\widetilde{SF}_k)_m.$$

We denote by $\widetilde{B}^+_k(\omega)$ the set of all elements in $B^+_k(\omega)$ such that the set of all classes in $\widetilde{QP}_k(\omega)$ represented by $x \in \widetilde{B}^+_k(\omega)$ is a basis of $\widetilde{QP}^+_k(\omega) = QP^+_k(\omega)/\widetilde{SF}_k(\omega)$.

By an argument analogous to the one in Section 3.2 and using Lemmas 3.2.1 and 3.2.2, we have determined a set of generators for $\mathrm{Ker}\big((\widetilde{Sq}^0_*)_{(5,20)}\big)$ consisting of 939 monomials (see Subsection 5.2). In [22], we have prove the following.

**Proposition 3.3.1.** *We have*

$$\dim \widetilde{QP}_5((3)|^4) = 154,\ \dim \widetilde{SF}_5((3)|^4) \leqslant 5.$$

By a direct computation using this generating set we have

$$\widetilde{B}^+_5((3)|^3|(1)|^2) = B(3) \cup \{a_j : 1 \leqslant j \leqslant 450\}$$

$$\widetilde{SF}_5((3)|^3|(1)|^2) = \langle\{[h_t] : 1 \leqslant t \leqslant 34\}\rangle,$$

where $h_t$ are the polynomials with the leading monomials $Y_t$ which are determined as follows:

$$\begin{array}{lll}
Y_1 = x_1^3x_2^3x_3^4x_4^7x_5^{28} & Y_2 = x_1^3x_2^3x_3^7x_4^4x_5^{28} & Y_3 = x_1^3x_2^3x_3^7x_4^{12}x_5^{20} \\
Y_4 = x_1^3x_2^3x_3^7x_4^{28}x_5^4 & Y_5 = x_1^3x_2^5x_3^{14}x_4^7x_5^{16} & Y_6 = x_1^3x_2^5x_3^{14}x_4^{16}x_5^7 \\
Y_7 = x_1^3x_2^5x_3^{14}x_4^{17}x_5^6 & Y_8 = x_1^3x_2^5x_3^{15}x_4^{16}x_5^6 & Y_9 = x_1^3x_2^7x_3^3x_4^4x_5^{28} \\
Y_{10} = x_1^3x_2^7x_3^3x_4^{12}x_5^{20} & Y_{11} = x_1^3x_2^7x_3^3x_4^{28}x_5^4 & Y_{12} = x_1^3x_2^7x_3^8x_4^5x_5^{22} \\
Y_{13} = x_1^3x_2^7x_3^8x_4^{21}x_5^6 & Y_{14} = x_1^3x_2^7x_3^{13}x_4^{16}x_5^6 & Y_{15} = x_1^3x_2^7x_3^{27}x_4^4x_5^4 \\
Y_{16} = x_1^3x_2^{13}x_3^6x_4^7x_5^{16} & Y_{17} = x_1^3x_2^{13}x_3^7x_4^6x_5^{16} & Y_{18} = x_1^3x_2^{15}x_3^5x_4^{16}x_5^6 \\
Y_{19} = x_1^7x_2^3x_3^3x_4^4x_5^{28} & Y_{20} = x_1^7x_2^3x_3^3x_4^{12}x_5^{20} & Y_{21} = x_1^7x_2^3x_3^3x_4^{28}x_5^4 \\
Y_{22} = x_1^7x_2^3x_3^8x_4^5x_5^{22} & Y_{23} = x_1^7x_2^3x_3^8x_4^{21}x_5^6 & Y_{24} = x_1^7x_2^3x_3^{13}x_4^{16}x_5^6 \\
Y_{25} = x_1^7x_2^3x_3^{27}x_4^4x_5^4 & Y_{26} = x_1^7x_2^7x_3^8x_4x_5^{22} & Y_{27} = x_1^7x_2^7x_3^8x_4^{17}x_5^6 \\
Y_{28} = x_1^7x_2^7x_3^9x_4^{16}x_5^6 & Y_{29} = x_1^7x_2^7x_3^{24}x_4x_5^6 & Y_{30} = x_1^7x_2^{11}x_3^5x_4^{16}x_5^6 \\
Y_{31} = x_1^7x_2^{11}x_3^{17}x_4^4x_5^6 & Y_{32} = x_1^7x_2^{11}x_3^{17}x_4^6x_5^4 & Y_{33} = x_1^7x_2^{27}x_3^3x_4^4x_5^4 \\
Y_{34} = x_1^{15}x_2^3x_3^5x_4^{16}x_5^6 & &
\end{array}$$

By combining the above results and the ones in Subsection 4.1 we get the following.

**Proposition 3.3.2.** *We have*

$$\begin{aligned}
&\dim \widetilde{QP}_5((3)|^3|(1)|^2) = 905,\ \dim \widetilde{SF}_5((3)|^3|(1)|^2) \leqslant 34,\\
&1690 \leqslant \dim(QP)_{45} \leqslant 1739.
\end{aligned}$$

**Remark 3.3.3.** Dr. Nguyen Khac Tin informs us that by using a computer program based on the algorithm of MAGMA, he obtains $\dim(QP)_{45} = 1731$. However, we have been unable to verify this result by hand computation.

# 4 Structures of $(QP_5)_{2^{d+1}+2^d-4}$ for $3 \leqslant d \leqslant 5$

In this section we present a sketch of structure of the space $(QP_5)_{2^{d+1}+2^d-4}$ with $3 \leqslant d \leqslant 5$. The detailed computations will be published elsewhere.

Let $x$ be an admissible monomial of degree $q = 2^{d+1} + 2^d - 4$ in $P_5$ with $d \geqslant 3$. By Lemma 2.14 in [19], $\omega_i(x) = 4$ for $1 \leqslant i \leqslant d-2$. Hence, $x = \bar{x}y^{2^{d-2}}$ with $\bar{x}$ a monomial of weight vector $(4)|^{d-2}$ and $y$ a monomial of degree 8 in $P_5$. By combining this and a result in [25] we easily obtain

$$\omega(x) = (4)|^{d-2}|(2)|(1)|^2 \text{ or } \omega(x) = (4)|^{d-2}|(2)|(3) \text{ or } \omega(x) = (4)|^{d-1}|(2). \quad (4.1)$$

This result is also presented in [8].

### 4.1 On a structure of the space $(QP_5)_{20}$

By (4.1), we have

$$(QP_5)_{20} \cong QP_5(4,2,1,1) \bigoplus QP_5(4,2,3) \bigoplus QP_5(4,4,2).$$

In [26] and [8], the authors say that

$$\dim QP_5(4,2,1,1) = 450,\ \dim QP_5(4,2,3) = 70,\ \dim QP_5(4,4,2) = 121.$$

However, the detailed computation is only presented for the space $QP_5(4,2,3)$. The computation of $QP_5(4,2,3)$ and $QP_5(4,4,2)$ is easy and the above dimensional results are true. However, the computation of $QP_5(4,2,1,1)$ is rather complicated. To determine a generating set for $QP_5(4,2,1,1)$ we need the following which is easy to checked the accuracy.

**Lemma 4.1.1.** *If $(i,j,t,u,v)$ is a permutation of $(1,2,3,4,5)$ such that $i < j < t < u$, then the monomials $x_i^2x_jx_tx_ux_v^3$ is strictly inadmissible.*

By a direct computation analogous to the one in Subsection 3.2 and using Lemma 4.1.1, we have determined a set of generators for $QP_5(4,2,1,1)$ consisting of 450 monomials (see Subsection 6.1). By using this generating set we get

$$\widetilde{SF}_5(4,2,1,1) = \langle\{[g_t] : 1 \leqslant t \leqslant 10\}\rangle,$$

where $g_t$ are the polynomials with the leading monomials $U_t$ which are determined as follows:

| | | | |
|---|---|---|---|
| $U_1 = x_1^3x_2^4x_3x_4^9x_5^3$ | $U_2 = x_1^3x_2^4x_3^9x_4x_5^3$ | $U_3 = x_1^3x_2^4x_3^9x_4^3x_5$ | $U_4 = x_1^3x_2^{12}x_3x_4x_5^3$ |
| $U_5 = x_1^3x_2^{12}x_3x_4^3x_5$ | $U_6 = x_1^3x_2^{12}x_3^3x_4x_5$ | $U_7 = x_1^7x_2^8x_3x_4x_5^3$ | $U_8 = x_1^7x_2^8x_3x_4^3x_5$ |
| $U_9 = x_1^7x_2^8x_3^3x_4x_5$ | $U_{10} = x_1^7x_2^9x_3^2x_4x_5$ | | |

If $\dim QP_5(4,2,1,1) = 450$, then $\dim \widetilde{SF}_5(4,2,1,1) = 10$. However, we have been unable to prove this by hand computation. Perhaps, this result be can only be checked by using a computer calculation.

From [20] we see that the monomials $x_1^3x_2^{12}x_3^3x_4$, $x_1^3x_2^{12}x_3x_4^3$, $x_1^7x_2^8x_3^3x_4$, $x_1^7x_2^8x_3x_4^3$, $x_1^7x_2^9x_3^2x_4$ are admissible. So, from Proposition 2.4.3, $U_t$, $4 \leqslant t \leqslant 10$, are admissible. Hence, from the above equalities and by a simple calculation, we obtain the following.

**Proposition 4.1.2.** *We have*

i) $\dim \widetilde{QP}_k(4,2,1,1) = 440$.

ii) $\widetilde{SF}_5(4,2,1,1)$ *is an $\mathbb{F}_2$-subspace of $QP_5(4,2,1,1)$ and*

$$7 \leqslant \dim \widetilde{SF}_5((4)|(2)|^3) \leqslant 10.$$

**Remark 4.1.3.** In [8], the author presents the detailed computation for the easiest case, $QP_5(4,2,3)$, and he says that "the remaining ones can be acquired through analogous computational techniques". However, Proposition 4.1.2 shows that his calculation techniques are not applicable to the case of $QP_5(4,2,1,1)$.

## 4.2 On a structure of the space $(QP_5)_{44}$

By (4.1), we have

$$(QP_5)_{44} \cong QP_5((4)|^2(2)|(1)|^2) \bigoplus QP_5((4)|^2|(2)|(3)) \bigoplus QP_5((4)|^3|(2)).$$

The authors of [26] and [8] say that by using computer calculation, they obtain the dimensional result that $\dim(QP_5)_{44} = 1426$. However, at the time of writing, we do not find any detailed results for this space. We need the following technical lemmas.

**Lemma 4.2.1** (See [10])**.** *Let $(i,j,t,u,v)$ is an arbitrary permutation of $(1,2,3,4,5)$. The following monomials are strictly inadmissible:*

$$x_i^2x_jx_t^3x_u^3x_v^3,\ x_i^3x_j^4x_t^7x_u^7x_v^7,\ i<j;\ x_i^7x_j^8x_t^7x_u^3x_v^3,\ i=1,\ j=2;\ x_1^7x_2^9x_3^6x_4^3x_5^3.$$

**Lemma 4.2.2** (See [10])**.** *Let $(i,j,t,u,v)$ is an arbitrary permutation of $(1,2,3,4,5)$. The following monomials are strictly inadmissible:*

$$\begin{aligned}
&x_i^3x_j^4x_t^{11}x_u^{11}x_v^{15},\ x_i^3x_j^8x_t^7x_u^{11}x_v^{15},\ x_i^3x_j^{12}x_t^7x_u^{11}x_v^{11},\ i<j\\
&x_i^7x_j^8x_t^7x_u^{11}x_v^{11},\ x_i^7x_j^9x_t^6x_u^{11}x_v^{11},\ x_i^7x_j^9x_t^7x_u^{11}x_v^{10},\ x_i^7x_j^{11}x_t^4x_u^{11}x_v^{11},\ i<j<t,\\
&x_i^7x_j^7x_t^8x_u^7x_v^{15},\ i<j<t<u.
\end{aligned}$$

**Lemma 4.2.3.** *Let $(i,j,t,u,v)$ is an arbitrary permutation of $(1,2,3,4,5)$. The following monomials are strictly inadmissible:*

$$\begin{aligned}
&x_i^3x_j^4x_t^3x_u^7x_v^{27},\ x_i^3x_j^4x_t^3x_u^{23}x_v^{11},\ x_i^3x_j^4x_t^3x_u^{15}x_v^{19},\ x_i^3x_j^{12}x_t^3x_u^7x_v^{19},\ 1<j<t,\ j<v;\\
&x_i^3x_j^4x_t^3x_u^3x_v^{31},\ x_i^3x_j^{28}x_t^3x_u^3x_v^7,\ x_i^3x_j^{12}x_t^3x_u^3x_v^{23},\ x_i^3x_j^{15}x_t^{20}x_u^3x_v^3,\ i<j<t<u,\\
&x_1^7x_2^{11}x_3^4x_4^3x_5^{19},\ x_1^7x_2^{11}x_3^4x_4^{19}x_5^3,\ x_1^7x_2^{11}x_3^{20}x_4^3x_5^3,\ x_1^7x_2^{27}x_3^4x_4^3x_5^3,\ x_1^{15}x_2^{16}x_3^3x_4^3x_5^7,\\
&x_1^{15}x_2^{16}x_3^3x_4^7x_5^3,\ x_1^{15}x_2^{16}x_3^7x_4^3x_5^3,\ x_1^{15}x_2^{17}x_3^6x_4^3x_5^3,\ x_1^{15}x_2^{19}x_3^4x_4^3x_5^3.
\end{aligned}$$

By a similar computation as in Subsection 3.2 and using Lemmas 4.2.1-4.2.3, we have determined a generating set for $(QP_5)_{44}$ consisting of 1426 monomials and proved that $\dim QP_5((4)|^3|(2)) = 176$, $\dim QP_5((4)|^2|(2)|(3)) = 110$, and $1095 \leqslant \dim QP_5((4)|^2(2)|(1)|^2) \leqslant 1140$. The monomials of degree 44 in this generating set are given in Subsection 6.2 and they are admissible. By a direct computation using this generating set we obtain

$$\widetilde{SF}_5((4)|^3|(2)) = 0; \quad \widetilde{SF}_5((4)|^2|(2)|(3)) = 0$$
$$\widetilde{SF}_5((4)|^2(2)|(1)|^2) = \langle\{[\bar{g}_s] : 1 \leqslant s \leqslant 45\}\rangle,$$

where $\bar{g}_s$ are the polynomials with the leading monomials $V_s$ which are determined as follows:

$$\begin{array}{lll}
V_1 = x_1^3x_2^7x_3^8x_4^{19}x_5^7 & V_2 = x_1^3x_2^7x_3^{24}x_4^3x_5^7 & V_3 = x_1^3x_2^7x_3^{24}x_4^7x_5^3 \\
V_4 = x_1^3x_2^{15}x_3^{16}x_4^3x_5^7 & V_5 = x_1^3x_2^{15}x_3^{16}x_4^7x_5^3 & V_6 = x_1^3x_2^{15}x_3^{19}x_4^4x_5^3 \\
V_7 = x_1^7x_2^3x_3^8x_4^{19}x_5^7 & V_8 = x_1^7x_2^3x_3^{24}x_4^3x_5^7 & V_9 = x_1^7x_2^3x_3^{24}x_4^7x_5^3 \\
V_{10} = x_1^7x_2^9x_3^2x_4^{19}x_5^7 & V_{11} = x_1^7x_2^9x_3^3x_4^{18}x_5^7 & V_{12} = x_1^7x_2^9x_3^3x_4^{19}x_5^6 \\
V_{13} = x_1^7x_2^9x_3^{18}x_4^3x_5^7 & V_{14} = x_1^7x_2^9x_3^{18}x_4^7x_5^3 & V_{15} = x_1^7x_2^9x_3^{19}x_4^2x_5^7 \\
V_{16} = x_1^7x_2^9x_3^{19}x_4^3x_5^6 & V_{17} = x_1^7x_2^9x_3^{19}x_4^6x_5^3 & V_{18} = x_1^7x_2^9x_3^{19}x_4^7x_5^2 \\
V_{19} = x_1^7x_2^{11}x_3^{16}x_4^3x_5^7 & V_{20} = x_1^7x_2^{11}x_3^{16}x_4^7x_5^3 & V_{21} = x_1^7x_2^{11}x_3^{19}x_4^4x_5^3 \\
V_{22} = x_1^7x_2^{15}x_3^{16}x_4^3x_5^3 & V_{23} = x_1^7x_2^{25}x_3^2x_4^3x_5^7 & V_{24} = x_1^7x_2^{25}x_3^2x_4^7x_5^3 \\
V_{25} = x_1^7x_2^{25}x_3^3x_4^2x_5^7 & V_{26} = x_1^7x_2^{25}x_3^3x_4^3x_5^6 & V_{27} = x_1^7x_2^{25}x_3^3x_4^6x_5^3 \\
V_{28} = x_1^7x_2^{25}x_3^3x_4^7x_5^2 & V_{29} = x_1^7x_2^{25}x_3^7x_4^2x_5^3 & V_{30} = x_1^7x_2^{25}x_3^7x_4^3x_5^2 \\
V_{31} = x_1^{15}x_2^3x_3^{16}x_4^3x_5^7 & V_{32} = x_1^{15}x_2^3x_3^{16}x_4^7x_5^3 & V_{33} = x_1^{15}x_2^3x_3^{19}x_4^4x_5^3 \\
V_{34} = x_1^{15}x_2^7x_3^{16}x_4^3x_5^3 & V_{35} = x_1^{15}x_2^{17}x_3^2x_4^3x_5^7 & V_{36} = x_1^{15}x_2^{17}x_3^2x_4^7x_5^3 \\
V_{37} = x_1^{15}x_2^{17}x_3^3x_4^2x_5^7 & V_{38} = x_1^{15}x_2^{17}x_3^3x_4^3x_5^6 & V_{39} = x_1^{15}x_2^{17}x_3^3x_4^6x_5^3 \\
V_{40} = x_1^{15}x_2^{17}x_3^3x_4^7x_5^2 & V_{41} = x_1^{15}x_2^{17}x_3^7x_4^2x_5^3 & V_{42} = x_1^{15}x_2^{17}x_3^7x_4^3x_5^2 \\
V_{43} = x_1^{15}x_2^{19}x_3^3x_4^4x_5^3 & V_{44} = x_1^{15}x_2^{19}x_3^5x_4^2x_5^3 & V_{45} = x_1^{15}x_2^{19}x_3^5x_4^3x_5^2
\end{array}$$

If $\dim(QP_5)_{44} = 1426$, then $\dim\widetilde{SF}_5((4)|^2(2)|(1)|^2) = 45$. However, this result can not be proved by hand computation. From the above result, we obtain the following.

**Proposition 4.2.4.** *We have*

i) $\dim\widetilde{QP}_k((4)|^2(2)|(1)|^2) = 1095$.

ii) $\widetilde{SF}_5((4)|^2(2)|(1)|^2)$ *is an* $\mathbb{F}_2$*-subspace of* $QP_5((4)|^2(2)|(1)|^2)$ *and*

$$\dim\widetilde{SF}_5((4)|^2(2)|(1)|^2) \leqslant 45.$$

## 4.3 On a structure of the space $(QP_5)_{92}$

By (4.1), we have

$$(QP_5)_{92} \cong QP_5((4)|^3(2)|(1)|^2) \bigoplus QP_5((4)|^3|(2)|(3)) \bigoplus QP_5((4)|^4|(2)).$$

The authors of [26] and [8] say that by using computer calculation, they obtains the dimensional result $\dim(QP_5)_{92} = 1706$. However, at the time of writing, the detailed computations were not available in their works. Moreover, we also do not find the detailed results for this space in any documents. To determined $(QP_5)_{92}$ we need some technical lemmas.

**Lemma 4.3.1** (See [10]). *Let $(i,j,t,u,v)$ is an arbitrary permutation of $(1,2,3,4,5)$. The following monomials are strictly inadmissible:*

$x_i^7x_j^7x_t^{15}x_u^{16}x_v^{15}$, $t<u<v$; $x_i^7x_j^{15}x_t^{17}x_u^6x_v^{15}$, $i,\ j<t=3$; $x_1^{15}x_2^{19}x_3^5x_4^{14}x_5^7$;
$x_i^7x_j^{15}x_t^{17}x_u^{14}x_v^7$; $(u,v)=(4,5)$, $x_i^{15}x_j^{19}x_t^5x_u^6x_v^{15}$, $(i,j)=(1,2)$.

**Lemma 4.3.2.** *Let $(i,j,t,u,v)$ is an arbitrary permutation of $(1,2,3,4,5)$. The following monomials are strictly inadmissible:*

$x_i^7x_j^7x_t^{15}x_u^{31}x_v^{32}$; $x_i^7x_j^{15}x_t^{23}x_u^{39}x_v^8$; $x_i^7x_j^{15}x_t^{31}x_u^{33}x_v^6$; $x_i^7x_j^{31}x_t^{39}x_u^9x_v^6$; $x_i^7x_j^7x_t^{31}x_u^{39}x_v^8$;
$x_i^7x_j^{15}x_t^{23}x_u^{15}x_v^{32}$, $j<t<u$; $x_i^7x_j^{31}x_t^{33}x_u^7x_v^{14}$, $i,\ j<t=3$;
$x_i^7x_j^{15}x_t^{17}x_u^{39}x_v^{14}$, $(u,v)=(4,5)$; $x_i^{15}x_j^{31}x_t^{35}x_u^5x_v^6$, $j<j<u<v$;
$x_i^{15}x_j^{19}x_t^5x_u^{14}x_v^{39}$, $x_i^{15}x_j^{19}x_t^7x_u^{37}x_v^{14}$, $x_i^{15}x_j^{51}x_t^5x_u^7x_v^{14}$, $x_i^{31}x_j^{35}x_t^5x_u^7x_v^{14}$, $(i,j)=(1,2)$;
$x_1^7x_2^{15}x_3^{49}x_4^7x_5^{14}$, $x_1^{15}x_2^7x_3^{49}x_4^7x_5^{14}$, $x_1^{15}x_2^{19}x_3^5x_4^{46}x_5^7$, $x_1^{15}x_2^{19}x_3^{39}x_4^{13}x_5^6$,
$x_1^{15}x_2^{23}x_3^{39}x_4^9x_5^6$, $x_1^{15}x_2^{51}x_3^7x_4^{13}x_5^6$, $x_1^{31}x_2^{35}x_3^7x_4^{13}x_5^6$, $x_1^{31}x_2^{39}x_3^{11}x_4^5x_5^6$,

**Lemma 4.3.3.** *Let $(i,j,t,u,v)$ is an arbitrary permutation of $(1,2,3,4,5)$. The following monomials are strictly inadmissible:*

$$x_i^{15}x_j^{15}x_t^{15}x_u^{16}x_v^{31},\ x_i^{15}x_j^{15}x_t^{16}x_u^{23}x_v^{23},\ x_1^{15}x_2^{15}x_3^{23}x_4^{17}x_5^{22}.$$

By using the above lemma and an analogous argument as in Subsection 3.2, we have determined a generating set for $(QP_5)_{92}$ consisting of 1711 monomials and proved that $\dim QP_5((4)|^4|(2)) = 186$, $\dim QP_5((4)|^2|(2)|(3)) = 124$ and $1395 \leqslant \dim QP_5((4)|^2(2)|(1)|^2) \leqslant 1401$. The monomials in this generating set are explicitly given in Subsection 6.3. By a direct computation using this generating set, we get

$$\widetilde{SF}_5((4)|^4|(2)) = 0;\ \ \widetilde{SF}_5((4)|^3|(2)|(3)) = 0$$
$$\widetilde{SF}_5((4)|^3(2)|(1)|^2) = \langle\{[\tilde g_u] : 1 \leqslant u \leqslant 6\}\rangle,$$

where $\tilde g_u$ are the polynomials with the leading monomials $W_u$ which are determined as follows:

$$W_1 = x_1^{15}x_2^{15}x_3^{23}x_4^{32}x_5^7 \quad W_2 = x_1^7x_2^{15}x_3^{17}x_4^7x_5^{46} \quad W_3 = x_1^{15}x_2^7x_3^{17}x_4^7x_5^{46}$$

By computing from the above result, we obtain the following.

**Proposition 4.3.4.** *We have*

i) $\dim \widetilde{QP}_k((4)|^3(2)|(1)|^2) = 1395$.
ii) $\widetilde{SF}_5((4)|^3(2)|(1)|^2)$ *is an* $\mathbb{F}_2$*-subspace of* $QP_5((4)|^3(2)|(1)|^2)$ *and*

$$\dim \widetilde{SF}_5((4)|^3(2)|(1)|^2) \leqslant 6.$$

From our computation of $\widetilde{SF}_5((4)|^3(2)|(1)|^2)$, we obtain the following.

**Proposition 4.3.5.** *If* $\dim(QP_5)_{92} = 1706$*, then* $[\tilde g_1] \neq 0$ *and*

$$\widetilde{SF}_5((4)|^3(2)|(1)|^2) = \langle[\tilde g_1]\rangle.$$

*Proof.* For $1 \leqslant s < 5$, denote by $\psi_s : P_5 \to P_5$ the homomorphism of $\mathcal{A}$-algebras given by

$$\psi_s(x_t) = \begin{cases} x_{s+1}, & \text{if } t = s,\\ x_s, & \text{if } t = s+1,\\ x_t, & \text{if } t \neq s.\end{cases}$$

Then, $\{\psi_1, \psi_2, \psi_3, \psi_4\}$ is a set of generators for the symmetric group $\Sigma_5 \subset GL_5$.

We have

$$\begin{aligned}\tilde g_2 = {}& x_1x_2^{7}x_3^{15}x_4^{7}x_5^{62} + x_1x_2^{15}x_3^{15}x_4^{7}x_5^{54} + x_1x_2^{15}x_3^{15}x_4^{22}x_5^{39} + x_1x_2^{15}x_3^{15}x_4^{23}x_5^{38}\\
&+ x_1^{3}x_2^{5}x_3^{15}x_4^{23}x_5^{46} + x_1^{3}x_2^{5}x_3^{15}x_4^{30}x_5^{39} + x_1^{3}x_2^{7}x_3^{7}x_4^{13}x_5^{62} + x_1^{3}x_2^{7}x_3^{7}x_4^{29}x_5^{46}\\
&+ x_1^{3}x_2^{7}x_3^{13}x_4^{23}x_5^{46} + x_1^{3}x_2^{7}x_3^{13}x_4^{30}x_5^{39} + x_1^{3}x_2^{7}x_3^{15}x_4^{13}x_5^{54} + x_1^{3}x_2^{7}x_3^{29}x_4^{7}x_5^{46}\\
&+ x_1^{3}x_2^{15}x_3^{5}x_4^{23}x_5^{46} + x_1^{3}x_2^{15}x_3^{5}x_4^{30}x_5^{39} + x_1^{3}x_2^{15}x_3^{7}x_4^{13}x_5^{54} + x_1^{3}x_2^{15}x_3^{7}x_4^{21}x_5^{46}\\
&+ x_1^{3}x_2^{15}x_3^{15}x_4^{21}x_5^{38} + x_1^{3}x_2^{15}x_3^{21}x_4^{7}x_5^{46} + x_1^{7}x_2x_3^{15}x_4^{23}x_5^{46} + x_1^{7}x_2x_3^{15}x_4^{30}x_5^{39}\\
&+ x_1^{7}x_2^{7}x_3^{7}x_4^{9}x_5^{62} + x_1^{7}x_2^{7}x_3^{7}x_4^{25}x_5^{46} + x_1^{7}x_2^{7}x_3^{9}x_4^{23}x_5^{46} + x_1^{7}x_2^{7}x_3^{9}x_4^{30}x_5^{39}\\
&+ x_1^{7}x_2^{7}x_3^{15}x_4^{7}x_5^{56} + x_1^{7}x_2^{7}x_3^{15}x_4^{9}x_5^{54} + x_1^{7}x_2^{7}x_3^{25}x_4^{7}x_5^{46} + x_1^{7}x_2^{15}x_3x_4^{23}x_5^{46}\\
&+ x_1^{7}x_2^{15}x_3x_4^{30}x_5^{39} + x_1^{7}x_2^{15}x_3^{7}x_4^{9}x_5^{54} + x_1^{7}x_2^{15}x_3^{7}x_4^{17}x_5^{46} + x_1^{7}x_2^{15}x_3^{15}x_4^{7}x_5^{48}\\
&+ x_1^{7}x_2^{15}x_3^{15}x_4^{16}x_5^{39} + x_1^{7}x_2^{15}x_3^{15}x_4^{17}x_5^{38} + x_1^{7}x_2^{15}x_3^{15}x_4^{23}x_5^{32} + x_1^{7}x_2^{15}x_3^{17}x_4^{7}x_5^{46},\end{aligned}$$

and $\tilde g_3 = \psi_1(\tilde g_2)$. Hence, if $[\tilde g_2] \neq 0$, then $[\tilde g_3] \neq 0$ and $\dim(QP_5)_{92} \geqslant 1707$. This is a contradiction. Therefore, $[\tilde g_2] = [\tilde g_3] = 0$.

We have

$$\begin{aligned}\tilde g_5 = {}& x_1x_2^{15}x_3^{7}x_4^{30}x_5^{39} + x_1x_2^{15}x_3^{23}x_4^{7}x_5^{46} + x_1^{3}x_2^{7}x_3^{7}x_4^{29}x_5^{46} + x_1^{3}x_2^{7}x_3^{13}x_4^{30}x_5^{39}\\
&+ x_1^{3}x_2^{7}x_3^{15}x_4^{13}x_5^{54} + x_1^{3}x_2^{7}x_3^{15}x_4^{21}x_5^{46} + x_1^{3}x_2^{7}x_3^{29}x_4^{7}x_5^{46} + x_1^{3}x_2^{13}x_3^{7}x_4^{30}x_5^{39}\\
&+ x_1^{3}x_2^{13}x_3^{23}x_4^{7}x_5^{46} + x_1^{3}x_2^{15}x_3^{7}x_4^{21}x_5^{46} + x_1^{3}x_2^{15}x_3^{13}x_4^{22}x_5^{39} + x_1^{3}x_2^{15}x_3^{21}x_4^{7}x_5^{46}\\
&+ x_1^{3}x_2^{15}x_3^{21}x_4^{14}x_5^{39} + x_1^{3}x_2^{15}x_3^{23}x_4^{5}x_5^{46} + x_1^{3}x_2^{15}x_3^{29}x_4^{6}x_5^{39} + x_1^{7}x_2^{7}x_3^{7}x_4^{9}x_5^{62}\\
&+ x_1^{7}x_2^{7}x_3^{9}x_4^{30}x_5^{39} + x_1^{7}x_2^{7}x_3^{11}x_4^{5}x_5^{62} + x_1^{7}x_2^{7}x_3^{11}x_4^{13}x_5^{54} + x_1^{7}x_2^{7}x_3^{15}x_4^{9}x_5^{54}\\
&+ x_1^{7}x_2^{7}x_3^{15}x_4^{17}x_5^{46} + x_1^{7}x_2^{7}x_3^{25}x_4^{7}x_5^{46} + x_1^{7}x_2^{7}x_3^{27}x_4^{5}x_5^{46} + x_1^{7}x_2^{11}x_3^{7}x_4^{21}x_5^{46}\\
&+ x_1^{7}x_2^{11}x_3^{13}x_4^{22}x_5^{39} + x_1^{7}x_2^{11}x_3^{21}x_4^{7}x_5^{46} + x_1^{7}x_2^{11}x_3^{21}x_4^{14}x_5^{39} + x_1^{7}x_2^{11}x_3^{23}x_4^{5}x_5^{46}\\
&+ x_1^{7}x_2^{11}x_3^{29}x_4^{6}x_5^{39} + x_1^{7}x_2^{15}x_3^{7}x_4x_5^{62} + x_1^{7}x_2^{15}x_3^{7}x_4^{9}x_5^{54} + x_1^{7}x_2^{15}x_3^{11}x_4^{5}x_5^{54}\\
&+ x_1^{7}x_2^{27}x_3^{7}x_4^{5}x_5^{46} + x_1^{15}x_2x_3^{7}x_4^{30}x_5^{39} + x_1^{15}x_2x_3^{23}x_4^{7}x_5^{46} + x_1^{15}x_2^{3}x_3^{7}x_4^{21}x_5^{46}\\
&+ x_1^{15}x_2^{3}x_3^{13}x_4^{22}x_5^{39} + x_1^{15}x_2^{3}x_3^{21}x_4^{7}x_5^{46} + x_1^{15}x_2^{3}x_3^{21}x_4^{14}x_5^{39} + x_1^{15}x_2^{3}x_3^{23}x_4^{5}x_5^{46}\end{aligned}$$

$$
\begin{aligned}
&+ x_1^{15}x_2^{3}x_3^{29}x_4^{6}x_5^{39} + x_1^{15}x_2^{7}x_3^{7}x_4^{9}x_5^{54} + x_1^{15}x_2^{7}x_3^{7}x_4^{17}x_5^{46} + x_1^{15}x_2^{7}x_3^{11}x_4^{5}x_5^{54}\\
&+ x_1^{15}x_2^{7}x_3^{19}x_4^{5}x_5^{46} + x_1^{15}x_2^{15}x_3^{7}x_4x_5^{54} + x_1^{15}x_2^{15}x_3^{7}x_4^{16}x_5^{39} + x_1^{15}x_2^{15}x_3^{17}x_4^{6}x_5^{39}\\
&+ x_1^{15}x_2^{15}x_3^{17}x_4^{7}x_5^{38} + x_1^{15}x_2^{15}x_3^{19}x_4^{5}x_5^{38} + x_1^{15}x_2^{15}x_3^{23}x_4x_5^{38} + x_1^{15}x_2^{15}x_3^{23}x_4^{7}x_5^{32}\\
&+ x_1^{15}x_2^{19}x_3^{7}x_4^{5}x_5^{46},
\end{aligned}
$$

and $\tilde{g}_4 \equiv \psi_3(\tilde{g}_5)$, $\tilde{g}_6 \equiv \psi_4(\tilde{g}_5)$. So, if $[\tilde{g}_5] \neq 0$, then $\dim(QP_5)_{92} \geqslant 1708$ and we have a contradiction. This implies $[\tilde{g}_4] = [\tilde{g}_5] = [\tilde{g}_6] = 0$. Combining these facts and the hypothesis that $\dim(QP_5)_{92} = 1706$, we must have $[\tilde{g}_1] \neq 0$ and the proposition is proved. $\square$

**Remark 4.3.6.** For any $d \geqslant 6$, by combining Theorem 1.3 and 1.4 in [20] we easily obtain

$$\dim(QP_5)_{2^{d+1}+2^d-4} = (2^5 - 1)\dim(QP_4)_8 = 31 \times 55 = 1705.$$

The author of [8] says that by using a computer program, he has checked the accuracy of the above result for any $d \geqslant 6$, but we do not believe that there is a computer program that can check this result for infinitely many values of $d$.

# 5 The admissible monomials of degree $2^{d+2} + 2^{d+1} - 3$

In this section, we list all elements in the set

$$B_5((3)|^d|(1)|^2) = B_5^0((3)|^d|(1)|^2) \bigcup B_5^+((3)|^d|(1)|^2)$$

for all $d \geqslant 3$.

By Kameko [4], we have $B_3^+((3)|^d|(1)|^2) = \{v_j = v_{d,j} : 1 \leqslant j \leqslant 6\}$, where

$$
\begin{array}{ll}
v_1 = x_1^{2^d-1}x_2^{2^d-1}x_3^{2^{d+2}-1} & v_2 = x_1^{2^d-1}x_2^{2^{d+1}-1}x_3^{2^{d+1}+2^d-1}\\
v_3 = x_1^{2^d-1}x_2^{2^{d+2}-1}x_3^{2^d-1} & v_4 = x_1^{2^{d+1}-1}x_2^{2^d-1}x_3^{2^{d+1}+2^d-1}\\
v_5 = x_1^{2^{d+1}-1}x_2^{2^{d+1}+2^d-1}x_3^{2^d-1} & v_6 = x_1^{2^{d+2}-1}x_2^{2^d-1}x_3^{2^d-1}
\end{array}
$$

From [20], for, $d \geqslant 4$, we have $B_4^+((3)|^d|(1)|^2) = \{w_j = w_{d,j} : 1 \leqslant j \leqslant 66\}$, where

$$\begin{array}{ll}
w_{1} = x_1x_2^{2^d-2}x_3^{2^d-1}x_4^{2^{d+2}-1} & w_{2} = x_1x_2^{2^d-2}x_3^{2^{d+1}-1}x_4^{2^{d+1}+2^d-1}\\
w_{3} = x_1x_2^{2^d-2}x_3^{2^{d+2}-1}x_4^{2^d-1} & w_{4} = x_1x_2^{2^d-1}x_3^{2^d-2}x_4^{2^{d+2}-1}\\
w_{5} = x_1x_2^{2^d-1}x_3^{2^d-1}x_4^{2^{d+2}-2} & w_{6} = x_1x_2^{2^d-1}x_3^{2^{d+1}-2}x_4^{2^{d+1}+2^d-1}\\
w_{7} = x_1x_2^{2^d-1}x_3^{2^{d+1}-1}x_4^{2^{d+1}+2^d-2} & w_{8} = x_1x_2^{2^d-1}x_3^{2^{d+2}-2}x_4^{2^d-1}\\
w_{9} = x_1x_2^{2^d-1}x_3^{2^{d+2}-1}x_4^{2^d-2} & w_{10} = x_1x_2^{2^{d+1}-2}x_3^{2^d-1}x_4^{2^{d+1}+2^d-1}\\
w_{11} = x_1x_2^{2^{d+1}-2}x_3^{2^{d+1}+2^d-1}x_4^{2^d-1} & w_{12} = x_1x_2^{2^{d+1}-1}x_3^{2^d-2}x_4^{2^{d+1}+2^d-1}\\
w_{13} = x_1x_2^{2^{d+1}-1}x_3^{2^d-1}x_4^{2^{d+1}+2^d-2} & w_{14} = x_1x_2^{2^{d+1}-1}x_3^{2^{d+1}+2^d-2}x_4^{2^d-1}\\
w_{15} = x_1x_2^{2^{d+1}-1}x_3^{2^{d+1}+2^d-1}x_4^{2^d-2} & w_{16} = x_1x_2^{2^{d+2}-2}x_3^{2^d-1}x_4^{2^d-1}\\
w_{17} = x_1x_2^{2^{d+2}-1}x_3^{2^d-2}x_4^{2^d-1} & w_{18} = x_1x_2^{2^{d+2}-1}x_3^{2^d-1}x_4^{2^d-2}\\
w_{19} = x_1^3x_2^{2^d-3}x_3^{2^d-2}x_4^{2^{d+2}-1} & w_{20} = x_1^3x_2^{2^d-3}x_3^{2^d-1}x_4^{2^{d+2}-2}\\
w_{21} = x_1^3x_2^{2^d-3}x_3^{2^{d+1}-2}x_4^{2^{d+1}+2^d-1} & w_{22} = x_1^3x_2^{2^d-3}x_3^{2^{d+1}-1}x_4^{2^{d+1}+2^d-2}\\
w_{23} = x_1^3x_2^{2^d-3}x_3^{2^{d+2}-2}x_4^{2^d-1} & w_{24} = x_1^3x_2^{2^d-3}x_3^{2^{d+2}-1}x_4^{2^d-2}\\
w_{25} = x_1^3x_2^{2^d-1}x_3^{2^d-3}x_4^{2^{d+2}-2} & w_{26} = x_1^3x_2^{2^d-1}x_3^{2^{d+1}-3}x_4^{2^{d+1}+2^d-2}\\
w_{27} = x_1^3x_2^{2^d-1}x_3^{2^{d+2}-3}x_4^{2^d-2} & w_{28} = x_1^3x_2^{2^{d+1}-3}x_3^{2^d-2}x_4^{2^{d+1}+2^d-1}\\
w_{29} = x_1^3x_2^{2^{d+1}-3}x_3^{2^d-1}x_4^{2^{d+1}+2^d-2} & w_{30} = x_1^3x_2^{2^{d+1}-3}x_3^{2^{d+1}+2^d-2}x_4^{2^d-1}\\
w_{31} = x_1^3x_2^{2^{d+1}-3}x_3^{2^{d+1}+2^d-1}x_4^{2^d-2} & w_{32} = x_1^3x_2^{2^{d+1}-1}x_3^{2^d-3}x_4^{2^{d+1}+2^d-2}\\
w_{33} = x_1^3x_2^{2^{d+1}-1}x_3^{2^{d+1}+2^d-3}x_4^{2^d-2} & w_{34} = x_1^3x_2^{2^{d+2}-3}x_3^{2^d-2}x_4^{2^d-1}\\
w_{35} = x_1^3x_2^{2^{d+2}-3}x_3^{2^d-1}x_4^{2^d-2} & w_{36} = x_1^3x_2^{2^{d+2}-1}x_3^{2^d-3}x_4^{2^d-2}\\
w_{37} = x_1^7x_2^{2^d-5}x_3^{2^d-3}x_4^{2^{d+2}-2} & w_{38} = x_1^7x_2^{2^d-5}x_3^{2^{d+1}-3}x_4^{2^{d+1}+2^d-2}\\
w_{39} = x_1^7x_2^{2^d-5}x_3^{2^{d+2}-3}x_4^{2^d-2} & w_{40} = x_1^7x_2^{2^{d+1}-5}x_3^{2^d-3}x_4^{2^{d+1}+2^d-2}\\
w_{41} = x_1^7x_2^{2^{d+1}-5}x_3^{2^{d+1}+2^d-3}x_4^{2^d-2} & w_{42} = x_1^7x_2^{2^{d+2}-5}x_3^{2^d-3}x_4^{2^d-2}\\
w_{43} = x_1^{2^d-1}x_2x_3^{2^d-2}x_4^{2^{d+2}-1} & w_{44} = x_1^{2^d-1}x_2x_3^{2^d-1}x_4^{2^{d+2}-2}\\
w_{45} = x_1^{2^d-1}x_2x_3^{2^{d+1}-2}x_4^{2^{d+1}+2^d-1} & w_{46} = x_1^{2^d-1}x_2x_3^{2^{d+1}-1}x_4^{2^{d+1}+2^d-2}\\
w_{47} = x_1^{2^d-1}x_2x_3^{2^{d+2}-2}x_4^{2^d-1} & w_{48} = x_1^{2^d-1}x_2x_3^{2^{d+2}-1}x_4^{2^d-2}\\
w_{49} = x_1^{2^d-1}x_2^3x_3^{2^d-3}x_4^{2^{d+2}-2} & w_{50} = x_1^{2^d-1}x_2^3x_3^{2^{d+1}-3}x_4^{2^{d+1}+2^d-2}\\
w_{51} = x_1^{2^d-1}x_2^3x_3^{2^{d+2}-3}x_4^{2^d-2} & w_{52} = x_1^{2^d-1}x_2^{2^d-1}x_3x_4^{2^{d+2}-2}\\
w_{53} = x_1^{2^d-1}x_2^{2^{d+1}-1}x_3x_4^{2^{d+1}+2^d-2} & w_{54} = x_1^{2^d-1}x_2^{2^{d+2}-1}x_3x_4^{2^d-2}\\
w_{55} = x_1^{2^{d+1}-1}x_2x_3^{2^d-2}x_4^{2^{d+1}+2^d-1} & w_{56} = x_1^{2^{d+1}-1}x_2x_3^{2^d-1}x_4^{2^{d+1}+2^d-2}\\
w_{57} = x_1^{2^{d+1}-1}x_2x_3^{2^{d+1}+2^d-2}x_4^{2^d-1} & w_{58} = x_1^{2^{d+1}-1}x_2x_3^{2^{d+1}+2^d-1}x_4^{2^d-2}\\
w_{59} = x_1^{2^{d+1}-1}x_2^3x_3^{2^d-3}x_4^{2^{d+1}+2^d-2} & w_{60} = x_1^{2^{d+1}-1}x_2^3x_3^{2^{d+1}+2^d-3}x_4^{2^d-2}\\
w_{61} = x_1^{2^{d+1}-1}x_2^{2^d-1}x_3x_4^{2^{d+1}+2^d-2} & w_{62} = x_1^{2^{d+1}-1}x_2^{2^{d+1}+2^d-1}x_3x_4^{2^d-2}\\
w_{63} = x_1^{2^{d+2}-1}x_2x_3^{2^d-2}x_4^{2^d-1} & w_{64} = x_1^{2^{d+2}-1}x_2x_3^{2^d-1}x_4^{2^d-2}\\
w_{65} = x_1^{2^{d+2}-1}x_2^3x_3^{2^d-3}x_4^{2^d-2} & w_{66} = x_1^{2^{d+2}-1}x_2^{2^d-1}x_3x_4^{2^d-2}
\end{array}$$

For $d = 4$, $B_5^+ = \{w_{4,j} : j \neq 37, 38, 39\} \cup \{w_{37}, w_{38}, w_{39}\}$ where

$$w_{37} = x_1^7x_2^7x_3^7x_4^{24},\ w_{38} = x_1^7x_2^7x_3^9x_4^{22},\ w_{39} = x_1^7x_2^7x_3^{25}x_4^6.$$

By Proposition 2.4.2, we have $|B_5^0((3)|^d|(1)|^2)| = \binom{5}{3} \times 6 + \binom{5}{4} \times 66 = 390$.

## 5.1 The cases $d \geqslant 4$

For $d \geqslant 4$, $B_5^+((3)|^d|(1)|^2 = B(d) \bigcup C(d)$, where $C(d)$ is the set of the monomials $a_j = a_{d,j}$, $1 \leqslant j \leqslant 475$, which are determined as follows:

For $d \geqslant 4$,

$$a_1 = x_1x_2x_3^{2^d-2}x_4^{2^d-1}x_5^{2^{d+2}-2} \qquad a_2 = x_1x_2x_3^{2^d-2}x_4^{2^{d+1}-2}x_5^{2^{d+1}+2^d-1}$$
$$a_3 = x_1x_2x_3^{2^d-2}x_4^{2^{d+1}-1}x_5^{2^{d+1}+2^d-2} \qquad a_4 = x_1x_2x_3^{2^d-2}x_4^{2^{d+2}-2}x_5^{2^d-1}$$
$$a_5 = x_1x_2x_3^{2^d-1}x_4^{2^d-2}x_5^{2^{d+2}-2} \qquad a_6 = x_1x_2x_3^{2^d-1}x_4^{2^{d+1}-2}x_5^{2^{d+1}+2^d-2}$$
$$a_7 = x_1x_2x_3^{2^d-1}x_4^{2^{d+2}-2}x_5^{2^d-2} \qquad a_8 = x_1x_2x_3^{2^{d+1}-2}x_4^{2^d-2}x_5^{2^{d+1}+2^d-1}$$
$$a_9 = x_1x_2x_3^{2^{d+1}-2}x_4^{2^d-1}x_5^{2^{d+1}+2^d-2} \qquad a_{10} = x_1x_2x_3^{2^{d+1}-2}x_4^{2^{d+1}+2^d-2}x_5^{2^d-1}$$
$$a_{11} = x_1x_2x_3^{2^{d+1}-2}x_4^{2^{d+1}+2^d-1}x_5^{2^d-2} \qquad a_{12} = x_1x_2x_3^{2^{d+1}-1}x_4^{2^d-2}x_5^{2^{d+1}+2^d-2}$$
$$a_{13} = x_1x_2x_3^{2^{d+1}-1}x_4^{2^{d+1}+2^d-2}x_5^{2^d-2} \qquad a_{14} = x_1x_2x_3^{2^{d+2}-2}x_4^{2^d-2}x_5^{2^d-1}$$
$$a_{15} = x_1x_2x_3^{2^{d+2}-2}x_4^{2^d-1}x_5^{2^d-2} \qquad a_{16} = x_1x_2^2x_3^{2^d-4}x_4^{2^{d+1}-1}x_5^{2^{d+1}+2^d-1}$$
$$a_{17} = x_1x_2^2x_3^{2^d-3}x_4^{2^d-1}x_5^{2^{d+2}-2} \qquad a_{18} = x_1x_2^2x_3^{2^d-3}x_4^{2^{d+1}-2}x_5^{2^{d+1}+2^d-1}$$
$$a_{19} = x_1x_2^2x_3^{2^d-3}x_4^{2^{d+1}-1}x_5^{2^{d+1}+2^d-2} \qquad a_{20} = x_1x_2^2x_3^{2^d-3}x_4^{2^{d+2}-2}x_5^{2^d-1}$$
$$a_{21} = x_1x_2^2x_3^{2^d-1}x_4^{2^d-3}x_5^{2^{d+2}-2} \qquad a_{22} = x_1x_2^2x_3^{2^d-1}x_4^{2^d-1}x_5^{2^{d+2}-4}$$
$$a_{23} = x_1x_2^2x_3^{2^d-1}x_4^{2^{d+1}-4}x_5^{2^{d+1}+2^d-1} \qquad a_{24} = x_1x_2^2x_3^{2^d-1}x_4^{2^{d+1}-3}x_5^{2^{d+1}+2^d-2}$$
$$a_{25} = x_1x_2^2x_3^{2^d-1}x_4^{2^{d+1}-1}x_5^{2^{d+1}+2^d-4} \qquad a_{26} = x_1x_2^2x_3^{2^d-1}x_4^{2^{d+2}-4}x_5^{2^d-1}$$
$$a_{27} = x_1x_2^2x_3^{2^d-1}x_4^{2^{d+2}-3}x_5^{2^d-2} \qquad a_{28} = x_1x_2^2x_3^{2^{d+1}-4}x_4^{2^d-1}x_5^{2^{d+1}+2^d-1}$$
$$a_{29} = x_1x_2^2x_3^{2^{d+1}-4}x_4^{2^{d+1}+2^d-1}x_5^{2^d-1} \qquad a_{30} = x_1x_2^2x_3^{2^{d+1}-3}x_4^{2^d-2}x_5^{2^{d+1}+2^d-1}$$
$$a_{31} = x_1x_2^2x_3^{2^{d+1}-3}x_4^{2^d-1}x_5^{2^{d+1}+2^d-2} \qquad a_{32} = x_1x_2^2x_3^{2^{d+1}-3}x_4^{2^{d+1}+2^d-2}x_5^{2^d-1}$$
$$a_{33} = x_1x_2^2x_3^{2^{d+1}-3}x_4^{2^{d+1}+2^d-1}x_5^{2^d-2} \qquad a_{34} = x_1x_2^2x_3^{2^{d+1}-1}x_4^{2^d-4}x_5^{2^{d+1}+2^d-1}$$
$$a_{35} = x_1x_2^2x_3^{2^{d+1}-1}x_4^{2^d-3}x_5^{2^{d+1}+2^d-2} \qquad a_{36} = x_1x_2^2x_3^{2^{d+1}-1}x_4^{2^d-1}x_5^{2^{d+1}+2^d-4}$$
$$a_{37} = x_1x_2^2x_3^{2^{d+1}-1}x_4^{2^{d+1}+2^d-4}x_5^{2^d-1} \qquad a_{38} = x_1x_2^2x_3^{2^{d+1}-1}x_4^{2^{d+1}+2^d-3}x_5^{2^d-2}$$
$$a_{39} = x_1x_2^2x_3^{2^{d+1}-1}x_4^{2^{d+1}+2^d-1}x_5^{2^d-4} \qquad a_{40} = x_1x_2^2x_3^{2^{d+2}-4}x_4^{2^d-1}x_5^{2^d-1}$$
$$a_{41} = x_1x_2^2x_3^{2^{d+2}-3}x_4^{2^d-2}x_5^{2^d-1} \qquad a_{42} = x_1x_2^2x_3^{2^{d+2}-3}x_4^{2^d-1}x_5^{2^d-2}$$
$$a_{43} = x_1x_2^3x_3^{2^d-4}x_4^{2^d-1}x_5^{2^{d+2}-2} \qquad a_{44} = x_1x_2^3x_3^{2^d-4}x_4^{2^{d+1}-2}x_5^{2^{d+1}+2^d-1}$$
$$a_{45} = x_1x_2^3x_3^{2^d-4}x_4^{2^{d+1}-1}x_5^{2^{d+1}+2^d-2} \qquad a_{46} = x_1x_2^3x_3^{2^d-4}x_4^{2^{d+2}-2}x_5^{2^d-1}$$
$$a_{47} = x_1x_2^3x_3^{2^d-3}x_4^{2^d-2}x_5^{2^{d+2}-2} \qquad a_{48} = x_1x_2^3x_3^{2^d-3}x_4^{2^{d+1}-2}x_5^{2^{d+1}+2^d-2}$$
$$a_{49} = x_1x_2^3x_3^{2^d-3}x_4^{2^{d+2}-2}x_5^{2^d-2} \qquad a_{50} = x_1x_2^3x_3^{2^d-2}x_4^{2^d-3}x_5^{2^{d+2}-2}$$
$$a_{51} = x_1x_2^3x_3^{2^d-2}x_4^{2^d-1}x_5^{2^{d+2}-4} \qquad a_{52} = x_1x_2^3x_3^{2^d-2}x_4^{2^{d+1}-4}x_5^{2^{d+1}+2^d-1}$$
$$a_{53} = x_1x_2^3x_3^{2^d-2}x_4^{2^{d+1}-3}x_5^{2^{d+1}+2^d-2} \qquad a_{54} = x_1x_2^3x_3^{2^d-2}x_4^{2^{d+1}-1}x_5^{2^{d+1}+2^d-4}$$
$$a_{55} = x_1x_2^3x_3^{2^d-2}x_4^{2^{d+2}-4}x_5^{2^d-1} \qquad a_{56} = x_1x_2^3x_3^{2^d-2}x_4^{2^{d+2}-3}x_5^{2^d-2}$$

$a_{57} = x_1x_2^3x_3^{2^d-1}x_4^{2^d-4}x_5^{2^{d+2}-2}$ $\quad$ $a_{58} = x_1x_2^3x_3^{2^d-1}x_4^{2^d-2}x_5^{2^{d+2}-4}$

$a_{59} = x_1x_2^3x_3^{2^d-1}x_4^{2^{d+1}-4}x_5^{2^{d+1}+2^d-2}$ $\quad$ $a_{60} = x_1x_2^3x_3^{2^d-1}x_4^{2^{d+1}-2}x_5^{2^{d+1}+2^d-4}$

$a_{61} = x_1x_2^3x_3^{2^d-1}x_4^{2^{d+2}-4}x_5^{2^d-2}$ $\quad$ $a_{62} = x_1x_2^3x_3^{2^d-1}x_4^{2^{d+2}-2}x_5^{2^d-4}$

$a_{63} = x_1x_2^3x_3^{2^{d+1}-4}x_4^{2^d-2}x_5^{2^{d+1}+2^d-1}$ $\quad$ $a_{64} = x_1x_2^3x_3^{2^{d+1}-4}x_4^{2^d-1}x_5^{2^{d+1}+2^d-2}$

$a_{65} = x_1x_2^3x_3^{2^{d+1}-4}x_4^{2^{d+1}+2^d-2}x_5^{2^d-1}$ $\quad$ $a_{66} = x_1x_2^3x_3^{2^{d+1}-4}x_4^{2^{d+1}+2^d-1}x_5^{2^d-2}$

$a_{67} = x_1x_2^3x_3^{2^{d+1}-3}x_4^{2^d-2}x_5^{2^{d+1}+2^d-2}$ $\quad$ $a_{68} = x_1x_2^3x_3^{2^{d+1}-3}x_4^{2^{d+1}+2^d-2}x_5^{2^d-2}$

$a_{69} = x_1x_2^3x_3^{2^{d+1}-2}x_4^{2^d-4}x_5^{2^{d+1}+2^d-1}$ $\quad$ $a_{70} = x_1x_2^3x_3^{2^{d+1}-2}x_4^{2^d-3}x_5^{2^{d+1}+2^d-2}$

$a_{71} = x_1x_2^3x_3^{2^{d+1}-2}x_4^{2^d-1}x_5^{2^{d+1}+2^d-4}$ $\quad$ $a_{72} = x_1x_2^3x_3^{2^{d+1}-2}x_4^{2^{d+1}+2^d-4}x_5^{2^d-1}$

$a_{73} = x_1x_2^3x_3^{2^{d+1}-2}x_4^{2^{d+1}+2^d-3}x_5^{2^d-2}$ $\quad$ $a_{74} = x_1x_2^3x_3^{2^{d+1}-2}x_4^{2^{d+1}+2^d-1}x_5^{2^d-4}$

$a_{75} = x_1x_2^3x_3^{2^{d+1}-1}x_4^{2^d-4}x_5^{2^{d+1}+2^d-2}$ $\quad$ $a_{76} = x_1x_2^3x_3^{2^{d+1}-1}x_4^{2^d-2}x_5^{2^{d+1}+2^d-4}$

$a_{77} = x_1x_2^3x_3^{2^{d+1}-1}x_4^{2^{d+1}+2^d-4}x_5^{2^d-2}$ $\quad$ $a_{78} = x_1x_2^3x_3^{2^{d+1}-1}x_4^{2^{d+1}+2^d-2}x_5^{2^d-4}$

$a_{79} = x_1x_2^3x_3^{2^{d+2}-4}x_4^{2^d-2}x_5^{2^d-1}$ $\quad$ $a_{80} = x_1x_2^3x_3^{2^{d+2}-4}x_4^{2^d-1}x_5^{2^d-2}$

$a_{81} = x_1x_2^3x_3^{2^{d+2}-3}x_4^{2^d-2}x_5^{2^d-2}$ $\quad$ $a_{82} = x_1x_2^3x_3^{2^{d+2}-2}x_4^{2^d-4}x_5^{2^d-1}$

$a_{83} = x_1x_2^3x_3^{2^{d+2}-2}x_4^{2^d-3}x_5^{2^d-2}$ $\quad$ $a_{84} = x_1x_2^3x_3^{2^{d+2}-2}x_4^{2^d-1}x_5^{2^d-4}$

$a_{85} = x_1x_2^6x_3^{2^d-5}x_4^{2^d-3}x_5^{2^{d+2}-2}$ $\quad$ $a_{86} = x_1x_2^6x_3^{2^d-5}x_4^{2^{d+1}-3}x_5^{2^{d+1}+2^d-2}$

$a_{87} = x_1x_2^6x_3^{2^d-5}x_4^{2^{d+2}-3}x_5^{2^d-2}$ $\quad$ $a_{88} = x_1x_2^6x_3^{2^{d+1}-5}x_4^{2^d-3}x_5^{2^{d+1}+2^d-2}$

$a_{89} = x_1x_2^6x_3^{2^{d+1}-5}x_4^{2^{d+1}+2^d-3}x_5^{2^d-2}$ $\quad$ $a_{90} = x_1x_2^6x_3^{2^{d+2}-5}x_4^{2^d-3}x_5^{2^d-2}$

$a_{91} = x_1x_2^7x_3^{2^d-6}x_4^{2^d-3}x_5^{2^{d+2}-2}$ $\quad$ $a_{92} = x_1x_2^7x_3^{2^d-6}x_4^{2^{d+1}-3}x_5^{2^{d+1}+2^d-2}$

$a_{93} = x_1x_2^7x_3^{2^d-6}x_4^{2^{d+2}-3}x_5^{2^d-2}$ $\quad$ $a_{94} = x_1x_2^7x_3^{2^d-5}x_4^{2^d-4}x_5^{2^{d+2}-2}$

$a_{95} = x_1x_2^7x_3^{2^d-5}x_4^{2^d-2}x_5^{2^{d+2}-4}$ $\quad$ $a_{96} = x_1x_2^7x_3^{2^d-5}x_4^{2^{d+1}-4}x_5^{2^{d+1}+2^d-2}$

$a_{97} = x_1x_2^7x_3^{2^d-5}x_4^{2^{d+1}-2}x_5^{2^{d+1}+2^d-4}$ $\quad$ $a_{98} = x_1x_2^7x_3^{2^d-5}x_4^{2^{d+2}-4}x_5^{2^d-2}$

$a_{99} = x_1x_2^7x_3^{2^d-5}x_4^{2^{d+2}-2}x_5^{2^d-4}$ $\quad$ $a_{100} = x_1x_2^7x_3^{2^{d+1}-6}x_4^{2^d-3}x_5^{2^{d+1}+2^d-2}$

$a_{101} = x_1x_2^7x_3^{2^{d+1}-6}x_4^{2^{d+1}+2^d-3}x_5^{2^d-2}$ $\quad$ $a_{102} = x_1x_2^7x_3^{2^{d+1}-5}x_4^{2^d-4}x_5^{2^{d+1}+2^d-2}$

$a_{103} = x_1x_2^7x_3^{2^{d+1}-5}x_4^{2^d-2}x_5^{2^{d+1}+2^d-4}$ $\quad$ $a_{104} = x_1x_2^7x_3^{2^{d+1}-5}x_4^{2^{d+1}+2^d-4}x_5^{2^d-2}$

$a_{105} = x_1x_2^7x_3^{2^{d+1}-5}x_4^{2^{d+1}+2^d-2}x_5^{2^d-4}$ $\quad$ $a_{106} = x_1x_2^7x_3^{2^{d+2}-6}x_4^{2^d-3}x_5^{2^d-2}$

$a_{107} = x_1x_2^7x_3^{2^{d+2}-5}x_4^{2^d-4}x_5^{2^d-2}$ $\quad$ $a_{108} = x_1x_2^7x_3^{2^{d+2}-5}x_4^{2^d-2}x_5^{2^d-4}$

$a_{109} = x_1x_2^{2^d-2}x_3x_4^{2^d-1}x_5^{2^{d+2}-2}$ $\quad$ $a_{110} = x_1x_2^{2^d-2}x_3x_4^{2^{d+1}-2}x_5^{2^{d+1}+2^d-1}$

$a_{111} = x_1x_2^{2^d-2}x_3x_4^{2^{d+1}-1}x_5^{2^{d+1}+2^d-2}$ $\quad$ $a_{112} = x_1x_2^{2^d-2}x_3x_4^{2^{d+2}-2}x_5^{2^d-1}$

$a_{113} = x_1x_2^{2^d-2}x_3^3x_4^{2^d-3}x_5^{2^{d+2}-2}$ $\quad$ $a_{114} = x_1x_2^{2^d-2}x_3^3x_4^{2^{d+1}-3}x_5^{2^{d+1}+2^d-2}$

$a_{115} = x_1x_2^{2^d-2}x_3^3x_4^{2^{d+2}-3}x_5^{2^d-2}$ $\quad$ $a_{116} = x_1x_2^{2^d-2}x_3^{2^d-1}x_4x_5^{2^{d+2}-2}$

$a_{117} = x_1x_2^{2^d-2}x_3^{2^{d+1}-1}x_4x_5^{2^{d+1}+2^d-2}$ $\quad$ $a_{118} = x_1x_2^{2^d-1}x_3x_4^{2^d-2}x_5^{2^{d+2}-2}$

$a_{119} = x_1x_2^{2^d-1}x_3x_4^{2^{d+1}-2}x_5^{2^{d+1}+2^d-2}$ $\quad$ $a_{120} = x_1x_2^{2^d-1}x_3x_4^{2^{d+2}-2}x_5^{2^d-2}$

$a_{121} = x_1x_2^{2^d-1}x_3^2x_4^{2^d-3}x_5^{2^{d+2}-2}$ $\quad$ $a_{122} = x_1x_2^{2^d-1}x_3^2x_4^{2^d-1}x_5^{2^{d+2}-4}$

$a_{123} = x_1x_2^{2^d-1}x_3^2x_4^{2^{d+1}-4}x_5^{2^{d+1}+2^d-1}$ $\quad$ $a_{124} = x_1x_2^{2^d-1}x_3^2x_4^{2^{d+1}-3}x_5^{2^{d+1}+2^d-2}$

$a_{125} = x_1x_2^{2^d-1}x_3^2x_4^{2^{d+1}-1}x_5^{2^{d+1}+2^d-4}$ $\quad$ $a_{126} = x_1x_2^{2^d-1}x_3^2x_4^{2^{d+2}-4}x_5^{2^d-1}$

$a_{127} = x_1x_2^{2^d-1}x_3^2x_4^{2^{d+2}-3}x_5^{2^d-2}$ $\quad$ $a_{128} = x_1x_2^{2^d-1}x_3^3x_4^{2^d-4}x_5^{2^{d+2}-2}$

$a_{129} = x_1x_2^{2^d-1}x_3^3x_4^{2^d-2}x_5^{2^{d+2}-4}$ $a_{130} = x_1x_2^{2^d-1}x_3^3x_4^{2^{d+1}-4}x_5^{2^{d+1}+2^d-2}$
$a_{131} = x_1x_2^{2^d-1}x_3^3x_4^{2^{d+1}-2}x_5^{2^{d+1}+2^d-4}$ $a_{132} = x_1x_2^{2^d-1}x_3^3x_4^{2^{d+2}-4}x_5^{2^d-2}$
$a_{133} = x_1x_2^{2^d-1}x_3^3x_4^{2^{d+2}-2}x_5^{2^d-4}$ $a_{134} = x_1x_2^{2^d-1}x_3^{2^d-2}x_4x_5^{2^{d+2}-2}$
$a_{135} = x_1x_2^{2^d-1}x_3^{2^d-1}x_4^2x_5^{2^{d+2}-4}$ $a_{136} = x_1x_2^{2^d-1}x_3^{2^{d+1}-2}x_4x_5^{2^{d+1}+2^d-2}$
$a_{137} = x_1x_2^{2^d-1}x_3^{2^{d+1}-1}x_4^2x_5^{2^{d+1}+2^d-4}$ $a_{138} = x_1x_2^{2^d-1}x_3^{2^{d+2}-2}x_4x_5^{2^d-2}$
$a_{139} = x_1x_2^{2^{d+1}-2}x_3x_4^{2^d-2}x_5^{2^{d+1}+2^d-1}$ $a_{140} = x_1x_2^{2^{d+1}-2}x_3x_4^{2^d-1}x_5^{2^{d+1}+2^d-2}$
$a_{141} = x_1x_2^{2^{d+1}-2}x_3x_4^{2^{d+1}+2^d-2}x_5^{2^d-1}$ $a_{142} = x_1x_2^{2^{d+1}-2}x_3x_4^{2^{d+1}+2^d-1}x_5^{2^d-2}$
$a_{143} = x_1x_2^{2^{d+1}-2}x_3^3x_4^{2^d-3}x_5^{2^{d+1}+2^d-2}$ $a_{144} = x_1x_2^{2^{d+1}-2}x_3^3x_4^{2^{d+1}+2^d-3}x_5^{2^d-2}$
$a_{145} = x_1x_2^{2^{d+1}-2}x_3^{2^d-1}x_4x_5^{2^{d+1}+2^d-2}$ $a_{146} = x_1x_2^{2^{d+1}-2}x_3^{2^{d+1}+2^d-1}x_4x_5^{2^d-2}$
$a_{147} = x_1x_2^{2^{d+1}-1}x_3x_4^{2^d-2}x_5^{2^{d+1}+2^d-2}$ $a_{148} = x_1x_2^{2^{d+1}-1}x_3x_4^{2^{d+1}+2^d-2}x_5^{2^d-2}$
$a_{149} = x_1x_2^{2^{d+1}-1}x_3^2x_4^{2^d-4}x_5^{2^{d+1}+2^d-1}$ $a_{150} = x_1x_2^{2^{d+1}-1}x_3^2x_4^{2^d-3}x_5^{2^{d+1}+2^d-2}$
$a_{151} = x_1x_2^{2^{d+1}-1}x_3^2x_4^{2^d-1}x_5^{2^{d+1}+2^d-4}$ $a_{152} = x_1x_2^{2^{d+1}-1}x_3^2x_4^{2^{d+1}+2^d-4}x_5^{2^d-1}$
$a_{153} = x_1x_2^{2^{d+1}-1}x_3^2x_4^{2^{d+1}+2^d-3}x_5^{2^d-2}$ $a_{154} = x_1x_2^{2^{d+1}-1}x_3^2x_4^{2^{d+1}+2^d-1}x_5^{2^d-4}$
$a_{155} = x_1x_2^{2^{d+1}-1}x_3^3x_4^{2^d-4}x_5^{2^{d+1}+2^d-2}$ $a_{156} = x_1x_2^{2^{d+1}-1}x_3^3x_4^{2^d-2}x_5^{2^{d+1}+2^d-4}$
$a_{157} = x_1x_2^{2^{d+1}-1}x_3^3x_4^{2^{d+1}+2^d-4}x_5^{2^d-2}$ $a_{158} = x_1x_2^{2^{d+1}-1}x_3^3x_4^{2^{d+1}+2^d-2}x_5^{2^d-4}$
$a_{159} = x_1x_2^{2^{d+1}-1}x_3^{2^d-2}x_4x_5^{2^{d+1}+2^d-2}$ $a_{160} = x_1x_2^{2^{d+1}-1}x_3^{2^d-1}x_4^2x_5^{2^{d+1}+2^d-4}$
$a_{161} = x_1x_2^{2^{d+1}-1}x_3^{2^{d+1}+2^d-2}x_4x_5^{2^d-2}$ $a_{162} = x_1x_2^{2^{d+1}-1}x_3^{2^{d+1}+2^d-1}x_4^2x_5^{2^d-4}$
$a_{163} = x_1x_2^{2^{d+2}-2}x_3x_4^{2^d-2}x_5^{2^d-1}$ $a_{164} = x_1x_2^{2^{d+2}-2}x_3x_4^{2^d-1}x_5^{2^d-2}$
$a_{165} = x_1x_2^{2^{d+2}-2}x_3^3x_4^{2^d-3}x_5^{2^d-2}$ $a_{166} = x_1x_2^{2^{d+2}-2}x_3^{2^d-1}x_4x_5^{2^d-2}$
$a_{167} = x_1^3x_2x_3^{2^d-4}x_4^{2^d-1}x_5^{2^{d+2}-2}$ $a_{168} = x_1^3x_2x_3^{2^d-4}x_4^{2^{d+1}-2}x_5^{2^{d+1}+2^d-1}$
$a_{169} = x_1^3x_2x_3^{2^d-4}x_4^{2^{d+1}-1}x_5^{2^{d+1}+2^d-2}$ $a_{170} = x_1^3x_2x_3^{2^d-4}x_4^{2^{d+2}-2}x_5^{2^d-1}$
$a_{171} = x_1^3x_2x_3^{2^d-3}x_4^{2^d-2}x_5^{2^{d+2}-2}$ $a_{172} = x_1^3x_2x_3^{2^d-3}x_4^{2^{d+1}-2}x_5^{2^{d+1}+2^d-2}$
$a_{173} = x_1^3x_2x_3^{2^d-3}x_4^{2^{d+2}-2}x_5^{2^d-2}$ $a_{174} = x_1^3x_2x_3^{2^d-2}x_4^{2^d-3}x_5^{2^{d+2}-2}$
$a_{175} = x_1^3x_2x_3^{2^d-2}x_4^{2^d-1}x_5^{2^{d+2}-4}$ $a_{176} = x_1^3x_2x_3^{2^d-2}x_4^{2^{d+1}-4}x_5^{2^{d+1}+2^d-1}$
$a_{177} = x_1^3x_2x_3^{2^d-2}x_4^{2^{d+1}-3}x_5^{2^{d+1}+2^d-2}$ $a_{178} = x_1^3x_2x_3^{2^d-2}x_4^{2^{d+1}-1}x_5^{2^{d+1}+2^d-4}$
$a_{179} = x_1^3x_2x_3^{2^d-2}x_4^{2^{d+2}-4}x_5^{2^d-1}$ $a_{180} = x_1^3x_2x_3^{2^d-2}x_4^{2^{d+2}-3}x_5^{2^d-2}$
$a_{181} = x_1^3x_2x_3^{2^d-1}x_4^{2^d-4}x_5^{2^{d+2}-2}$ $a_{182} = x_1^3x_2x_3^{2^d-1}x_4^{2^d-2}x_5^{2^{d+2}-4}$
$a_{183} = x_1^3x_2x_3^{2^d-1}x_4^{2^{d+1}-4}x_5^{2^{d+1}+2^d-2}$ $a_{184} = x_1^3x_2x_3^{2^d-1}x_4^{2^{d+1}-2}x_5^{2^{d+1}+2^d-4}$
$a_{185} = x_1^3x_2x_3^{2^d-1}x_4^{2^{d+2}-4}x_5^{2^d-2}$ $a_{186} = x_1^3x_2x_3^{2^d-1}x_4^{2^{d+2}-2}x_5^{2^d-4}$
$a_{187} = x_1^3x_2x_3^{2^{d+1}-4}x_4^{2^d-2}x_5^{2^{d+1}+2^d-1}$ $a_{188} = x_1^3x_2x_3^{2^{d+1}-4}x_4^{2^d-1}x_5^{2^{d+1}+2^d-2}$
$a_{189} = x_1^3x_2x_3^{2^{d+1}-4}x_4^{2^{d+1}+2^d-2}x_5^{2^d-1}$ $a_{190} = x_1^3x_2x_3^{2^{d+1}-4}x_4^{2^{d+1}+2^d-1}x_5^{2^d-2}$
$a_{191} = x_1^3x_2x_3^{2^{d+1}-3}x_4^{2^d-2}x_5^{2^{d+1}+2^d-2}$ $a_{192} = x_1^3x_2x_3^{2^{d+1}-3}x_4^{2^{d+1}+2^d-2}x_5^{2^d-2}$
$a_{193} = x_1^3x_2x_3^{2^{d+1}-2}x_4^{2^d-4}x_5^{2^{d+1}+2^d-1}$ $a_{194} = x_1^3x_2x_3^{2^{d+1}-2}x_4^{2^d-3}x_5^{2^{d+1}+2^d-2}$
$a_{195} = x_1^3x_2x_3^{2^{d+1}-2}x_4^{2^d-1}x_5^{2^{d+1}+2^d-4}$ $a_{196} = x_1^3x_2x_3^{2^{d+1}-2}x_4^{2^{d+1}+2^d-4}x_5^{2^d-1}$
$a_{197} = x_1^3x_2x_3^{2^{d+1}-2}x_4^{2^{d+1}+2^d-3}x_5^{2^d-2}$ $a_{198} = x_1^3x_2x_3^{2^{d+1}-2}x_4^{2^{d+1}+2^d-1}x_5^{2^d-4}$

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\begin{aligned}
&a_{199} = x_1^3x_2x_3^{2^{d+1}-1}x_4^{2^d-4}x_5^{2^{d+1}+2^d-2} && a_{200} = x_1^3x_2x_3^{2^{d+1}-1}x_4^{2^d-2}x_5^{2^{d+1}+2^d-4}\\
&a_{201} = x_1^3x_2x_3^{2^{d+1}-1}x_4^{2^{d+1}+2^d-4}x_5^{2^d-2} && a_{202} = x_1^3x_2x_3^{2^{d+1}-1}x_4^{2^{d+1}+2^d-2}x_5^{2^d-4}\\
&a_{203} = x_1^3x_2x_3^{2^{d+2}-4}x_4^{2^d-2}x_5^{2^d-1} && a_{204} = x_1^3x_2x_3^{2^{d+2}-4}x_4^{2^d-1}x_5^{2^d-2}\\
&a_{205} = x_1^3x_2x_3^{2^{d+2}-3}x_4^{2^d-2}x_5^{2^d-2} && a_{206} = x_1^3x_2x_3^{2^{d+2}-2}x_4^{2^d-4}x_5^{2^d-1}\\
&a_{207} = x_1^3x_2x_3^{2^{d+2}-2}x_4^{2^d-3}x_5^{2^d-2} && a_{208} = x_1^3x_2x_3^{2^{d+2}-2}x_4^{2^d-1}x_5^{2^d-4}\\
&a_{209} = x_1^3x_2^3x_3^{2^d-4}x_4^{2^d-3}x_5^{2^{d+2}-2} && a_{210} = x_1^3x_2^3x_3^{2^d-4}x_4^{2^{d+1}-3}x_5^{2^{d+1}+2^d-2}\\
&a_{211} = x_1^3x_2^3x_3^{2^d-4}x_4^{2^{d+2}-3}x_5^{2^d-2} && a_{212} = x_1^3x_2^3x_3^{2^d-3}x_4^{2^d-4}x_5^{2^{d+2}-2}\\
&a_{213} = x_1^3x_2^3x_3^{2^d-3}x_4^{2^d-2}x_5^{2^{d+2}-4} && a_{214} = x_1^3x_2^3x_3^{2^d-3}x_4^{2^{d+1}-4}x_5^{2^{d+1}+2^d-2}\\
&a_{215} = x_1^3x_2^3x_3^{2^d-3}x_4^{2^{d+1}-2}x_5^{2^{d+1}+2^d-4} && a_{216} = x_1^3x_2^3x_3^{2^d-3}x_4^{2^{d+2}-4}x_5^{2^d-2}\\
&a_{217} = x_1^3x_2^3x_3^{2^d-3}x_4^{2^{d+2}-2}x_5^{2^d-4} && a_{218} = x_1^3x_2^3x_3^{2^{d+1}-4}x_4^{2^d-3}x_5^{2^{d+1}+2^d-2}\\
&a_{219} = x_1^3x_2^3x_3^{2^{d+1}-4}x_4^{2^{d+1}+2^d-3}x_5^{2^d-2} && a_{220} = x_1^3x_2^3x_3^{2^{d+1}-3}x_4^{2^d-4}x_5^{2^{d+1}+2^d-2}\\
&a_{221} = x_1^3x_2^3x_3^{2^{d+1}-3}x_4^{2^d-2}x_5^{2^{d+1}+2^d-4} && a_{222} = x_1^3x_2^3x_3^{2^{d+1}-3}x_4^{2^{d+1}+2^d-4}x_5^{2^d-2}\\
&a_{223} = x_1^3x_2^3x_3^{2^{d+1}-3}x_4^{2^{d+1}+2^d-2}x_5^{2^d-4} && a_{224} = x_1^3x_2^3x_3^{2^{d+2}-4}x_4^{2^d-3}x_5^{2^d-2}\\
&a_{225} = x_1^3x_2^3x_3^{2^{d+2}-3}x_4^{2^d-4}x_5^{2^d-2} && a_{226} = x_1^3x_2^3x_3^{2^{d+2}-3}x_4^{2^d-2}x_5^{2^d-4}\\
&a_{227} = x_1^3x_2^4x_3^{2^d-5}x_4^{2^d-3}x_5^{2^{d+2}-2} && a_{228} = x_1^3x_2^4x_3^{2^d-5}x_4^{2^{d+1}-3}x_5^{2^{d+1}+2^d-2}\\
&a_{229} = x_1^3x_2^4x_3^{2^d-5}x_4^{2^{d+2}-3}x_5^{2^d-2} && a_{230} = x_1^3x_2^4x_3^{2^{d+1}-5}x_4^{2^d-3}x_5^{2^{d+1}+2^d-2}\\
&a_{231} = x_1^3x_2^4x_3^{2^{d+1}-5}x_4^{2^{d+1}+2^d-3}x_5^{2^d-2} && a_{232} = x_1^3x_2^4x_3^{2^{d+2}-5}x_4^{2^d-3}x_5^{2^d-2}\\
&a_{233} = x_1^3x_2^5x_3^{2^d-6}x_4^{2^d-3}x_5^{2^{d+2}-2} && a_{234} = x_1^3x_2^5x_3^{2^d-6}x_4^{2^d-1}x_5^{2^{d+2}-4}\\
&a_{235} = x_1^3x_2^5x_3^{2^d-6}x_4^{2^{d+1}-4}x_5^{2^{d+1}+2^d-1} && a_{236} = x_1^3x_2^5x_3^{2^d-6}x_4^{2^{d+1}-3}x_5^{2^{d+1}+2^d-2}\\
&a_{237} = x_1^3x_2^5x_3^{2^d-6}x_4^{2^{d+1}-1}x_5^{2^{d+1}+2^d-4} && a_{238} = x_1^3x_2^5x_3^{2^d-6}x_4^{2^{d+2}-4}x_5^{2^d-1}\\
&a_{239} = x_1^3x_2^5x_3^{2^d-6}x_4^{2^{d+2}-3}x_5^{2^d-2} && a_{240} = x_1^3x_2^5x_3^{2^d-5}x_4^{2^d-4}x_5^{2^{d+2}-2}\\
&a_{241} = x_1^3x_2^5x_3^{2^d-5}x_4^{2^d-2}x_5^{2^{d+2}-4} && a_{242} = x_1^3x_2^5x_3^{2^d-5}x_4^{2^{d+1}-4}x_5^{2^{d+1}+2^d-2}\\
&a_{243} = x_1^3x_2^5x_3^{2^d-5}x_4^{2^{d+1}-2}x_5^{2^{d+1}+2^d-4} && a_{244} = x_1^3x_2^5x_3^{2^d-5}x_4^{2^{d+2}-4}x_5^{2^d-2}\\
&a_{245} = x_1^3x_2^5x_3^{2^d-5}x_4^{2^{d+2}-2}x_5^{2^d-4} && a_{246} = x_1^3x_2^5x_3^{2^d-1}x_4^{2^d-6}x_5^{2^{d+2}-4}\\
&a_{247} = x_1^3x_2^5x_3^{2^d-1}x_4^{2^{d+1}-6}x_5^{2^{d+1}+2^d-4} && a_{248} = x_1^3x_2^5x_3^{2^d-1}x_4^{2^{d+2}-6}x_5^{2^d-4}\\
&a_{249} = x_1^3x_2^5x_3^{2^{d+1}-6}x_4^{2^d-4}x_5^{2^{d+1}+2^d-1} && a_{250} = x_1^3x_2^5x_3^{2^{d+1}-6}x_4^{2^d-3}x_5^{2^{d+1}+2^d-2}\\
&a_{251} = x_1^3x_2^5x_3^{2^{d+1}-6}x_4^{2^d-1}x_5^{2^{d+1}+2^d-4} && a_{252} = x_1^3x_2^5x_3^{2^{d+1}-6}x_4^{2^{d+1}+2^d-4}x_5^{2^d-1}\\
&a_{253} = x_1^3x_2^5x_3^{2^{d+1}-6}x_4^{2^{d+1}+2^d-3}x_5^{2^d-2} && a_{254} = x_1^3x_2^5x_3^{2^{d+1}-6}x_4^{2^{d+1}+2^d-1}x_5^{2^d-4}\\
&a_{255} = x_1^3x_2^5x_3^{2^{d+1}-5}x_4^{2^d-4}x_5^{2^{d+1}+2^d-2} && a_{256} = x_1^3x_2^5x_3^{2^{d+1}-5}x_4^{2^d-2}x_5^{2^{d+1}+2^d-4}\\
&a_{257} = x_1^3x_2^5x_3^{2^{d+1}-5}x_4^{2^{d+1}+2^d-4}x_5^{2^d-2} && a_{258} = x_1^3x_2^5x_3^{2^{d+1}-5}x_4^{2^{d+1}+2^d-2}x_5^{2^d-4}\\
&a_{259} = x_1^3x_2^5x_3^{2^{d+1}-1}x_4^{2^d-6}x_5^{2^{d+1}+2^d-4} && a_{260} = x_1^3x_2^5x_3^{2^{d+1}-1}x_4^{2^{d+1}+2^d-6}x_5^{2^d-4}\\
&a_{261} = x_1^3x_2^5x_3^{2^{d+2}-6}x_4^{2^d-4}x_5^{2^d-1} && a_{262} = x_1^3x_2^5x_3^{2^{d+2}-6}x_4^{2^d-3}x_5^{2^d-2}\\
&a_{263} = x_1^3x_2^5x_3^{2^{d+2}-6}x_4^{2^d-1}x_5^{2^d-4} && a_{264} = x_1^3x_2^5x_3^{2^{d+2}-5}x_4^{2^d-4}x_5^{2^d-2}\\
&a_{265} = x_1^3x_2^5x_3^{2^{d+2}-5}x_4^{2^d-2}x_5^{2^d-4} && a_{266} = x_1^3x_2^7x_3^{2^d-7}x_4^{2^d-4}x_5^{2^{d+2}-2}\\
&a_{267} = x_1^3x_2^7x_3^{2^d-7}x_4^{2^d-2}x_5^{2^{d+2}-4} && a_{268} = x_1^3x_2^7x_3^{2^d-7}x_4^{2^{d+1}-4}x_5^{2^{d+1}+2^d-2}
\end{aligned}
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$a_{269} = x_1^3x_2^7x_3^{2^d-7}x_4^{2^{d+1}-2}x_5^{2^{d+1}+2^d-4}$ $a_{270} = x_1^3x_2^7x_3^{2^d-7}x_4^{2^{d+2}-4}x_5^{2^d-2}$
$a_{271} = x_1^3x_2^7x_3^{2^d-7}x_4^{2^{d+2}-2}x_5^{2^d-4}$ $a_{272} = x_1^3x_2^7x_3^{2^d-3}x_4^{2^d-6}x_5^{2^{d+2}-4}$
$a_{273} = x_1^3x_2^7x_3^{2^d-3}x_4^{2^{d+1}-6}x_5^{2^{d+1}+2^d-4}$ $a_{274} = x_1^3x_2^7x_3^{2^d-3}x_4^{2^{d+2}-6}x_5^{2^d-4}$
$a_{275} = x_1^3x_2^7x_3^{2^{d+1}-7}x_4^{2^d-4}x_5^{2^{d+1}+2^d-2}$ $a_{276} = x_1^3x_2^7x_3^{2^{d+1}-7}x_4^{2^d-2}x_5^{2^{d+1}+2^d-4}$
$a_{277} = x_1^3x_2^7x_3^{2^{d+1}-7}x_4^{2^{d+1}+2^d-4}x_5^{2^d-2}$ $a_{278} = x_1^3x_2^7x_3^{2^{d+1}-7}x_4^{2^{d+1}+2^d-2}x_5^{2^d-4}$
$a_{279} = x_1^3x_2^7x_3^{2^{d+1}-3}x_4^{2^d-6}x_5^{2^{d+1}+2^d-4}$ $a_{280} = x_1^3x_2^7x_3^{2^{d+1}-3}x_4^{2^{d+1}+2^d-6}x_5^{2^d-4}$
$a_{281} = x_1^3x_2^7x_3^{2^{d+2}-7}x_4^{2^d-4}x_5^{2^d-2}$ $a_{282} = x_1^3x_2^7x_3^{2^{d+2}-7}x_4^{2^d-2}x_5^{2^d-4}$
$a_{283} = x_1^3x_2^7x_3^{2^{d+2}-3}x_4^{2^d-6}x_5^{2^d-4}$ $a_{284} = x_1^3x_2^{2^d-3}x_3x_4^{2^d-2}x_5^{2^{d+2}-2}$
$a_{285} = x_1^3x_2^{2^d-3}x_3x_4^{2^{d+1}-2}x_5^{2^{d+1}+2^d-2}$ $a_{286} = x_1^3x_2^{2^d-3}x_3x_4^{2^{d+2}-2}x_5^{2^d-2}$
$a_{287} = x_1^3x_2^{2^d-3}x_3^2x_4^{2^d-3}x_5^{2^{d+2}-2}$ $a_{288} = x_1^3x_2^{2^d-3}x_3^2x_4^{2^d-1}x_5^{2^{d+2}-4}$
$a_{289} = x_1^3x_2^{2^d-3}x_3^2x_4^{2^{d+1}-4}x_5^{2^{d+1}+2^d-1}$ $a_{290} = x_1^3x_2^{2^d-3}x_3^2x_4^{2^{d+1}-3}x_5^{2^{d+1}+2^d-2}$
$a_{291} = x_1^3x_2^{2^d-3}x_3^2x_4^{2^{d+1}-1}x_5^{2^{d+1}+2^d-4}$ $a_{292} = x_1^3x_2^{2^d-3}x_3^2x_4^{2^{d+2}-4}x_5^{2^d-1}$
$a_{293} = x_1^3x_2^{2^d-3}x_3^2x_4^{2^{d+2}-3}x_5^{2^d-2}$ $a_{294} = x_1^3x_2^{2^d-3}x_3^3x_4^{2^d-4}x_5^{2^{d+2}-2}$
$a_{295} = x_1^3x_2^{2^d-3}x_3^3x_4^{2^d-2}x_5^{2^{d+2}-4}$ $a_{296} = x_1^3x_2^{2^d-3}x_3^3x_4^{2^{d+1}-4}x_5^{2^{d+1}+2^d-2}$
$a_{297} = x_1^3x_2^{2^d-3}x_3^3x_4^{2^{d+1}-2}x_5^{2^{d+1}+2^d-4}$ $a_{298} = x_1^3x_2^{2^d-3}x_3^3x_4^{2^{d+2}-4}x_5^{2^d-2}$
$a_{299} = x_1^3x_2^{2^d-3}x_3^3x_4^{2^{d+2}-2}x_5^{2^d-4}$ $a_{300} = x_1^3x_2^{2^d-3}x_3^{2^d-2}x_4x_5^{2^{d+2}-2}$
$a_{301} = x_1^3x_2^{2^d-3}x_3^{2^d-1}x_4^2x_5^{2^{d+2}-4}$ $a_{302} = x_1^3x_2^{2^d-3}x_3^{2^{d+1}-2}x_4x_5^{2^{d+1}+2^d-2}$
$a_{303} = x_1^3x_2^{2^d-3}x_3^{2^{d+1}-1}x_4^2x_5^{2^{d+1}+2^d-4}$ $a_{304} = x_1^3x_2^{2^d-3}x_3^{2^{d+2}-2}x_4x_5^{2^d-2}$
$a_{305} = x_1^3x_2^{2^d-1}x_3x_4^{2^d-4}x_5^{2^{d+2}-2}$ $a_{306} = x_1^3x_2^{2^d-1}x_3x_4^{2^d-2}x_5^{2^{d+2}-4}$
$a_{307} = x_1^3x_2^{2^d-1}x_3x_4^{2^{d+1}-4}x_5^{2^{d+1}+2^d-2}$ $a_{308} = x_1^3x_2^{2^d-1}x_3x_4^{2^{d+1}-2}x_5^{2^{d+1}+2^d-4}$
$a_{309} = x_1^3x_2^{2^d-1}x_3x_4^{2^{d+2}-4}x_5^{2^d-2}$ $a_{310} = x_1^3x_2^{2^d-1}x_3x_4^{2^{d+2}-2}x_5^{2^d-4}$
$a_{311} = x_1^3x_2^{2^d-1}x_3^5x_4^{2^d-6}x_5^{2^{d+2}-4}$ $a_{312} = x_1^3x_2^{2^d-1}x_3^5x_4^{2^{d+1}-6}x_5^{2^{d+1}+2^d-4}$
$a_{313} = x_1^3x_2^{2^d-1}x_3^5x_4^{2^{d+2}-6}x_5^{2^d-4}$ $a_{314} = x_1^3x_2^{2^d-1}x_3^{2^d-3}x_4^2x_5^{2^{d+2}-4}$
$a_{315} = x_1^3x_2^{2^d-1}x_3^{2^{d+1}-3}x_4^2x_5^{2^{d+1}+2^d-4}$ $a_{316} = x_1^3x_2^{2^d-1}x_3^{2^{d+2}-3}x_4^2x_5^{2^d-4}$
$a_{317} = x_1^3x_2^{2^{d+1}-3}x_3x_4^{2^d-2}x_5^{2^{d+1}+2^d-2}$ $a_{318} = x_1^3x_2^{2^{d+1}-3}x_3x_4^{2^{d+1}+2^d-2}x_5^{2^d-2}$
$a_{319} = x_1^3x_2^{2^{d+1}-3}x_3^2x_4^{2^d-4}x_5^{2^{d+1}+2^d-1}$ $a_{320} = x_1^3x_2^{2^{d+1}-3}x_3^2x_4^{2^d-3}x_5^{2^{d+1}+2^d-2}$
$a_{321} = x_1^3x_2^{2^{d+1}-3}x_3^2x_4^{2^d-1}x_5^{2^{d+1}+2^d-4}$ $a_{322} = x_1^3x_2^{2^{d+1}-3}x_3^2x_4^{2^{d+1}+2^d-4}x_5^{2^d-1}$
$a_{323} = x_1^3x_2^{2^{d+1}-3}x_3^2x_4^{2^{d+1}+2^d-3}x_5^{2^d-2}$ $a_{324} = x_1^3x_2^{2^{d+1}-3}x_3^2x_4^{2^{d+1}+2^d-1}x_5^{2^d-4}$
$a_{325} = x_1^3x_2^{2^{d+1}-3}x_3^3x_4^{2^d-4}x_5^{2^{d+1}+2^d-2}$ $a_{326} = x_1^3x_2^{2^{d+1}-3}x_3^3x_4^{2^d-2}x_5^{2^{d+1}+2^d-4}$
$a_{327} = x_1^3x_2^{2^{d+1}-3}x_3^3x_4^{2^{d+1}+2^d-4}x_5^{2^d-2}$ $a_{328} = x_1^3x_2^{2^{d+1}-3}x_3^3x_4^{2^{d+1}+2^d-2}x_5^{2^d-4}$
$a_{329} = x_1^3x_2^{2^{d+1}-3}x_3^{2^d-2}x_4x_5^{2^{d+1}+2^d-2}$ $a_{330} = x_1^3x_2^{2^{d+1}-3}x_3^{2^d-1}x_4^2x_5^{2^{d+1}+2^d-4}$
$a_{331} = x_1^3x_2^{2^{d+1}-3}x_3^{2^{d+1}+2^d-2}x_4x_5^{2^d-2}$ $a_{332} = x_1^3x_2^{2^{d+1}-3}x_3^{2^{d+1}+2^d-1}x_4^2x_5^{2^d-4}$
$a_{333} = x_1^3x_2^{2^{d+1}-1}x_3x_4^{2^d-4}x_5^{2^{d+1}+2^d-2}$ $a_{334} = x_1^3x_2^{2^{d+1}-1}x_3x_4^{2^d-2}x_5^{2^{d+1}+2^d-4}$
$a_{335} = x_1^3x_2^{2^{d+1}-1}x_3x_4^{2^{d+1}+2^d-4}x_5^{2^d-2}$ $a_{336} = x_1^3x_2^{2^{d+1}-1}x_3x_4^{2^{d+1}+2^d-2}x_5^{2^d-4}$
$a_{337} = x_1^3x_2^{2^{d+1}-1}x_3^5x_4^{2^d-6}x_5^{2^{d+1}+2^d-4}$ $a_{338} = x_1^3x_2^{2^{d+1}-1}x_3^5x_4^{2^{d+1}+2^d-6}x_5^{2^d-4}$

$a_{339} = x_1^3x_2^{2^{d+1}-1}x_3^{2^d-3}x_4^2x_5^{2^{d+1}+2^d-4}$ $a_{340} = x_1^3x_2^{2^{d+1}-1}x_3^{2^{d+1}+2^d-3}x_4^2x_5^{2^d-4}$
$a_{341} = x_1^3x_2^{2^{d+2}-3}x_3x_4^{2^d-2}x_5^{2^d-2}$ $a_{342} = x_1^3x_2^{2^{d+2}-3}x_3^2x_4^{2^d-4}x_5^{2^d-1}$
$a_{343} = x_1^3x_2^{2^{d+2}-3}x_3^2x_4^{2^d-3}x_5^{2^d-2}$ $a_{344} = x_1^3x_2^{2^{d+2}-3}x_3^2x_4^{2^d-1}x_5^{2^d-4}$
$a_{345} = x_1^3x_2^{2^{d+2}-3}x_3^3x_4^{2^d-4}x_5^{2^d-2}$ $a_{346} = x_1^3x_2^{2^{d+2}-3}x_3^3x_4^{2^d-2}x_5^{2^d-4}$
$a_{347} = x_1^3x_2^{2^{d+2}-3}x_3^{2^d-2}x_4x_5^{2^d-2}$ $a_{348} = x_1^3x_2^{2^{d+2}-3}x_3^{2^d-1}x_4^2x_5^{2^d-4}$
$a_{349} = x_1^7x_2x_3^{2^d-6}x_4^{2^d-3}x_5^{2^{d+2}-2}$ $a_{350} = x_1^7x_2x_3^{2^d-6}x_4^{2^{d+1}-3}x_5^{2^{d+1}+2^d-2}$
$a_{351} = x_1^7x_2x_3^{2^d-6}x_4^{2^{d+2}-3}x_5^{2^d-2}$ $a_{352} = x_1^7x_2x_3^{2^d-5}x_4^{2^d-4}x_5^{2^{d+2}-2}$
$a_{353} = x_1^7x_2x_3^{2^d-5}x_4^{2^d-2}x_5^{2^{d+2}-4}$ $a_{354} = x_1^7x_2x_3^{2^d-5}x_4^{2^{d+1}-4}x_5^{2^{d+1}+2^d-2}$
$a_{355} = x_1^7x_2x_3^{2^d-5}x_4^{2^{d+1}-2}x_5^{2^{d+1}+2^d-4}$ $a_{356} = x_1^7x_2x_3^{2^d-5}x_4^{2^{d+2}-4}x_5^{2^d-2}$
$a_{357} = x_1^7x_2x_3^{2^d-5}x_4^{2^{d+2}-2}x_5^{2^d-4}$ $a_{358} = x_1^7x_2x_3^{2^{d+1}-6}x_4^{2^d-3}x_5^{2^{d+1}+2^d-2}$
$a_{359} = x_1^7x_2x_3^{2^{d+1}-6}x_4^{2^{d+1}+2^d-3}x_5^{2^d-2}$ $a_{360} = x_1^7x_2x_3^{2^{d+1}-5}x_4^{2^d-4}x_5^{2^{d+1}+2^d-2}$
$a_{361} = x_1^7x_2x_3^{2^{d+1}-5}x_4^{2^d-2}x_5^{2^{d+1}+2^d-4}$ $a_{362} = x_1^7x_2x_3^{2^{d+1}-5}x_4^{2^{d+1}+2^d-4}x_5^{2^d-2}$
$a_{363} = x_1^7x_2x_3^{2^{d+1}-5}x_4^{2^{d+1}+2^d-2}x_5^{2^d-4}$ $a_{364} = x_1^7x_2x_3^{2^{d+2}-6}x_4^{2^d-3}x_5^{2^d-2}$
$a_{365} = x_1^7x_2x_3^{2^{d+2}-5}x_4^{2^d-4}x_5^{2^d-2}$ $a_{366} = x_1^7x_2x_3^{2^{d+2}-5}x_4^{2^d-2}x_5^{2^d-4}$
$a_{367} = x_1^7x_2^3x_3^{2^d-7}x_4^{2^d-4}x_5^{2^{d+2}-2}$ $a_{368} = x_1^7x_2^3x_3^{2^d-7}x_4^{2^d-2}x_5^{2^{d+2}-4}$
$a_{369} = x_1^7x_2^3x_3^{2^d-7}x_4^{2^{d+1}-4}x_5^{2^{d+1}+2^d-2}$ $a_{370} = x_1^7x_2^3x_3^{2^d-7}x_4^{2^{d+1}-2}x_5^{2^{d+1}+2^d-4}$
$a_{371} = x_1^7x_2^3x_3^{2^d-7}x_4^{2^{d+2}-4}x_5^{2^d-2}$ $a_{372} = x_1^7x_2^3x_3^{2^d-7}x_4^{2^{d+2}-2}x_5^{2^d-4}$
$a_{373} = x_1^7x_2^3x_3^{2^d-3}x_4^{2^d-6}x_5^{2^{d+2}-4}$ $a_{374} = x_1^7x_2^3x_3^{2^d-3}x_4^{2^{d+1}-6}x_5^{2^{d+1}+2^d-4}$
$a_{375} = x_1^7x_2^3x_3^{2^d-3}x_4^{2^{d+2}-6}x_5^{2^d-4}$ $a_{376} = x_1^7x_2^3x_3^{2^{d+1}-7}x_4^{2^d-4}x_5^{2^{d+1}+2^d-2}$
$a_{377} = x_1^7x_2^3x_3^{2^{d+1}-7}x_4^{2^d-2}x_5^{2^{d+1}+2^d-4}$ $a_{378} = x_1^7x_2^3x_3^{2^{d+1}-7}x_4^{2^{d+1}+2^d-4}x_5^{2^d-2}$
$a_{379} = x_1^7x_2^3x_3^{2^{d+1}-7}x_4^{2^{d+1}+2^d-2}x_5^{2^d-4}$ $a_{380} = x_1^7x_2^3x_3^{2^{d+1}-3}x_4^{2^d-6}x_5^{2^{d+1}+2^d-4}$
$a_{381} = x_1^7x_2^3x_3^{2^{d+1}-3}x_4^{2^{d+1}+2^d-6}x_5^{2^d-4}$ $a_{382} = x_1^7x_2^3x_3^{2^{d+2}-7}x_4^{2^d-4}x_5^{2^d-2}$
$a_{383} = x_1^7x_2^3x_3^{2^{d+2}-7}x_4^{2^d-2}x_5^{2^d-4}$ $a_{384} = x_1^7x_2^3x_3^{2^{d+2}-3}x_4^{2^d-6}x_5^{2^d-4}$
$a_{385} = x_1^7x_2^{11}x_3^{2^d-11}x_4^{2^d-6}x_5^{2^{d+2}-4}$ $a_{386} = x_1^7x_2^{11}x_3^{2^d-11}x_4^{2^{d+1}-6}x_5^{2^{d+1}+2^d-4}$
$a_{387} = x_1^7x_2^{11}x_3^{2^d-11}x_4^{2^{d+2}-6}x_5^{2^d-4}$ $a_{388} = x_1^7x_2^{11}x_3^{2^{d+1}-11}x_4^{2^d-6}x_5^{2^{d+1}+2^d-4}$
$a_{389} = x_1^7x_2^{11}x_3^{2^{d+1}-11}x_4^{2^{d+1}+2^d-6}x_5^{2^d-4}$ $a_{390} = x_1^7x_2^{11}x_3^{2^{d+2}-11}x_4^{2^d-6}x_5^{2^d-4}$
$a_{391} = x_1^7x_2^{2^d-5}x_3x_4^{2^d-4}x_5^{2^{d+2}-2}$ $a_{392} = x_1^7x_2^{2^d-5}x_3x_4^{2^d-2}x_5^{2^{d+2}-4}$
$a_{393} = x_1^7x_2^{2^d-5}x_3x_4^{2^{d+1}-4}x_5^{2^{d+1}+2^d-2}$ $a_{394} = x_1^7x_2^{2^d-5}x_3x_4^{2^{d+1}-2}x_5^{2^{d+1}+2^d-4}$
$a_{395} = x_1^7x_2^{2^d-5}x_3x_4^{2^{d+2}-4}x_5^{2^d-2}$ $a_{396} = x_1^7x_2^{2^d-5}x_3x_4^{2^{d+2}-2}x_5^{2^d-4}$
$a_{397} = x_1^7x_2^{2^d-5}x_3^{2^d-3}x_4^2x_5^{2^{d+2}-4}$ $a_{398} = x_1^7x_2^{2^d-5}x_3^{2^{d+1}-3}x_4^2x_5^{2^{d+1}+2^d-4}$
$a_{399} = x_1^7x_2^{2^d-5}x_3^{2^{d+2}-3}x_4^2x_5^{2^d-4}$ $a_{400} = x_1^7x_2^{2^{d+1}-5}x_3x_4^{2^d-4}x_5^{2^{d+1}+2^d-2}$
$a_{401} = x_1^7x_2^{2^{d+1}-5}x_3x_4^{2^d-2}x_5^{2^{d+1}+2^d-4}$ $a_{402} = x_1^7x_2^{2^{d+1}-5}x_3x_4^{2^{d+1}+2^d-4}x_5^{2^d-2}$
$a_{403} = x_1^7x_2^{2^{d+1}-5}x_3x_4^{2^{d+1}+2^d-2}x_5^{2^d-4}$ $a_{404} = x_1^7x_2^{2^{d+1}-5}x_3^5x_4^{2^d-6}x_5^{2^{d+1}+2^d-4}$
$a_{405} = x_1^7x_2^{2^{d+1}-5}x_3^5x_4^{2^{d+1}+2^d-6}x_5^{2^d-4}$ $a_{406} = x_1^7x_2^{2^{d+1}-5}x_3^{2^d-3}x_4^2x_5^{2^{d+1}+2^d-4}$
$a_{407} = x_1^7x_2^{2^{d+1}-5}x_3^{2^{d+1}+2^d-3}x_4^2x_5^{2^d-4}$ $a_{408} = x_1^7x_2^{2^{d+2}-5}x_3x_4^{2^d-4}x_5^{2^d-2}$
$a_{409} = x_1^7x_2^{2^{d+2}-5}x_3x_4^{2^d-2}x_5^{2^d-4}$ $a_{410} = x_1^7x_2^{2^{d+2}-5}x_3^5x_4^{2^d-6}x_5^{2^d-4}$

$$a_{411} = x_1^{7}x_2^{2^{d+2}-5}x_3^{2^{d}-3}x_4^{2}x_5^{2^{d}-4} \qquad a_{412} = x_1^{2^{d}-1}x_2x_3x_4^{2^{d}-2}x_5^{2^{d+2}-2}$$
$$a_{413} = x_1^{2^{d}-1}x_2x_3x_4^{2^{d+1}-2}x_5^{2^{d+1}+2^{d}-2} \qquad a_{414} = x_1^{2^{d}-1}x_2x_3x_4^{2^{d+2}-2}x_5^{2^{d}-2}$$
$$a_{415} = x_1^{2^{d}-1}x_2x_3^{2}x_4^{2^{d}-3}x_5^{2^{d+2}-2} \qquad a_{416} = x_1^{2^{d}-1}x_2x_3^{2}x_4^{2^{d}-1}x_5^{2^{d+2}-4}$$
$$a_{417} = x_1^{2^{d}-1}x_2x_3^{2}x_4^{2^{d+1}-4}x_5^{2^{d+1}+2^{d}-1} \qquad a_{418} = x_1^{2^{d}-1}x_2x_3^{2}x_4^{2^{d+1}-3}x_5^{2^{d+1}+2^{d}-2}$$
$$a_{419} = x_1^{2^{d}-1}x_2x_3^{2}x_4^{2^{d+1}-1}x_5^{2^{d+1}+2^{d}-4} \qquad a_{420} = x_1^{2^{d}-1}x_2x_3^{2}x_4^{2^{d+2}-4}x_5^{2^{d}-1}$$
$$a_{421} = x_1^{2^{d}-1}x_2x_3^{2}x_4^{2^{d+2}-3}x_5^{2^{d}-2} \qquad a_{422} = x_1^{2^{d}-1}x_2x_3^{3}x_4^{2^{d}-4}x_5^{2^{d+2}-2}$$
$$a_{423} = x_1^{2^{d}-1}x_2x_3^{3}x_4^{2^{d}-2}x_5^{2^{d+2}-4} \qquad a_{424} = x_1^{2^{d}-1}x_2x_3^{3}x_4^{2^{d+1}-4}x_5^{2^{d+1}+2^{d}-2}$$
$$a_{425} = x_1^{2^{d}-1}x_2x_3^{3}x_4^{2^{d+1}-2}x_5^{2^{d+1}+2^{d}-4} \qquad a_{426} = x_1^{2^{d}-1}x_2x_3^{3}x_4^{2^{d+2}-4}x_5^{2^{d}-2}$$
$$a_{427} = x_1^{2^{d}-1}x_2x_3^{3}x_4^{2^{d+2}-2}x_5^{2^{d}-4} \qquad a_{428} = x_1^{2^{d}-1}x_2x_3^{2^{d}-2}x_4x_5^{2^{d+2}-2}$$
$$a_{429} = x_1^{2^{d}-1}x_2x_3^{2^{d}-1}x_4^{2}x_5^{2^{d+2}-4} \qquad a_{430} = x_1^{2^{d}-1}x_2x_3^{2^{d+1}-2}x_4x_5^{2^{d+1}+2^{d}-2}$$
$$a_{431} = x_1^{2^{d}-1}x_2x_3^{2^{d+1}-1}x_4^{2}x_5^{2^{d+1}+2^{d}-4} \qquad a_{432} = x_1^{2^{d}-1}x_2x_3^{2^{d+2}-2}x_4x_5^{2^{d}-2}$$
$$a_{433} = x_1^{2^{d}-1}x_2^{3}x_3x_4^{2^{d}-4}x_5^{2^{d+2}-2} \qquad a_{434} = x_1^{2^{d}-1}x_2^{3}x_3x_4^{2^{d}-2}x_5^{2^{d+2}-4}$$
$$a_{435} = x_1^{2^{d}-1}x_2^{3}x_3x_4^{2^{d+1}-4}x_5^{2^{d+1}+2^{d}-2} \qquad a_{436} = x_1^{2^{d}-1}x_2^{3}x_3x_4^{2^{d+1}-2}x_5^{2^{d+1}+2^{d}-4}$$
$$a_{437} = x_1^{2^{d}-1}x_2^{3}x_3x_4^{2^{d+2}-4}x_5^{2^{d}-2} \qquad a_{438} = x_1^{2^{d}-1}x_2^{3}x_3x_4^{2^{d+2}-2}x_5^{2^{d}-4}$$
$$a_{439} = x_1^{2^{d}-1}x_2^{3}x_3^{5}x_4^{2^{d}-6}x_5^{2^{d+2}-4} \qquad a_{440} = x_1^{2^{d}-1}x_2^{3}x_3^{5}x_4^{2^{d+1}-6}x_5^{2^{d+1}+2^{d}-4}$$
$$a_{441} = x_1^{2^{d}-1}x_2^{3}x_3^{5}x_4^{2^{d+2}-6}x_5^{2^{d}-4} \qquad a_{442} = x_1^{2^{d}-1}x_2^{3}x_3^{2^{d}-3}x_4^{2}x_5^{2^{d+2}-4}$$
$$a_{443} = x_1^{2^{d}-1}x_2^{3}x_3^{2^{d+1}-3}x_4^{2}x_5^{2^{d+1}+2^{d}-4} \qquad a_{444} = x_1^{2^{d}-1}x_2^{3}x_3^{2^{d+2}-3}x_4^{2}x_5^{2^{d}-4}$$
$$a_{445} = x_1^{2^{d}-1}x_2^{2^{d}-1}x_3x_4^{2}x_5^{2^{d+2}-4} \qquad a_{446} = x_1^{2^{d}-1}x_2^{2^{d+1}-1}x_3x_4^{2}x_5^{2^{d+1}+2^{d}-4}$$
$$a_{447} = x_1^{2^{d+1}-1}x_2x_3x_4^{2^{d}-2}x_5^{2^{d+1}+2^{d}-2} \qquad a_{448} = x_1^{2^{d+1}-1}x_2x_3x_4^{2^{d+1}+2^{d}-2}x_5^{2^{d}-2}$$
$$a_{449} = x_1^{2^{d+1}-1}x_2x_3^{2}x_4^{2^{d}-4}x_5^{2^{d+1}+2^{d}-1} \qquad a_{450} = x_1^{2^{d+1}-1}x_2x_3^{2}x_4^{2^{d}-3}x_5^{2^{d+1}+2^{d}-2}$$
$$a_{451} = x_1^{2^{d+1}-1}x_2x_3^{2}x_4^{2^{d}-1}x_5^{2^{d+1}+2^{d}-4} \qquad a_{452} = x_1^{2^{d+1}-1}x_2x_3^{2}x_4^{2^{d+1}+2^{d}-4}x_5^{2^{d}-1}$$
$$a_{453} = x_1^{2^{d+1}-1}x_2x_3^{2}x_4^{2^{d+1}+2^{d}-3}x_5^{2^{d}-2} \qquad a_{454} = x_1^{2^{d+1}-1}x_2x_3^{2}x_4^{2^{d+1}+2^{d}-1}x_5^{2^{d}-4}$$
$$a_{455} = x_1^{2^{d+1}-1}x_2x_3^{3}x_4^{2^{d}-4}x_5^{2^{d+1}+2^{d}-2} \qquad a_{456} = x_1^{2^{d+1}-1}x_2x_3^{3}x_4^{2^{d}-2}x_5^{2^{d+1}+2^{d}-4}$$
$$a_{457} = x_1^{2^{d+1}-1}x_2x_3^{3}x_4^{2^{d+1}+2^{d}-4}x_5^{2^{d}-2} \qquad a_{458} = x_1^{2^{d+1}-1}x_2x_3^{3}x_4^{2^{d+1}+2^{d}-2}x_5^{2^{d}-4}$$
$$a_{459} = x_1^{2^{d+1}-1}x_2x_3^{2^{d}-2}x_4x_5^{2^{d+1}+2^{d}-2} \qquad a_{460} = x_1^{2^{d+1}-1}x_2x_3^{2^{d}-1}x_4^{2}x_5^{2^{d+1}+2^{d}-4}$$
$$a_{461} = x_1^{2^{d+1}-1}x_2x_3^{2^{d+1}+2^{d}-2}x_4x_5^{2^{d}-2} \qquad a_{462} = x_1^{2^{d+1}-1}x_2x_3^{2^{d+1}+2^{d}-1}x_4^{2}x_5^{2^{d}-4}$$
$$a_{463} = x_1^{2^{d+1}-1}x_2^{3}x_3x_4^{2^{d}-4}x_5^{2^{d+1}+2^{d}-2} \qquad a_{464} = x_1^{2^{d+1}-1}x_2^{3}x_3x_4^{2^{d}-2}x_5^{2^{d+1}+2^{d}-4}$$
$$a_{465} = x_1^{2^{d+1}-1}x_2^{3}x_3x_4^{2^{d+1}+2^{d}-4}x_5^{2^{d}-2} \qquad a_{466} = x_1^{2^{d+1}-1}x_2^{3}x_3x_4^{2^{d+1}+2^{d}-2}x_5^{2^{d}-4}$$
$$a_{467} = x_1^{2^{d+1}-1}x_2^{3}x_3^{5}x_4^{2^{d}-6}x_5^{2^{d+1}+2^{d}-4} \qquad a_{468} = x_1^{2^{d+1}-1}x_2^{3}x_3^{5}x_4^{2^{d+1}+2^{d}-6}x_5^{2^{d}-4}$$
$$a_{469} = x_1^{2^{d+1}-1}x_2^{3}x_3^{2^{d}-3}x_4^{2}x_5^{2^{d+1}+2^{d}-4} \qquad a_{470} = x_1^{2^{d+1}-1}x_2^{3}x_3^{2^{d+1}+2^{d}-3}x_4^{2}x_5^{2^{d}-4}$$
$$a_{471} = x_1^{2^{d+1}-1}x_2^{2^{d}-1}x_3x_4^{2}x_5^{2^{d+1}+2^{d}-4} \qquad a_{472} = x_1^{2^{d+1}-1}x_2^{2^{d+1}+2^{d}-1}x_3x_4^{2}x_5^{2^{d}-4}$$

For $d = 4$,

$$a_{473} = x_1^{7}x_2^{11}x_3^{13}x_4^{14}x_5^{48},\ a_{474} = x_1^{7}x_2^{11}x_3^{13}x_4^{18}x_5^{44},\ a_{475} = x_1^{7}x_2^{11}x_3^{13}x_4^{50}x_5^{12}.$$

For $d \geqslant 5$,

$a_{473} = x_1^7x_2^{2^d-5}x_3^5x_4^{2^d-6}x_5^{2^{d+2}-4}$ $\qquad$ $a_{474} = x_1^7x_2^{2^d-5}x_3^5x_4^{2^{d+1}-6}x_5^{2^{d+1}+2^d-4}$
$a_{475} = x_1^7x_2^{2^d-5}x_3^5x_4^{2^{d+2}-6}x_5^{2^d-4}$

## 5.2 The cases $d = 3$

For $d = 3$, consider the monomials $a_t = a_{3,t}$ as in Subsection 5.1 with $d = 3$, $1 \leqslant t \leqslant 84$ and $B(3)$ as in the proof of Theorem 1.3. Then, we have

$$\widetilde{B}_5^+((3)|^3|(1)|^2) = B(3)\bigcup\{a_{3,j} : 1 \leqslant j \leqslant 84\} \cup C(3),$$

where $C(3)$ is the set of the monomials $a_j$, $85 \leqslant j \leqslant 450$, which are determined as follows:

| | | | |
|---|---|---|---|
| 85. $x_1x_2^6x_3x_4^7x_5^{30}$ | 86. $x_1x_2^6x_3x_4^{14}x_5^{23}$ | 87. $x_1x_2^6x_3x_4^{15}x_5^{22}$ | 88. $x_1x_2^6x_3x_4^{30}x_5^7$ |
| 89. $x_1x_2^6x_3^3x_4^5x_5^{30}$ | 90. $x_1x_2^6x_3^3x_4^{13}x_5^{22}$ | 91. $x_1x_2^6x_3^3x_4^{29}x_5^6$ | 92. $x_1x_2^6x_3^7x_4x_5^{30}$ |
| 93. $x_1x_2^6x_3^7x_4^7x_5^{24}$ | 94. $x_1x_2^6x_3^7x_4^9x_5^{22}$ | 95. $x_1x_2^6x_3^7x_4^{25}x_5^6$ | 96. $x_1x_2^6x_3^{11}x_4^5x_5^{22}$ |
| 97. $x_1x_2^6x_3^{11}x_4^{21}x_5^6$ | 98. $x_1x_2^6x_3^{15}x_4x_5^{22}$ | 99. $x_1x_2^6x_3^{27}x_4^5x_5^6$ | 100. $x_1x_2^7x_3x_4^6x_5^{30}$ |
| 101. $x_1x_2^7x_3x_4^{14}x_5^{22}$ | 102. $x_1x_2^7x_3x_4^{30}x_5^6$ | 103. $x_1x_2^7x_3^2x_4^5x_5^{30}$ | 104. $x_1x_2^7x_3^2x_4^7x_5^{28}$ |
| 105. $x_1x_2^7x_3^2x_4^{12}x_5^{23}$ | 106. $x_1x_2^7x_3^2x_4^{13}x_5^{22}$ | 107. $x_1x_2^7x_3^2x_4^{15}x_5^{20}$ | 108. $x_1x_2^7x_3^2x_4^{28}x_5^7$ |
| 109. $x_1x_2^7x_3^2x_4^{29}x_5^6$ | 110. $x_1x_2^7x_3^3x_4^4x_5^{30}$ | 111. $x_1x_2^7x_3^3x_4^6x_5^{28}$ | 112. $x_1x_2^7x_3^3x_4^{12}x_5^{22}$ |
| 113. $x_1x_2^7x_3^3x_4^{14}x_5^{20}$ | 114. $x_1x_2^7x_3^3x_4^{28}x_5^6$ | 115. $x_1x_2^7x_3^3x_4^{30}x_5^4$ | 116. $x_1x_2^7x_3^6x_4x_5^{30}$ |
| 117. $x_1x_2^7x_3^6x_4^7x_5^{24}$ | 118. $x_1x_2^7x_3^6x_4^9x_5^{22}$ | 119. $x_1x_2^7x_3^6x_4^{25}x_5^6$ | 120. $x_1x_2^7x_3^7x_4^2x_5^{28}$ |
| 121. $x_1x_2^7x_3^7x_4^6x_5^{24}$ | 122. $x_1x_2^7x_3^7x_4^8x_5^{22}$ | 123. $x_1x_2^7x_3^7x_4^{10}x_5^{20}$ | 124. $x_1x_2^7x_3^7x_4^{14}x_5^{16}$ |
| 125. $x_1x_2^7x_3^7x_4^{24}x_5^6$ | 126. $x_1x_2^7x_3^7x_4^{26}x_5^4$ | 127. $x_1x_2^7x_3^{10}x_4^5x_5^{22}$ | 128. $x_1x_2^7x_3^{10}x_4^{21}x_5^6$ |
| 129. $x_1x_2^7x_3^{11}x_4^4x_5^{22}$ | 130. $x_1x_2^7x_3^{11}x_4^6x_5^{20}$ | 131. $x_1x_2^7x_3^{11}x_4^{20}x_5^6$ | 132. $x_1x_2^7x_3^{11}x_4^{22}x_5^4$ |
| 133. $x_1x_2^7x_3^{14}x_4x_5^{22}$ | 134. $x_1x_2^7x_3^{15}x_4^2x_5^{20}$ | 135. $x_1x_2^7x_3^{26}x_4^5x_5^6$ | 136. $x_1x_2^7x_3^{27}x_4^4x_5^6$ |
| 137. $x_1x_2^7x_3^{27}x_4^6x_5^4$ | 138. $x_1x_2^7x_3^{30}x_4x_5^6$ | 139. $x_1x_2^{14}x_3x_4^6x_5^{23}$ | 140. $x_1x_2^{14}x_3x_4^7x_5^{22}$ |
| 141. $x_1x_2^{14}x_3x_4^{22}x_5^7$ | 142. $x_1x_2^{14}x_3x_4^{23}x_5^6$ | 143. $x_1x_2^{14}x_3^3x_4^5x_5^{22}$ | 144. $x_1x_2^{14}x_3^3x_4^{21}x_5^6$ |
| 145. $x_1x_2^{14}x_3^7x_4x_5^{22}$ | 146. $x_1x_2^{14}x_3^{23}x_4x_5^6$ | 147. $x_1x_2^{15}x_3x_4^6x_5^{22}$ | 148. $x_1x_2^{15}x_3x_4^{22}x_5^6$ |
| 149. $x_1x_2^{15}x_3^2x_4^4x_5^{23}$ | 150. $x_1x_2^{15}x_3^2x_4^5x_5^{22}$ | 151. $x_1x_2^{15}x_3^2x_4^7x_5^{20}$ | 152. $x_1x_2^{15}x_3^2x_4^{20}x_5^7$ |
| 153. $x_1x_2^{15}x_3^2x_4^{21}x_5^6$ | 154. $x_1x_2^{15}x_3^2x_4^{23}x_5^4$ | 155. $x_1x_2^{15}x_3^3x_4^4x_5^{22}$ | 156. $x_1x_2^{15}x_3^3x_4^6x_5^{20}$ |
| 157. $x_1x_2^{15}x_3^3x_4^{20}x_5^6$ | 158. $x_1x_2^{15}x_3^3x_4^{22}x_5^4$ | 159. $x_1x_2^{15}x_3^6x_4x_5^{22}$ | 160. $x_1x_2^{15}x_3^7x_4^2x_5^{20}$ |
| 161. $x_1x_2^{15}x_3^{22}x_4x_5^6$ | 162. $x_1x_2^{15}x_3^{23}x_4^2x_5^4$ | 163. $x_1x_2^{30}x_3x_4^6x_5^7$ | 164. $x_1x_2^{30}x_3x_4^7x_5^6$ |
| 165. $x_1x_2^{30}x_3^3x_4^5x_5^6$ | 166. $x_1x_2^{30}x_3^7x_4x_5^6$ | 167. $x_1^3x_2x_3^4x_4^7x_5^{30}$ | 168. $x_1^3x_2x_3^4x_4^{14}x_5^{23}$ |
| 169. $x_1^3x_2x_3^4x_4^{15}x_5^{22}$ | 170. $x_1^3x_2x_3^4x_4^{30}x_5^7$ | 171. $x_1^3x_2x_3^5x_4^6x_5^{30}$ | 172. $x_1^3x_2x_3^5x_4^{14}x_5^{22}$ |
| 173. $x_1^3x_2x_3^5x_4^{30}x_5^6$ | 174. $x_1^3x_2x_3^6x_4^5x_5^{30}$ | 175. $x_1^3x_2x_3^6x_4^7x_5^{28}$ | 176. $x_1^3x_2x_3^6x_4^{12}x_5^{23}$ |
| 177. $x_1^3x_2x_3^6x_4^{13}x_5^{22}$ | 178. $x_1^3x_2x_3^6x_4^{15}x_5^{20}$ | 179. $x_1^3x_2x_3^6x_4^{28}x_5^7$ | 180. $x_1^3x_2x_3^6x_4^{29}x_5^6$ |
| 181. $x_1^3x_2x_3^7x_4^4x_5^{30}$ | 182. $x_1^3x_2x_3^7x_4^6x_5^{28}$ | 183. $x_1^3x_2x_3^7x_4^{12}x_5^{22}$ | 184. $x_1^3x_2x_3^7x_4^{14}x_5^{20}$ |
| 185. $x_1^3x_2x_3^7x_4^{28}x_5^6$ | 186. $x_1^3x_2x_3^7x_4^{30}x_5^4$ | 187. $x_1^3x_2x_3^{12}x_4^6x_5^{23}$ | 188. $x_1^3x_2x_3^{12}x_4^7x_5^{22}$ |
| 189. $x_1^3x_2x_3^{12}x_4^{22}x_5^7$ | 190. $x_1^3x_2x_3^{12}x_4^{23}x_5^6$ | 191. $x_1^3x_2x_3^{13}x_4^6x_5^{22}$ | 192. $x_1^3x_2x_3^{13}x_4^{22}x_5^6$ |
| 193. $x_1^3x_2x_3^{14}x_4^4x_5^{23}$ | 194. $x_1^3x_2x_3^{14}x_4^5x_5^{22}$ | 195. $x_1^3x_2x_3^{14}x_4^7x_5^{20}$ | 196. $x_1^3x_2x_3^{14}x_4^{20}x_5^7$ |
| 197. $x_1^3x_2x_3^{14}x_4^{21}x_5^6$ | 198. $x_1^3x_2x_3^{14}x_4^{23}x_5^4$ | 199. $x_1^3x_2x_3^{15}x_4^4x_5^{22}$ | 200. $x_1^3x_2x_3^{15}x_4^6x_5^{20}$ |

201. $x_1^3x_2x_3^{15}x_4^{20}x_5^6$ 202. $x_1^3x_2x_3^{15}x_4^{22}x_5^4$ 203. $x_1^3x_2x_3^{28}x_4^6x_5^7$ 204. $x_1^3x_2x_3^{28}x_4^7x_5^6$
205. $x_1^3x_2x_3^{29}x_4^6x_5^6$ 206. $x_1^3x_2x_3^{30}x_4^4x_5^7$ 207. $x_1^3x_2x_3^{30}x_4^5x_5^6$ 208. $x_1^3x_2x_3^{30}x_4^7x_5^4$
209. $x_1^3x_2^3x_3^4x_4^5x_5^{30}$ 210. $x_1^3x_2^3x_3^4x_4^{13}x_5^{22}$ 211. $x_1^3x_2^3x_3^4x_4^{29}x_5^6$ 212. $x_1^3x_2^3x_3^5x_4^4x_5^{30}$
213. $x_1^3x_2^3x_3^5x_4^6x_5^{28}$ 214. $x_1^3x_2^3x_3^5x_4^{12}x_5^{22}$ 215. $x_1^3x_2^3x_3^5x_4^{14}x_5^{20}$ 216. $x_1^3x_2^3x_3^5x_4^{28}x_5^6$
217. $x_1^3x_2^3x_3^5x_4^{30}x_5^4$ 218. $x_1^3x_2^3x_3^{12}x_4^5x_5^{22}$ 219. $x_1^3x_2^3x_3^{12}x_4^{21}x_5^6$ 220. $x_1^3x_2^3x_3^{13}x_4^4x_5^{22}$
221. $x_1^3x_2^3x_3^{13}x_4^6x_5^{20}$ 222. $x_1^3x_2^3x_3^{13}x_4^{20}x_5^6$ 223. $x_1^3x_2^3x_3^{13}x_4^{22}x_5^4$ 224. $x_1^3x_2^3x_3^{28}x_4^5x_5^6$
225. $x_1^3x_2^3x_3^{29}x_4^4x_5^6$ 226. $x_1^3x_2^3x_3^{29}x_4^6x_5^4$ 227. $x_1^3x_2^4x_3^3x_4^5x_5^{30}$ 228. $x_1^3x_2^4x_3^3x_4^{13}x_5^{22}$
229. $x_1^3x_2^4x_3^3x_4^{29}x_5^6$ 230. $x_1^3x_2^4x_3^{11}x_4^5x_5^{22}$ 231. $x_1^3x_2^4x_3^{11}x_4^{21}x_5^6$ 232. $x_1^3x_2^4x_3^{27}x_4^5x_5^6$
233. $x_1^3x_2^5x_3x_4^6x_5^{30}$ 234. $x_1^3x_2^5x_3x_4^{14}x_5^{22}$ 235. $x_1^3x_2^5x_3x_4^{30}x_5^6$ 236. $x_1^3x_2^5x_3^2x_4^5x_5^{30}$
237. $x_1^3x_2^5x_3^2x_4^7x_5^{28}$ 238. $x_1^3x_2^5x_3^2x_4^{12}x_5^{23}$ 239. $x_1^3x_2^5x_3^2x_4^{13}x_5^{22}$ 240. $x_1^3x_2^5x_3^2x_4^{15}x_5^{20}$
241. $x_1^3x_2^5x_3^2x_4^{28}x_5^7$ 242. $x_1^3x_2^5x_3^2x_4^{29}x_5^6$ 243. $x_1^3x_2^5x_3^3x_4^4x_5^{30}$ 244. $x_1^3x_2^5x_3^3x_4^6x_5^{28}$
245. $x_1^3x_2^5x_3^3x_4^{12}x_5^{22}$ 246. $x_1^3x_2^5x_3^3x_4^{14}x_5^{20}$ 247. $x_1^3x_2^5x_3^3x_4^{28}x_5^6$ 248. $x_1^3x_2^5x_3^3x_4^{30}x_5^4$
249. $x_1^3x_2^5x_3^6x_4x_5^{30}$ 250. $x_1^3x_2^5x_3^6x_4^7x_5^{24}$ 251. $x_1^3x_2^5x_3^6x_4^8x_5^{23}$ 252. $x_1^3x_2^5x_3^6x_4^9x_5^{22}$
253. $x_1^3x_2^5x_3^6x_4^{15}x_5^{16}$ 254. $x_1^3x_2^5x_3^6x_4^{24}x_5^7$ 255. $x_1^3x_2^5x_3^6x_4^{25}x_5^6$ 256. $x_1^3x_2^5x_3^7x_4^2x_5^{28}$
257. $x_1^3x_2^5x_3^7x_4^6x_5^{24}$ 258. $x_1^3x_2^5x_3^7x_4^8x_5^{22}$ 259. $x_1^3x_2^5x_3^7x_4^{10}x_5^{20}$ 260. $x_1^3x_2^5x_3^7x_4^{14}x_5^{16}$
261. $x_1^3x_2^5x_3^7x_4^{24}x_5^6$ 262. $x_1^3x_2^5x_3^7x_4^{26}x_5^4$ 263. $x_1^3x_2^5x_3^{10}x_4^4x_5^{23}$ 264. $x_1^3x_2^5x_3^{10}x_4^5x_5^{22}$
265. $x_1^3x_2^5x_3^{10}x_4^7x_5^{20}$ 266. $x_1^3x_2^5x_3^{10}x_4^{20}x_5^7$ 267. $x_1^3x_2^5x_3^{10}x_4^{21}x_5^6$ 268. $x_1^3x_2^5x_3^{10}x_4^{23}x_5^4$
269. $x_1^3x_2^5x_3^{11}x_4^4x_5^{22}$ 270. $x_1^3x_2^5x_3^{11}x_4^6x_5^{20}$ 271. $x_1^3x_2^5x_3^{11}x_4^{20}x_5^6$ 272. $x_1^3x_2^5x_3^{11}x_4^{22}x_5^4$
273. $x_1^3x_2^5x_3^{14}x_4x_5^{22}$ 274. $x_1^3x_2^5x_3^{15}x_4^2x_5^{20}$ 275. $x_1^3x_2^5x_3^{15}x_4^6x_5^{16}$ 276. $x_1^3x_2^5x_3^{15}x_4^{18}x_5^4$
277. $x_1^3x_2^5x_3^{26}x_4^4x_5^7$ 278. $x_1^3x_2^5x_3^{26}x_4^5x_5^6$ 279. $x_1^3x_2^5x_3^{26}x_4^7x_5^4$ 280. $x_1^3x_2^5x_3^{27}x_4^4x_5^6$
281. $x_1^3x_2^5x_3^{27}x_4^6x_5^4$ 282. $x_1^3x_2^5x_3^{30}x_4x_5^6$ 283. $x_1^3x_2^7x_3x_4^4x_5^{30}$ 284. $x_1^3x_2^7x_3x_4^6x_5^{28}$
285. $x_1^3x_2^7x_3x_4^{12}x_5^{22}$ 286. $x_1^3x_2^7x_3x_4^{14}x_5^{20}$ 287. $x_1^3x_2^7x_3x_4^{28}x_5^6$ 288. $x_1^3x_2^7x_3x_4^{30}x_5^4$
289. $x_1^3x_2^7x_3^5x_4^2x_5^{28}$ 290. $x_1^3x_2^7x_3^5x_4^6x_5^{24}$ 291. $x_1^3x_2^7x_3^5x_4^8x_5^{22}$ 292. $x_1^3x_2^7x_3^5x_4^{10}x_5^{20}$
293. $x_1^3x_2^7x_3^5x_4^{14}x_5^{16}$ 294. $x_1^3x_2^7x_3^5x_4^{24}x_5^6$ 295. $x_1^3x_2^7x_3^5x_4^{26}x_5^4$ 296. $x_1^3x_2^7x_3^9x_4^4x_5^{22}$
297. $x_1^3x_2^7x_3^9x_4^6x_5^{20}$ 298. $x_1^3x_2^7x_3^9x_4^{20}x_5^6$ 299. $x_1^3x_2^7x_3^9x_4^{22}x_5^4$ 300. $x_1^3x_2^7x_3^{13}x_4^2x_5^{20}$
301. $x_1^3x_2^7x_3^{13}x_4^6x_5^{16}$ 302. $x_1^3x_2^7x_3^{13}x_4^{18}x_5^4$ 303. $x_1^3x_2^7x_3^{25}x_4^4x_5^6$ 304. $x_1^3x_2^7x_3^{25}x_4^6x_5^4$
305. $x_1^3x_2^7x_3^{29}x_4^2x_5^4$ 306. $x_1^3x_2^{13}x_3x_4^6x_5^{22}$ 307. $x_1^3x_2^{13}x_3x_4^{22}x_5^6$ 308. $x_1^3x_2^{13}x_3^2x_4^4x_5^{23}$
309. $x_1^3x_2^{13}x_3^2x_4^5x_5^{22}$ 310. $x_1^3x_2^{13}x_3^2x_4^7x_5^{20}$ 311. $x_1^3x_2^{13}x_3^2x_4^{20}x_5^7$ 312. $x_1^3x_2^{13}x_3^2x_4^{21}x_5^6$
313. $x_1^3x_2^{13}x_3^2x_4^{23}x_5^4$ 314. $x_1^3x_2^{13}x_3^3x_4^4x_5^{22}$ 315. $x_1^3x_2^{13}x_3^3x_4^6x_5^{20}$ 316. $x_1^3x_2^{13}x_3^3x_4^{20}x_5^6$
317. $x_1^3x_2^{13}x_3^3x_4^{22}x_5^4$ 318. $x_1^3x_2^{13}x_3^6x_4x_5^{22}$ 319. $x_1^3x_2^{13}x_3^7x_4^2x_5^{20}$ 320. $x_1^3x_2^{13}x_3^{22}x_4x_5^6$
321. $x_1^3x_2^{13}x_3^{23}x_4^2x_5^4$ 322. $x_1^3x_2^{15}x_3x_4^4x_5^{22}$ 323. $x_1^3x_2^{15}x_3x_4^6x_5^{20}$ 324. $x_1^3x_2^{15}x_3x_4^{20}x_5^6$
325. $x_1^3x_2^{15}x_3x_4^{22}x_5^4$ 326. $x_1^3x_2^{15}x_3^5x_4^2x_5^{20}$ 327. $x_1^3x_2^{15}x_3^5x_4^6x_5^{16}$ 328. $x_1^3x_2^{15}x_3^5x_4^{18}x_5^4$
329. $x_1^3x_2^{15}x_3^{21}x_4^2x_5^4$ 330. $x_1^3x_2^{29}x_3x_4^6x_5^6$ 331. $x_1^3x_2^{29}x_3^2x_4^4x_5^7$ 332. $x_1^3x_2^{29}x_3^2x_4^5x_5^6$
333. $x_1^3x_2^{29}x_3^2x_4^7x_5^4$ 334. $x_1^3x_2^{29}x_3^3x_4^4x_5^6$ 335. $x_1^3x_2^{29}x_3^3x_4^6x_5^4$ 336. $x_1^3x_2^{29}x_3^6x_4x_5^6$
337. $x_1^3x_2^{29}x_3^7x_4^2x_5^4$ 338. $x_1^7x_2x_3x_4^6x_5^{30}$ 339. $x_1^7x_2x_3x_4^{14}x_5^{22}$ 340. $x_1^7x_2x_3x_4^{30}x_5^6$
341. $x_1^7x_2x_3^2x_4^5x_5^{30}$ 342. $x_1^7x_2x_3^2x_4^7x_5^{28}$ 343. $x_1^7x_2x_3^2x_4^{12}x_5^{23}$ 344. $x_1^7x_2x_3^2x_4^{13}x_5^{22}$
345. $x_1^7x_2x_3^2x_4^{15}x_5^{20}$ 346. $x_1^7x_2x_3^2x_4^{28}x_5^7$ 347. $x_1^7x_2x_3^2x_4^{29}x_5^6$ 348. $x_1^7x_2x_3^3x_4^4x_5^{30}$
349. $x_1^7x_2x_3^3x_4^6x_5^{28}$ 350. $x_1^7x_2x_3^3x_4^{12}x_5^{22}$ 351. $x_1^7x_2x_3^3x_4^{14}x_5^{20}$ 352. $x_1^7x_2x_3^3x_4^{28}x_5^6$
353. $x_1^7x_2x_3^3x_4^{30}x_5^4$ 354. $x_1^7x_2x_3^6x_4x_5^{30}$ 355. $x_1^7x_2x_3^6x_4^7x_5^{24}$ 356. $x_1^7x_2x_3^6x_4^9x_5^{22}$
357. $x_1^7x_2x_3^6x_4^{25}x_5^6$ 358. $x_1^7x_2x_3^7x_4^2x_5^{28}$ 359. $x_1^7x_2x_3^7x_4^6x_5^{24}$ 360. $x_1^7x_2x_3^7x_4^8x_5^{22}$
361. $x_1^7x_2x_3^7x_4^{10}x_5^{20}$ 362. $x_1^7x_2x_3^7x_4^{14}x_5^{16}$ 363. $x_1^7x_2x_3^7x_4^{24}x_5^6$ 364. $x_1^7x_2x_3^7x_4^{26}x_5^4$
365. $x_1^7x_2x_3^{10}x_4^5x_5^{22}$ 366. $x_1^7x_2x_3^{10}x_4^{21}x_5^6$ 367. $x_1^7x_2x_3^{11}x_4^4x_5^{22}$ 368. $x_1^7x_2x_3^{11}x_4^6x_5^{20}$
369. $x_1^7x_2x_3^{11}x_4^{20}x_5^6$ 370. $x_1^7x_2x_3^{11}x_4^{22}x_5^4$ 371. $x_1^7x_2x_3^{14}x_4x_5^{22}$ 372. $x_1^7x_2x_3^{15}x_4^2x_5^{20}$

| | | | |
|---|---|---|---|
| 373. $x_1^7x_2x_3^{26}x_4^5x_5^6$ | 374. $x_1^7x_2x_3^{27}x_4^4x_5^6$ | 375. $x_1^7x_2x_3^{27}x_4^6x_5^4$ | 376. $x_1^7x_2x_3^{30}x_4x_5^6$ |
| 377. $x_1^7x_2^3x_3x_4^4x_5^{30}$ | 378. $x_1^7x_2^3x_3x_4^6x_5^{28}$ | 379. $x_1^7x_2^3x_3x_4^{12}x_5^{22}$ | 380. $x_1^7x_2^3x_3x_4^{14}x_5^{20}$ |
| 381. $x_1^7x_2^3x_3x_4^{28}x_5^6$ | 382. $x_1^7x_2^3x_3x_4^{30}x_5^4$ | 383. $x_1^7x_2^3x_3^5x_4^2x_5^{28}$ | 384. $x_1^7x_2^3x_3^5x_4^6x_5^{24}$ |
| 385. $x_1^7x_2^3x_3^5x_4^8x_5^{22}$ | 386. $x_1^7x_2^3x_3^5x_4^{10}x_5^{20}$ | 387. $x_1^7x_2^3x_3^5x_4^{14}x_5^{16}$ | 388. $x_1^7x_2^3x_3^5x_4^{24}x_5^6$ |
| 389. $x_1^7x_2^3x_3^5x_4^{26}x_5^4$ | 390. $x_1^7x_2^3x_3^9x_4^4x_5^{22}$ | 391. $x_1^7x_2^3x_3^9x_4^6x_5^{20}$ | 392. $x_1^7x_2^3x_3^9x_4^{20}x_5^6$ |
| 393. $x_1^7x_2^3x_3^9x_4^{22}x_5^4$ | 394. $x_1^7x_2^3x_3^{13}x_4^2x_5^{20}$ | 395. $x_1^7x_2^3x_3^{13}x_4^6x_5^{16}$ | 396. $x_1^7x_2^3x_3^{13}x_4^{18}x_5^4$ |
| 397. $x_1^7x_2^3x_3^{25}x_4^4x_5^6$ | 398. $x_1^7x_2^3x_3^{25}x_4^6x_5^4$ | 399. $x_1^7x_2^3x_3^{29}x_4^2x_5^4$ | 400. $x_1^7x_2^7x_3x_4^2x_5^{28}$ |
| 401. $x_1^7x_2^7x_3x_4^6x_5^{24}$ | 402. $x_1^7x_2^7x_3x_4^8x_5^{22}$ | 403. $x_1^7x_2^7x_3x_4^{10}x_5^{20}$ | 404. $x_1^7x_2^7x_3x_4^{14}x_5^{16}$ |
| 405. $x_1^7x_2^7x_3x_4^{24}x_5^6$ | 406. $x_1^7x_2^7x_3x_4^{26}x_5^4$ | 407. $x_1^7x_2^7x_3^7x_4^8x_5^{16}$ | 408. $x_1^7x_2^7x_3^9x_4^2x_5^{20}$ |
| 409. $x_1^7x_2^7x_3^9x_4^6x_5^{16}$ | 410. $x_1^7x_2^7x_3^9x_4^{18}x_5^4$ | 411. $x_1^7x_2^7x_3^{25}x_4^2x_5^4$ | 412. $x_1^7x_2^9x_3^3x_4^6x_5^{20}$ |
| 413. $x_1^7x_2^{11}x_3x_4^4x_5^{22}$ | 414. $x_1^7x_2^{11}x_3x_4^6x_5^{20}$ | 415. $x_1^7x_2^{11}x_3x_4^{20}x_5^6$ | 416. $x_1^7x_2^{11}x_3x_4^{22}x_5^4$ |
| 417. $x_1^7x_2^{11}x_3^5x_4^2x_5^{20}$ | 418. $x_1^7x_2^{11}x_3^5x_4^6x_5^{16}$ | 419. $x_1^7x_2^{11}x_3^5x_4^{18}x_5^4$ | 420. $x_1^7x_2^{11}x_3^{21}x_4^2x_5^4$ |
| 421. $x_1^7x_2^{15}x_3x_4^2x_5^{20}$ | 422. $x_1^7x_2^{27}x_3x_4^4x_5^6$ | 423. $x_1^7x_2^{27}x_3x_4^6x_5^4$ | 424. $x_1^7x_2^{27}x_3^5x_4^2x_5^4$ |
| 425. $x_1^{15}x_2x_3x_4^6x_5^{22}$ | 426. $x_1^{15}x_2x_3x_4^{22}x_5^6$ | 427. $x_1^{15}x_2x_3^2x_4^4x_5^{23}$ | 428. $x_1^{15}x_2x_3^2x_4^5x_5^{22}$ |
| 429. $x_1^{15}x_2x_3^2x_4^7x_5^{20}$ | 430. $x_1^{15}x_2x_3^2x_4^{20}x_5^7$ | 431. $x_1^{15}x_2x_3^2x_4^{21}x_5^6$ | 432. $x_1^{15}x_2x_3^2x_4^{23}x_5^4$ |
| 433. $x_1^{15}x_2x_3^3x_4^4x_5^{22}$ | 434. $x_1^{15}x_2x_3^3x_4^6x_5^{20}$ | 435. $x_1^{15}x_2x_3^3x_4^{20}x_5^6$ | 436. $x_1^{15}x_2x_3^3x_4^{22}x_5^4$ |
| 437. $x_1^{15}x_2x_3^6x_4x_5^{22}$ | 438. $x_1^{15}x_2x_3^7x_4^2x_5^{20}$ | 439. $x_1^{15}x_2x_3^{22}x_4x_5^6$ | 440. $x_1^{15}x_2x_3^{23}x_4^2x_5^4$ |
| 441. $x_1^{15}x_2^3x_3x_4^4x_5^{22}$ | 442. $x_1^{15}x_2^3x_3x_4^6x_5^{20}$ | 443. $x_1^{15}x_2^3x_3x_4^{20}x_5^6$ | 444. $x_1^{15}x_2^3x_3x_4^{22}x_5^4$ |
| 445. $x_1^{15}x_2^3x_3^5x_4^2x_5^{20}$ | 446. $x_1^{15}x_2^3x_3^5x_4^6x_5^{16}$ | 447. $x_1^{15}x_2^3x_3^5x_4^{18}x_5^4$ | 448. $x_1^{15}x_2^3x_3^{21}x_4^2x_5^4$ |
| 449. $x_1^{15}x_2^7x_3x_4^2x_5^{20}$ | 450. $x_1^{15}x_2^{23}x_3x_4^2x_5^4$ | | |

Then, $B_5^0((3)|^3|(1)|^2)\bigcup \widetilde{B}_5^+((3)|^3|(1)|^2)\bigcup\{Y_t : 1 \leqslant t \leqslant 34\}$ is a set of generators for $\mathrm{Ker}\big((\widetilde{Sq}^0_*)_{(5,20)}\big)$. Here, the monomials $Y_t$ are given in Subsection 3.3.

# 6 The admissible monomials of degree $2^{d+1}+2^d-4$ for $3 \leqslant d \leqslant 5$

In this subsection we list the needed admissible monomials of degree $m = 2^{d+1} + 2^d - 4$ in $P_5$ for $3 \leqslant d \leqslant 5$. The elements in $B_5^0(m)$ can easy determined by using the results in [7], [4], [20] and Proposition 2.4.2. We have $|B_5^0((4)|^{d-2}|(2)|(1)|^2)| = 225$, $|B_5^0((4)|^{d-2}|(2)|(3))| = 20$, $|B_5^0((4)|^{d-1}|(2))| = 30$, hence $|B_5^0(m)| = 275$ for any $d \geqslant 3$. So, we list only the elements in $B_5^+(m)$.

## 6.1 The case $d = 3$

$\widetilde{B}_5^+(4,2,1,1) = \{a_t = a_{2,t} : 1 \leqslant t \leqslant 215\}$, where

| | | | |
|---|---|---|---|
| 1. $x_1x_2x_3x_4^3x_5^{14}$ | 2. $x_1x_2x_3x_4^6x_5^{11}$ | 3. $x_1x_2x_3x_4^7x_5^{10}$ | 4. $x_1x_2x_3x_4^{14}x_5^3$ |
| 5. $x_1x_2x_3^2x_4^3x_5^{13}$ | 6. $x_1x_2x_3^2x_4^5x_5^{11}$ | 7. $x_1x_2x_3^2x_4^7x_5^9$ | 8. $x_1x_2x_3^2x_4^{13}x_5^3$ |

9. $x_1x_2x_3^3x_4x_5^{14}$
10. $x_1x_2x_3^3x_4^2x_5^{13}$
11. $x_1x_2x_3^3x_4^3x_5^{12}$
12. $x_1x_2x_3^3x_4^4x_5^{11}$
13. $x_1x_2x_3^3x_4^5x_5^{10}$
14. $x_1x_2x_3^3x_4^6x_5^9$
15. $x_1x_2x_3^3x_4^7x_5^8$
16. $x_1x_2x_3^3x_4^{12}x_5^3$
17. $x_1x_2x_3^3x_4^{13}x_5^2$
18. $x_1x_2x_3^3x_4^{14}x_5$
19. $x_1x_2x_3^6x_4x_5^{11}$
20. $x_1x_2x_3^6x_4^3x_5^9$
21. $x_1x_2x_3^6x_4^9x_5^3$
22. $x_1x_2x_3^6x_4^{11}x_5$
23. $x_1x_2x_3^7x_4x_5^{10}$
24. $x_1x_2x_3^7x_4^2x_5^9$
25. $x_1x_2x_3^7x_4^3x_5^8$
26. $x_1x_2x_3^7x_4^8x_5^3$
27. $x_1x_2x_3^7x_4^9x_5^2$
28. $x_1x_2x_3^7x_4^{10}x_5$
29. $x_1x_2x_3^{14}x_4x_5^3$
30. $x_1x_2x_3^{14}x_4^3x_5$
31. $x_1x_2^2x_3x_4^3x_5^{13}$
32. $x_1x_2^2x_3x_4^5x_5^{11}$
33. $x_1x_2^2x_3x_4^7x_5^9$
34. $x_1x_2^2x_3x_4^{13}x_5^3$
35. $x_1x_2^2x_3^3x_4x_5^{13}$
36. $x_1x_2^2x_3^3x_4^5x_5^9$
37. $x_1x_2^2x_3^3x_4^{13}x_5$
38. $x_1x_2^2x_3^5x_4x_5^{11}$
39. $x_1x_2^2x_3^5x_4^3x_5^9$
40. $x_1x_2^2x_3^5x_4^9x_5^3$
41. $x_1x_2^2x_3^5x_4^{11}x_5$
42. $x_1x_2^2x_3^7x_4x_5^9$
43. $x_1x_2^2x_3^7x_4^9x_5$
44. $x_1x_2^2x_3^{13}x_4x_5^3$
45. $x_1x_2^2x_3^{13}x_4^3x_5$
46. $x_1x_2^3x_3x_4x_5^{14}$
47. $x_1x_2^3x_3x_4^2x_5^{13}$
48. $x_1x_2^3x_3x_4^3x_5^{12}$
49. $x_1x_2^3x_3x_4^4x_5^{11}$
50. $x_1x_2^3x_3x_4^5x_5^{10}$
51. $x_1x_2^3x_3x_4^6x_5^9$
52. $x_1x_2^3x_3x_4^7x_5^8$
53. $x_1x_2^3x_3x_4^{12}x_5^3$
54. $x_1x_2^3x_3x_4^{13}x_5^2$
55. $x_1x_2^3x_3x_4^{14}x_5$
56. $x_1x_2^3x_3^2x_4x_5^{13}$
57. $x_1x_2^3x_3^2x_4^5x_5^9$
58. $x_1x_2^3x_3^2x_4^{13}x_5$
59. $x_1x_2^3x_3^3x_4x_5^{12}$
60. $x_1x_2^3x_3^3x_4^4x_5^9$
61. $x_1x_2^3x_3^3x_4^5x_5^8$
62. $x_1x_2^3x_3^3x_4^{12}x_5$
63. $x_1x_2^3x_3^4x_4x_5^{11}$
64. $x_1x_2^3x_3^4x_4^3x_5^9$
65. $x_1x_2^3x_3^4x_4^9x_5^3$
66. $x_1x_2^3x_3^4x_4^{11}x_5$
67. $x_1x_2^3x_3^5x_4x_5^{10}$
68. $x_1x_2^3x_3^5x_4^2x_5^9$
69. $x_1x_2^3x_3^5x_4^3x_5^8$
70. $x_1x_2^3x_3^5x_4^8x_5^3$
71. $x_1x_2^3x_3^5x_4^9x_5^2$
72. $x_1x_2^3x_3^5x_4^{10}x_5$
73. $x_1x_2^3x_3^6x_4x_5^9$
74. $x_1x_2^3x_3^6x_4^9x_5$
75. $x_1x_2^3x_3^7x_4x_5^8$
76. $x_1x_2^3x_3^7x_4^8x_5$
77. $x_1x_2^3x_3^{12}x_4x_5^3$
78. $x_1x_2^3x_3^{12}x_4^3x_5$
79. $x_1x_2^3x_3^{13}x_4x_5^2$
80. $x_1x_2^3x_3^{13}x_4^2x_5$
81. $x_1x_2^3x_3^{14}x_4x_5$
82. $x_1x_2^6x_3x_4x_5^{11}$
83. $x_1x_2^6x_3x_4^3x_5^9$
84. $x_1x_2^6x_3x_4^9x_5^3$
85. $x_1x_2^6x_3x_4^{11}x_5$
86. $x_1x_2^6x_3^3x_4x_5^9$
87. $x_1x_2^6x_3^3x_4^9x_5$
88. $x_1x_2^6x_3^9x_4x_5^3$
89. $x_1x_2^6x_3^9x_4^3x_5$
90. $x_1x_2^6x_3^{11}x_4x_5$
91. $x_1x_2^7x_3x_4x_5^{10}$
92. $x_1x_2^7x_3x_4^2x_5^9$
93. $x_1x_2^7x_3x_4^3x_5^8$
94. $x_1x_2^7x_3x_4^8x_5^3$
95. $x_1x_2^7x_3x_4^9x_5^2$
96. $x_1x_2^7x_3x_4^{10}x_5$
97. $x_1x_2^7x_3^2x_4x_5^9$
98. $x_1x_2^7x_3^2x_4^9x_5$
99. $x_1x_2^7x_3^3x_4x_5^8$
100. $x_1x_2^7x_3^3x_4^8x_5$
101. $x_1x_2^7x_3^8x_4x_5^3$
102. $x_1x_2^7x_3^8x_4^3x_5$
103. $x_1x_2^7x_3^9x_4x_5^2$
104. $x_1x_2^7x_3^9x_4^2x_5$
105. $x_1x_2^7x_3^{10}x_4x_5$
106. $x_1x_2^{14}x_3x_4x_5^3$
107. $x_1x_2^{14}x_3x_4^3x_5$
108. $x_1x_2^{14}x_3^3x_4x_5$
109. $x_1^3x_2x_3x_4x_5^{14}$
110. $x_1^3x_2x_3x_4^2x_5^{13}$
111. $x_1^3x_2x_3x_4^3x_5^{12}$
112. $x_1^3x_2x_3x_4^4x_5^{11}$
113. $x_1^3x_2x_3x_4^5x_5^{10}$
114. $x_1^3x_2x_3x_4^6x_5^9$
115. $x_1^3x_2x_3x_4^7x_5^8$
116. $x_1^3x_2x_3x_4^{12}x_5^3$
117. $x_1^3x_2x_3x_4^{13}x_5^2$
118. $x_1^3x_2x_3x_4^{14}x_5$
119. $x_1^3x_2x_3^2x_4x_5^{13}$
120. $x_1^3x_2x_3^2x_4^5x_5^9$
121. $x_1^3x_2x_3^2x_4^{13}x_5$
122. $x_1^3x_2x_3^3x_4x_5^{12}$
123. $x_1^3x_2x_3^3x_4^4x_5^9$
124. $x_1^3x_2x_3^3x_4^5x_5^8$
125. $x_1^3x_2x_3^3x_4^{12}x_5$
126. $x_1^3x_2x_3^4x_4x_5^{11}$
127. $x_1^3x_2x_3^4x_4^3x_5^9$
128. $x_1^3x_2x_3^4x_4^9x_5^3$
129. $x_1^3x_2x_3^4x_4^{11}x_5$
130. $x_1^3x_2x_3^5x_4x_5^{10}$
131. $x_1^3x_2x_3^5x_4^2x_5^9$
132. $x_1^3x_2x_3^5x_4^3x_5^8$
133. $x_1^3x_2x_3^5x_4^8x_5^3$
134. $x_1^3x_2x_3^5x_4^9x_5^2$
135. $x_1^3x_2x_3^5x_4^{10}x_5$
136. $x_1^3x_2x_3^6x_4x_5^9$
137. $x_1^3x_2x_3^6x_4^9x_5$
138. $x_1^3x_2x_3^7x_4x_5^8$
139. $x_1^3x_2x_3^7x_4^8x_5$
140. $x_1^3x_2x_3^{12}x_4x_5^3$
141. $x_1^3x_2x_3^{12}x_4^3x_5$
142. $x_1^3x_2x_3^{13}x_4x_5^2$
143. $x_1^3x_2x_3^{13}x_4^2x_5$
144. $x_1^3x_2x_3^{14}x_4x_5$
145. $x_1^3x_2^3x_3x_4x_5^{12}$
146. $x_1^3x_2^3x_3x_4^4x_5^9$
147. $x_1^3x_2^3x_3x_4^5x_5^8$
148. $x_1^3x_2^3x_3x_4^{12}x_5$
149. $x_1^3x_2^3x_3^4x_4x_5^9$
150. $x_1^3x_2^3x_3^4x_4^9x_5$
151. $x_1^3x_2^3x_3^5x_4x_5^8$
152. $x_1^3x_2^3x_3^5x_4^8x_5$
153. $x_1^3x_2^3x_3^{12}x_4x_5$
154. $x_1^3x_2^4x_3x_4x_5^{11}$
155. $x_1^3x_2^4x_3x_4^3x_5^9$
156. $x_1^3x_2^4x_3x_4^{11}x_5$
157. $x_1^3x_2^4x_3^3x_4x_5^9$
158. $x_1^3x_2^4x_3^3x_4^9x_5$
159. $x_1^3x_2^4x_3^{11}x_4x_5$
160. $x_1^3x_2^5x_3x_4x_5^{10}$
161. $x_1^3x_2^5x_3x_4^2x_5^9$
162. $x_1^3x_2^5x_3x_4^3x_5^8$
163. $x_1^3x_2^5x_3x_4^8x_5^3$
164. $x_1^3x_2^5x_3x_4^9x_5^2$
165. $x_1^3x_2^5x_3x_4^{10}x_5$
166. $x_1^3x_2^5x_3^2x_4x_5^9$
167. $x_1^3x_2^5x_3^2x_4^9x_5$
168. $x_1^3x_2^5x_3^3x_4x_5^8$
169. $x_1^3x_2^5x_3^3x_4^8x_5$
170. $x_1^3x_2^5x_3^8x_4x_5^3$
171. $x_1^3x_2^5x_3^8x_4^3x_5$
172. $x_1^3x_2^5x_3^9x_4x_5^2$
173. $x_1^3x_2^5x_3^9x_4^2x_5$
174. $x_1^3x_2^5x_3^{10}x_4x_5$
175. $x_1^3x_2^7x_3x_4x_5^8$
176. $x_1^3x_2^7x_3x_4^8x_5$
177. $x_1^3x_2^7x_3^8x_4x_5$
178. $x_1^3x_2^{13}x_3x_4x_5^2$
179. $x_1^3x_2^{13}x_3x_4^2x_5$
180. $x_1^3x_2^{13}x_3^2x_4x_5$

| | | | |
|---|---|---|---|
| 181. $x_1^7x_2x_3x_4x_5^{10}$ | 182. $x_1^7x_2x_3x_4^2x_5^9$ | 183. $x_1^7x_2x_3x_4^3x_5^8$ | 184. $x_1^7x_2x_3x_4^8x_5^3$ |
| 185. $x_1^7x_2x_3x_4^9x_5^2$ | 186. $x_1^7x_2x_3x_4^{10}x_5$ | 187. $x_1^7x_2x_3^2x_4x_5^9$ | 188. $x_1^7x_2x_3^2x_4^9x_5$ |
| 189. $x_1^7x_2x_3^3x_4x_5^8$ | 190. $x_1^7x_2x_3^3x_4^8x_5$ | 191. $x_1^7x_2x_3^8x_4x_5^3$ | 192. $x_1^7x_2x_3^8x_4^3x_5$ |
| 193. $x_1^7x_2x_3^9x_4x_5^2$ | 194. $x_1^7x_2x_3^9x_4^2x_5$ | 195. $x_1^7x_2x_3^{10}x_4x_5$ | 196. $x_1^7x_2^3x_3x_4x_5^8$ |
| 197. $x_1^7x_2^3x_3x_4^8x_5$ | 198. $x_1^7x_2^3x_3^8x_4x_5$ | 199. $x_1^7x_2^9x_3x_4x_5^2$ | 200. $x_1^7x_2^9x_3x_4^2x_5$ |
| 201. $x_1x_2x_3x_4^{15}x_5^2$ | 202. $x_1x_2x_3x_4^2x_5^{15}$ | 203. $x_1x_2x_3^{15}x_4x_5^2$ | 204. $x_1x_2x_3^{15}x_4^2x_5$ |
| 205. $x_1x_2x_3^2x_4x_5^{15}$ | 206. $x_1x_2x_3^2x_4^{15}x_5$ | 207. $x_1x_2^{15}x_3x_4x_5^2$ | 208. $x_1x_2^{15}x_3x_4^2x_5$ |
| 209. $x_1x_2^{15}x_3^2x_4x_5$ | 210. $x_1x_2^2x_3x_4x_5^{15}$ | 211. $x_1x_2^2x_3x_4^{15}x_5$ | 212. $x_1x_2^2x_3^{15}x_4x_5$ |
| 213. $x_1^{15}x_2x_3x_4x_5^2$ | 214. $x_1^{15}x_2x_3x_4^2x_5$ | 215. $x_1^{15}x_2x_3^2x_4x_5$ | |

We have $|B_5^+(4,2,3)| = 50$ and its elements have been explicitly determined in [8].

$B_5^+(4,4,2) = \{\bar{a}_t : 1 \leqslant t \leqslant 91\}$, where

| | | | |
|---|---|---|---|
| 1. $x_1x_2^2x_3^3x_4^7x_5^7$ | 2. $x_1x_2^2x_3^7x_4^3x_5^7$ | 3. $x_1x_2^2x_3^7x_4^7x_5^3$ | 4. $x_1x_2^3x_3^2x_4^7x_5^7$ |
| 5. $x_1x_2^3x_3^3x_4^6x_5^7$ | 6. $x_1x_2^3x_3^3x_4^7x_5^6$ | 7. $x_1x_2^3x_3^6x_4^3x_5^7$ | 8. $x_1x_2^3x_3^6x_4^7x_5^3$ |
| 9. $x_1x_2^3x_3^7x_4^2x_5^7$ | 10. $x_1x_2^3x_3^7x_4^3x_5^6$ | 11. $x_1x_2^3x_3^7x_4^6x_5^3$ | 12. $x_1x_2^3x_3^7x_4^7x_5^2$ |
| 13. $x_1x_2^6x_3^3x_4^3x_5^7$ | 14. $x_1x_2^6x_3^3x_4^7x_5^3$ | 15. $x_1x_2^6x_3^7x_4^3x_5^3$ | 16. $x_1x_2^7x_3^2x_4^3x_5^7$ |
| 17. $x_1x_2^7x_3^2x_4^7x_5^3$ | 18. $x_1x_2^7x_3^3x_4^2x_5^7$ | 19. $x_1x_2^7x_3^3x_4^3x_5^6$ | 20. $x_1x_2^7x_3^3x_4^6x_5^3$ |
| 21. $x_1x_2^7x_3^3x_4^7x_5^2$ | 22. $x_1x_2^7x_3^6x_4^3x_5^3$ | 23. $x_1x_2^7x_3^7x_4^2x_5^3$ | 24. $x_1x_2^7x_3^7x_4^3x_5^2$ |
| 25. $x_1^3x_2x_3^2x_4^7x_5^7$ | 26. $x_1^3x_2x_3^3x_4^6x_5^7$ | 27. $x_1^3x_2x_3^3x_4^7x_5^6$ | 28. $x_1^3x_2x_3^6x_4^3x_5^7$ |
| 29. $x_1^3x_2x_3^6x_4^7x_5^3$ | 30. $x_1^3x_2x_3^7x_4^2x_5^7$ | 31. $x_1^3x_2x_3^7x_4^3x_5^6$ | 32. $x_1^3x_2x_3^7x_4^6x_5^3$ |
| 33. $x_1^3x_2x_3^7x_4^7x_5^2$ | 34. $x_1^3x_2^3x_3x_4^6x_5^7$ | 35. $x_1^3x_2^3x_3x_4^7x_5^6$ | 36. $x_1^3x_2^3x_3^3x_4^4x_5^7$ |
| 37. $x_1^3x_2^3x_3^3x_4^5x_5^6$ | 38. $x_1^3x_2^3x_3^3x_4^7x_5^4$ | 39. $x_1^3x_2^3x_3^4x_4^3x_5^7$ | 40. $x_1^3x_2^3x_3^4x_4^7x_5^3$ |
| 41. $x_1^3x_2^3x_3^5x_4^2x_5^7$ | 42. $x_1^3x_2^3x_3^5x_4^3x_5^6$ | 43. $x_1^3x_2^3x_3^5x_4^6x_5^3$ | 44. $x_1^3x_2^3x_3^5x_4^7x_5^2$ |
| 45. $x_1^3x_2^3x_3^7x_4x_5^6$ | 46. $x_1^3x_2^3x_3^7x_4^3x_5^4$ | 47. $x_1^3x_2^3x_3^7x_4^4x_5^3$ | 48. $x_1^3x_2^3x_3^7x_4^5x_5^2$ |
| 49. $x_1^3x_2^5x_3^2x_4^3x_5^7$ | 50. $x_1^3x_2^5x_3^2x_4^7x_5^3$ | 51. $x_1^3x_2^5x_3^3x_4^2x_5^7$ | 52. $x_1^3x_2^5x_3^3x_4^3x_5^6$ |
| 53. $x_1^3x_2^5x_3^3x_4^6x_5^3$ | 54. $x_1^3x_2^5x_3^3x_4^7x_5^2$ | 55. $x_1^3x_2^5x_3^6x_4^3x_5^3$ | 56. $x_1^3x_2^5x_3^7x_4^2x_5^3$ |
| 57. $x_1^3x_2^5x_3^7x_4^3x_5^2$ | 58. $x_1^3x_2^7x_3x_4^2x_5^7$ | 59. $x_1^3x_2^7x_3x_4^3x_5^6$ | 60. $x_1^3x_2^7x_3x_4^6x_5^3$ |
| 61. $x_1^3x_2^7x_3x_4^7x_5^2$ | 62. $x_1^3x_2^7x_3^3x_4x_5^6$ | 63. $x_1^3x_2^7x_3^3x_4^3x_5^4$ | 64. $x_1^3x_2^7x_3^3x_4^4x_5^3$ |
| 65. $x_1^3x_2^7x_3^3x_4^5x_5^2$ | 66. $x_1^3x_2^7x_3^5x_4^2x_5^3$ | 67. $x_1^3x_2^7x_3^5x_4^3x_5^2$ | 68. $x_1^3x_2^7x_3^7x_4x_5^2$ |
| 69. $x_1^7x_2x_3^2x_4^3x_5^7$ | 70. $x_1^7x_2x_3^2x_4^7x_5^3$ | 71. $x_1^7x_2x_3^3x_4^2x_5^7$ | 72. $x_1^7x_2x_3^3x_4^3x_5^6$ |
| 73. $x_1^7x_2x_3^3x_4^6x_5^3$ | 74. $x_1^7x_2x_3^3x_4^7x_5^2$ | 75. $x_1^7x_2x_3^6x_4^3x_5^3$ | 76. $x_1^7x_2x_3^7x_4^2x_5^3$ |
| 77. $x_1^7x_2x_3^7x_4^3x_5^2$ | 78. $x_1^7x_2^3x_3x_4^2x_5^7$ | 79. $x_1^7x_2^3x_3x_4^3x_5^6$ | 80. $x_1^7x_2^3x_3x_4^6x_5^3$ |
| 81. $x_1^7x_2^3x_3x_4^7x_5^2$ | 82. $x_1^7x_2^3x_3^3x_4x_5^6$ | 83. $x_1^7x_2^3x_3^3x_4^3x_5^4$ | 84. $x_1^7x_2^3x_3^3x_4^4x_5^3$ |
| 85. $x_1^7x_2^3x_3^3x_4^5x_5^2$ | 86. $x_1^7x_2^3x_3^5x_4^2x_5^3$ | 87. $x_1^7x_2^3x_3^5x_4^3x_5^2$ | 88. $x_1^7x_2^3x_3^7x_4x_5^2$ |
| 89. $x_1^7x_2^7x_3x_4^2x_5^3$ | 90. $x_1^7x_2^7x_3x_4^3x_5^2$ | 91. $x_1^7x_2^7x_3^3x_4x_5^2$ | |

## 6.2 The case $d = 4$

$\widetilde{B}_5^+((4)|^2(2)|(1)|^2) = \{b_t : 1 \leqslant t \leqslant 870\}$, where

| | | | |
|---|---|---|---|
| 1. $x_1x_2^2x_3^3x_4^7x_5^{31}$ | 2. $x_1x_2^2x_3^3x_4^{31}x_5^7$ | 3. $x_1x_2^2x_3^7x_4^3x_5^{31}$ | 4. $x_1x_2^2x_3^7x_4^{31}x_5^3$ |
| 5. $x_1x_2^2x_3^{31}x_4^3x_5^7$ | 6. $x_1x_2^2x_3^{31}x_4^7x_5^3$ | 7. $x_1x_2^3x_3^2x_4^7x_5^{31}$ | 8. $x_1x_2^3x_3^2x_4^{31}x_5^7$ |

9. $x_1x_2^3x_3^7x_4^2x_5^{31}$ 10. $x_1x_2^3x_3^7x_4^{31}x_5^2$ 11. $x_1x_2^3x_3^{31}x_4^2x_5^7$ 12. $x_1x_2^3x_3^{31}x_4^7x_5^2$
13. $x_1x_2^7x_3^2x_4^3x_5^{31}$ 14. $x_1x_2^7x_3^2x_4^{31}x_5^3$ 15. $x_1x_2^7x_3^3x_4^2x_5^{31}$ 16. $x_1x_2^7x_3^3x_4^{31}x_5^2$
17. $x_1x_2^7x_3^{31}x_4^2x_5^3$ 18. $x_1x_2^7x_3^{31}x_4^3x_5^2$ 19. $x_1x_2^{31}x_3^2x_4^3x_5^7$ 20. $x_1x_2^{31}x_3^2x_4^7x_5^3$
21. $x_1x_2^{31}x_3^3x_4^2x_5^7$ 22. $x_1x_2^{31}x_3^3x_4^7x_5^2$ 23. $x_1x_2^{31}x_3^7x_4^2x_5^3$ 24. $x_1x_2^{31}x_3^7x_4^3x_5^2$
25. $x_1^3x_2x_3^2x_4^7x_5^{31}$ 26. $x_1^3x_2x_3^2x_4^{31}x_5^7$ 27. $x_1^3x_2x_3^7x_4^2x_5^{31}$ 28. $x_1^3x_2x_3^7x_4^{31}x_5^2$
29. $x_1^3x_2x_3^{31}x_4^2x_5^7$ 30. $x_1^3x_2x_3^{31}x_4^7x_5^2$ 31. $x_1^3x_2^7x_3x_4^2x_5^{31}$ 32. $x_1^3x_2^7x_3x_4^{31}x_5^2$
33. $x_1^3x_2^7x_3^{31}x_4x_5^2$ 34. $x_1^3x_2^{31}x_3x_4^2x_5^7$ 35. $x_1^3x_2^{31}x_3x_4^7x_5^2$ 36. $x_1^3x_2^{31}x_3^7x_4x_5^2$
37. $x_1^7x_2x_3^2x_4^3x_5^{31}$ 38. $x_1^7x_2x_3^2x_4^{31}x_5^3$ 39. $x_1^7x_2x_3^3x_4^2x_5^{31}$ 40. $x_1^7x_2x_3^3x_4^{31}x_5^2$
41. $x_1^7x_2x_3^{31}x_4^2x_5^3$ 42. $x_1^7x_2x_3^{31}x_4^3x_5^2$ 43. $x_1^7x_2^3x_3x_4^2x_5^{31}$ 44. $x_1^7x_2^3x_3x_4^{31}x_5^2$
45. $x_1^7x_2^3x_3^{31}x_4x_5^2$ 46. $x_1^7x_2^{31}x_3x_4^2x_5^3$ 47. $x_1^7x_2^{31}x_3x_4^3x_5^2$ 48. $x_1^7x_2^{31}x_3^3x_4x_5^2$
49. $x_1^{31}x_2x_3^2x_4^3x_5^7$ 50. $x_1^{31}x_2x_3^2x_4^7x_5^3$ 51. $x_1^{31}x_2x_3^3x_4^2x_5^7$ 52. $x_1^{31}x_2x_3^3x_4^7x_5^2$
53. $x_1^{31}x_2x_3^7x_4^2x_5^3$ 54. $x_1^{31}x_2x_3^7x_4^3x_5^2$ 55. $x_1^{31}x_2^3x_3x_4^2x_5^7$ 56. $x_1^{31}x_2^3x_3x_4^7x_5^2$
57. $x_1^{31}x_2^3x_3^7x_4x_5^2$ 58. $x_1^{31}x_2^7x_3x_4^2x_5^3$ 59. $x_1^{31}x_2^7x_3x_4^3x_5^2$ 60. $x_1^{31}x_2^7x_3^3x_4x_5^2$
61. $x_1x_2^2x_3^3x_4^{15}x_5^{23}$ 62. $x_1x_2^2x_3^{15}x_4^3x_5^{23}$ 63. $x_1x_2^2x_3^{15}x_4^{23}x_5^3$ 64. $x_1x_2^3x_3^2x_4^{15}x_5^{23}$
65. $x_1x_2^3x_3^{15}x_4^2x_5^{23}$ 66. $x_1x_2^3x_3^{15}x_4^{23}x_5^2$ 67. $x_1x_2^{15}x_3^2x_4^3x_5^{23}$ 68. $x_1x_2^{15}x_3^2x_4^{23}x_5^3$
69. $x_1x_2^{15}x_3^3x_4^2x_5^{23}$ 70. $x_1x_2^{15}x_3^3x_4^{23}x_5^2$ 71. $x_1x_2^{15}x_3^{23}x_4^2x_5^3$ 72. $x_1x_2^{15}x_3^{23}x_4^3x_5^2$
73. $x_1^3x_2x_3^2x_4^{15}x_5^{23}$ 74. $x_1^3x_2x_3^{15}x_4^2x_5^{23}$ 75. $x_1^3x_2x_3^{15}x_4^{23}x_5^2$ 76. $x_1^3x_2^{15}x_3x_4^2x_5^{23}$
77. $x_1^3x_2^{15}x_3x_4^{23}x_5^2$ 78. $x_1^3x_2^{15}x_3^{23}x_4x_5^2$ 79. $x_1^{15}x_2x_3^2x_4^3x_5^{23}$ 80. $x_1^{15}x_2x_3^2x_4^{23}x_5^3$
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349. $x_1x_2^7x_3^{11}x_4^{22}x_5^3$ 350. $x_1^3x_2x_3^7x_4^{11}x_5^{22}$ 351. $x_1^3x_2^7x_3x_4^{11}x_5^{22}$ 352. $x_1^3x_2^7x_3^{11}x_4x_5^{22}$

353. $x_1^{7}x_2x_3^{3}x_4^{11}x_5^{22}$ 354. $x_1^{7}x_2x_3^{11}x_4^{3}x_5^{22}$ 355. $x_1^{7}x_2x_3^{11}x_4^{22}x_5^{3}$ 356. $x_1^{7}x_2^{3}x_3x_4^{11}x_5^{22}$
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593. $x_1^{31}x_2^{3}x_3^{3}x_4^{4}x_5^{3}$ 594. $x_1^{3}x_2^{3}x_3^{3}x_4^{5}x_5^{30}$ 595. $x_1^{3}x_2^{3}x_3^{5}x_4^{3}x_5^{30}$ 596. $x_1^{3}x_2^{3}x_3^{5}x_4^{30}x_5^{3}$
597. $x_1^{3}x_2^{5}x_3^{3}x_4^{3}x_5^{30}$ 598. $x_1^{3}x_2^{5}x_3^{3}x_4^{30}x_5^{3}$ 599. $x_1^{3}x_2^{5}x_3^{30}x_4^{3}x_5^{3}$ 600. $x_1^{3}x_2^{3}x_3^{3}x_4^{29}x_5^{6}$
601. $x_1^{3}x_2^{3}x_3^{29}x_4^{3}x_5^{6}$ 602. $x_1^{3}x_2^{3}x_3^{29}x_4^{6}x_5^{3}$ 603. $x_1^{3}x_2^{29}x_3^{3}x_4^{3}x_5^{6}$ 604. $x_1^{3}x_2^{29}x_3^{3}x_4^{6}x_5^{3}$
605. $x_1^{3}x_2^{29}x_3^{6}x_4^{3}x_5^{3}$ 606. $x_1^{3}x_2^{3}x_3^{3}x_4^{7}x_5^{28}$ 607. $x_1^{3}x_2^{3}x_3^{3}x_4^{28}x_5^{7}$ 608. $x_1^{3}x_2^{3}x_3^{7}x_4^{3}x_5^{28}$
609. $x_1^{3}x_2^{3}x_3^{7}x_4^{28}x_5^{3}$ 610. $x_1^{3}x_2^{3}x_3^{28}x_4^{3}x_5^{7}$ 611. $x_1^{3}x_2^{3}x_3^{28}x_4^{7}x_5^{3}$ 612. $x_1^{3}x_2^{7}x_3^{3}x_4^{3}x_5^{28}$
613. $x_1^{3}x_2^{7}x_3^{3}x_4^{28}x_5^{3}$ 614. $x_1^{7}x_2^{3}x_3^{3}x_4^{3}x_5^{28}$ 615. $x_1^{7}x_2^{3}x_3^{3}x_4^{28}x_5^{3}$ 616. $x_1^{3}x_2^{3}x_3^{3}x_4^{12}x_5^{23}$
617. $x_1^{3}x_2^{3}x_3^{12}x_4^{3}x_5^{23}$ 618. $x_1^{3}x_2^{3}x_3^{12}x_4^{23}x_5^{3}$ 619. $x_1^{3}x_2^{3}x_3^{3}x_4^{13}x_5^{22}$ 620. $x_1^{3}x_2^{3}x_3^{13}x_4^{3}x_5^{22}$
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625. $x_1^{3}x_2^{3}x_3^{3}x_4^{15}x_5^{20}$ 626. $x_1^{3}x_2^{3}x_3^{15}x_4^{3}x_5^{20}$ 627. $x_1^{3}x_2^{3}x_3^{15}x_4^{20}x_5^{3}$ 628. $x_1^{3}x_2^{15}x_3^{3}x_4^{3}x_5^{20}$
629. $x_1^{3}x_2^{15}x_3^{3}x_4^{20}x_5^{3}$ 630. $x_1^{15}x_2^{3}x_3^{3}x_4^{3}x_5^{20}$ 631. $x_1^{15}x_2^{3}x_3^{3}x_4^{20}x_5^{3}$ 632. $x_1^{3}x_2^{3}x_3^{4}x_4^{7}x_5^{27}$
633. $x_1^{3}x_2^{3}x_3^{4}x_4^{27}x_5^{7}$ 634. $x_1^{3}x_2^{3}x_3^{7}x_4^{4}x_5^{27}$ 635. $x_1^{3}x_2^{3}x_3^{7}x_4^{27}x_5^{4}$ 636. $x_1^{3}x_2^{7}x_3^{3}x_4^{4}x_5^{27}$
637. $x_1^{3}x_2^{7}x_3^{3}x_4^{27}x_5^{4}$ 638. $x_1^{3}x_2^{7}x_3^{27}x_4^{3}x_5^{4}$ 639. $x_1^{3}x_2^{7}x_3^{27}x_4^{4}x_5^{3}$ 640. $x_1^{7}x_2^{3}x_3^{3}x_4^{4}x_5^{27}$
641. $x_1^{7}x_2^{3}x_3^{3}x_4^{27}x_5^{4}$ 642. $x_1^{7}x_2^{3}x_3^{27}x_4^{3}x_5^{4}$ 643. $x_1^{7}x_2^{3}x_3^{27}x_4^{4}x_5^{3}$ 644. $x_1^{7}x_2^{27}x_3^{3}x_4^{3}x_5^{4}$
645. $x_1^{7}x_2^{27}x_3^{3}x_4^{4}x_5^{3}$ 646. $x_1^{3}x_2^{3}x_3^{4}x_4^{11}x_5^{23}$ 647. $x_1^{3}x_2^{3}x_3^{4}x_4^{15}x_5^{19}$ 648. $x_1^{3}x_2^{3}x_3^{15}x_4^{4}x_5^{19}$
649. $x_1^{3}x_2^{3}x_3^{15}x_4^{19}x_5^{4}$ 650. $x_1^{3}x_2^{15}x_3^{3}x_4^{4}x_5^{19}$ 651. $x_1^{3}x_2^{15}x_3^{3}x_4^{19}x_5^{4}$ 652. $x_1^{3}x_2^{15}x_3^{19}x_4^{3}x_5^{4}$
653. $x_1^{15}x_2^{3}x_3^{3}x_4^{4}x_5^{19}$ 654. $x_1^{15}x_2^{3}x_3^{3}x_4^{19}x_5^{4}$ 655. $x_1^{15}x_2^{3}x_3^{19}x_4^{3}x_5^{4}$ 656. $x_1^{15}x_2^{19}x_3^{3}x_4^{3}x_5^{4}$
657. $x_1^{3}x_2^{3}x_3^{5}x_4^{6}x_5^{27}$ 658. $x_1^{3}x_2^{3}x_3^{5}x_4^{27}x_5^{6}$ 659. $x_1^{3}x_2^{5}x_3^{3}x_4^{6}x_5^{27}$ 660. $x_1^{3}x_2^{5}x_3^{3}x_4^{27}x_5^{6}$
661. $x_1^{3}x_2^{5}x_3^{6}x_4^{3}x_5^{27}$ 662. $x_1^{3}x_2^{5}x_3^{6}x_4^{27}x_5^{3}$ 663. $x_1^{3}x_2^{5}x_3^{27}x_4^{3}x_5^{6}$ 664. $x_1^{3}x_2^{5}x_3^{27}x_4^{6}x_5^{3}$
665. $x_1^{3}x_2^{3}x_3^{5}x_4^{7}x_5^{26}$ 666. $x_1^{3}x_2^{3}x_3^{5}x_4^{26}x_5^{7}$ 667. $x_1^{3}x_2^{3}x_3^{7}x_4^{5}x_5^{26}$ 668. $x_1^{3}x_2^{5}x_3^{3}x_4^{7}x_5^{26}$
669. $x_1^{3}x_2^{5}x_3^{3}x_4^{26}x_5^{7}$ 670. $x_1^{3}x_2^{5}x_3^{7}x_4^{3}x_5^{26}$ 671. $x_1^{3}x_2^{5}x_3^{7}x_4^{26}x_5^{3}$ 672. $x_1^{3}x_2^{5}x_3^{26}x_4^{3}x_5^{7}$
673. $x_1^{3}x_2^{5}x_3^{26}x_4^{7}x_5^{3}$ 674. $x_1^{3}x_2^{7}x_3^{3}x_4^{5}x_5^{26}$ 675. $x_1^{3}x_2^{7}x_3^{5}x_4^{3}x_5^{26}$ 676. $x_1^{3}x_2^{7}x_3^{5}x_4^{26}x_5^{3}$
677. $x_1^{7}x_2^{3}x_3^{3}x_4^{5}x_5^{26}$ 678. $x_1^{7}x_2^{3}x_3^{5}x_4^{3}x_5^{26}$ 679. $x_1^{7}x_2^{3}x_3^{5}x_4^{26}x_5^{3}$ 680. $x_1^{3}x_2^{3}x_3^{5}x_4^{10}x_5^{23}$
681. $x_1^{3}x_2^{5}x_3^{3}x_4^{10}x_5^{23}$ 682. $x_1^{3}x_2^{5}x_3^{10}x_4^{3}x_5^{23}$ 683. $x_1^{3}x_2^{5}x_3^{10}x_4^{23}x_5^{3}$ 684. $x_1^{3}x_2^{3}x_3^{5}x_4^{11}x_5^{22}$
685. $x_1^{3}x_2^{5}x_3^{3}x_4^{11}x_5^{22}$ 686. $x_1^{3}x_2^{5}x_3^{11}x_4^{3}x_5^{22}$ 687. $x_1^{3}x_2^{5}x_3^{11}x_4^{22}x_5^{3}$ 688. $x_1^{3}x_2^{3}x_3^{5}x_4^{14}x_5^{19}$
689. $x_1^{3}x_2^{5}x_3^{3}x_4^{14}x_5^{19}$ 690. $x_1^{3}x_2^{5}x_3^{14}x_4^{3}x_5^{19}$ 691. $x_1^{3}x_2^{5}x_3^{14}x_4^{19}x_5^{3}$ 692. $x_1^{3}x_2^{3}x_3^{5}x_4^{15}x_5^{18}$
693. $x_1^{3}x_2^{3}x_3^{15}x_4^{5}x_5^{18}$ 694. $x_1^{3}x_2^{5}x_3^{3}x_4^{15}x_5^{18}$ 695. $x_1^{3}x_2^{5}x_3^{15}x_4^{3}x_5^{18}$ 696. $x_1^{3}x_2^{5}x_3^{15}x_4^{18}x_5^{3}$

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|---|---|---|---|
| 697. $x_1^{3}x_2^{15}x_3^{3}x_4^{5}x_5^{18}$ | 698. $x_1^{3}x_2^{15}x_3^{5}x_4^{3}x_5^{18}$ | 699. $x_1^{3}x_2^{15}x_3^{5}x_4^{18}x_5^{3}$ | 700. $x_1^{15}x_2^{3}x_3^{3}x_4^{5}x_5^{18}$ |
| 701. $x_1^{15}x_2^{3}x_3^{5}x_4^{3}x_5^{18}$ | 702. $x_1^{15}x_2^{3}x_3^{5}x_4^{18}x_5^{3}$ | 703. $x_1^{3}x_2^{3}x_3^{7}x_4^{25}x_5^{6}$ | 704. $x_1^{3}x_2^{7}x_3^{3}x_4^{25}x_5^{6}$ |
| 705. $x_1^{3}x_2^{7}x_3^{25}x_4^{3}x_5^{6}$ | 706. $x_1^{3}x_2^{7}x_3^{25}x_4^{6}x_5^{3}$ | 707. $x_1^{7}x_2^{3}x_3^{3}x_4^{25}x_5^{6}$ | 708. $x_1^{7}x_2^{3}x_3^{25}x_4^{3}x_5^{6}$ |
| 709. $x_1^{7}x_2^{3}x_3^{25}x_4^{6}x_5^{3}$ | 710. $x_1^{3}x_2^{3}x_3^{13}x_4^{6}x_5^{19}$ | 711. $x_1^{3}x_2^{3}x_3^{13}x_4^{19}x_5^{6}$ | 712. $x_1^{3}x_2^{13}x_3^{3}x_4^{6}x_5^{19}$ |
| 713. $x_1^{3}x_2^{13}x_3^{3}x_4^{19}x_5^{6}$ | 714. $x_1^{3}x_2^{13}x_3^{6}x_4^{3}x_5^{19}$ | 715. $x_1^{3}x_2^{13}x_3^{6}x_4^{19}x_5^{3}$ | 716. $x_1^{3}x_2^{13}x_3^{19}x_4^{3}x_5^{6}$ |
| 717. $x_1^{3}x_2^{13}x_3^{19}x_4^{6}x_5^{3}$ | 718. $x_1^{3}x_2^{3}x_3^{15}x_4^{17}x_5^{6}$ | 719. $x_1^{3}x_2^{15}x_3^{3}x_4^{17}x_5^{6}$ | 720. $x_1^{3}x_2^{15}x_3^{17}x_4^{3}x_5^{6}$ |
| 721. $x_1^{3}x_2^{15}x_3^{17}x_4^{6}x_5^{3}$ | 722. $x_1^{15}x_2^{3}x_3^{3}x_4^{17}x_5^{6}$ | 723. $x_1^{15}x_2^{3}x_3^{17}x_4^{3}x_5^{6}$ | 724. $x_1^{15}x_2^{3}x_3^{17}x_4^{6}x_5^{3}$ |
| 725. $x_1^{3}x_2^{3}x_3^{7}x_4^{7}x_5^{24}$ | 726. $x_1^{3}x_2^{3}x_3^{7}x_4^{24}x_5^{7}$ | 727. $x_1^{3}x_2^{7}x_3^{3}x_4^{7}x_5^{24}$ | 728. $x_1^{3}x_2^{7}x_3^{3}x_4^{24}x_5^{7}$ |
| 729. $x_1^{3}x_2^{7}x_3^{7}x_4^{3}x_5^{24}$ | 730. $x_1^{3}x_2^{7}x_3^{7}x_4^{24}x_5^{3}$ | 731. $x_1^{7}x_2^{3}x_3^{3}x_4^{7}x_5^{24}$ | 732. $x_1^{7}x_2^{3}x_3^{3}x_4^{24}x_5^{7}$ |
| 733. $x_1^{7}x_2^{3}x_3^{7}x_4^{3}x_5^{24}$ | 734. $x_1^{7}x_2^{3}x_3^{7}x_4^{24}x_5^{3}$ | 735. $x_1^{7}x_2^{7}x_3^{3}x_4^{3}x_5^{24}$ | 736. $x_1^{7}x_2^{7}x_3^{3}x_4^{24}x_5^{3}$ |
| 737. $x_1^{7}x_2^{7}x_3^{24}x_4^{3}x_5^{3}$ | 738. $x_1^{3}x_2^{3}x_3^{7}x_4^{8}x_5^{23}$ | 739. $x_1^{3}x_2^{7}x_3^{3}x_4^{8}x_5^{23}$ | 740. $x_1^{3}x_2^{7}x_3^{8}x_4^{3}x_5^{23}$ |
| 741. $x_1^{3}x_2^{7}x_3^{8}x_4^{23}x_5^{3}$ | 742. $x_1^{7}x_2^{3}x_3^{3}x_4^{8}x_5^{23}$ | 743. $x_1^{7}x_2^{3}x_3^{8}x_4^{3}x_5^{23}$ | 744. $x_1^{7}x_2^{3}x_3^{8}x_4^{23}x_5^{3}$ |
| 745. $x_1^{3}x_2^{3}x_3^{7}x_4^{9}x_5^{22}$ | 746. $x_1^{3}x_2^{7}x_3^{3}x_4^{9}x_5^{22}$ | 747. $x_1^{3}x_2^{7}x_3^{9}x_4^{3}x_5^{22}$ | 748. $x_1^{3}x_2^{7}x_3^{9}x_4^{22}x_5^{3}$ |
| 749. $x_1^{7}x_2^{3}x_3^{3}x_4^{9}x_5^{22}$ | 750. $x_1^{7}x_2^{3}x_3^{9}x_4^{3}x_5^{22}$ | 751. $x_1^{7}x_2^{3}x_3^{9}x_4^{22}x_5^{3}$ | 752. $x_1^{7}x_2^{9}x_3^{3}x_4^{3}x_5^{22}$ |
| 753. $x_1^{7}x_2^{9}x_3^{3}x_4^{22}x_5^{3}$ | 754. $x_1^{3}x_2^{3}x_3^{7}x_4^{11}x_5^{20}$ | 755. $x_1^{3}x_2^{7}x_3^{3}x_4^{11}x_5^{20}$ | 756. $x_1^{3}x_2^{7}x_3^{11}x_4^{3}x_5^{20}$ |
| 757. $x_1^{3}x_2^{7}x_3^{11}x_4^{20}x_5^{3}$ | 758. $x_1^{7}x_2^{3}x_3^{3}x_4^{11}x_5^{20}$ | 759. $x_1^{7}x_2^{3}x_3^{11}x_4^{3}x_5^{20}$ | 760. $x_1^{7}x_2^{3}x_3^{11}x_4^{20}x_5^{3}$ |
| 761. $x_1^{7}x_2^{11}x_3^{3}x_4^{3}x_5^{20}$ | 762. $x_1^{7}x_2^{11}x_3^{3}x_4^{20}x_5^{3}$ | 763. $x_1^{3}x_2^{3}x_3^{7}x_4^{12}x_5^{19}$ | 764. $x_1^{3}x_2^{3}x_3^{12}x_4^{7}x_5^{19}$ |
| 765. $x_1^{3}x_2^{3}x_3^{12}x_4^{19}x_5^{7}$ | 766. $x_1^{3}x_2^{7}x_3^{3}x_4^{12}x_5^{19}$ | 767. $x_1^{7}x_2^{3}x_3^{3}x_4^{12}x_5^{19}$ | 768. $x_1^{3}x_2^{3}x_3^{7}x_4^{13}x_5^{18}$ |
| 769. $x_1^{3}x_2^{3}x_3^{13}x_4^{7}x_5^{18}$ | 770. $x_1^{3}x_2^{3}x_3^{13}x_4^{18}x_5^{7}$ | 771. $x_1^{3}x_2^{7}x_3^{3}x_4^{13}x_5^{18}$ | 772. $x_1^{3}x_2^{7}x_3^{13}x_4^{3}x_5^{18}$ |
| 773. $x_1^{3}x_2^{7}x_3^{13}x_4^{18}x_5^{3}$ | 774. $x_1^{3}x_2^{13}x_3^{3}x_4^{7}x_5^{18}$ | 775. $x_1^{3}x_2^{13}x_3^{3}x_4^{18}x_5^{7}$ | 776. $x_1^{3}x_2^{13}x_3^{7}x_4^{3}x_5^{18}$ |
| 777. $x_1^{3}x_2^{13}x_3^{7}x_4^{18}x_5^{3}$ | 778. $x_1^{3}x_2^{13}x_3^{18}x_4^{3}x_5^{7}$ | 779. $x_1^{3}x_2^{13}x_3^{18}x_4^{7}x_5^{3}$ | 780. $x_1^{7}x_2^{3}x_3^{3}x_4^{13}x_5^{18}$ |
| 781. $x_1^{7}x_2^{3}x_3^{13}x_4^{3}x_5^{18}$ | 782. $x_1^{7}x_2^{3}x_3^{13}x_4^{18}x_5^{3}$ | 783. $x_1^{3}x_2^{3}x_3^{7}x_4^{15}x_5^{16}$ | 784. $x_1^{3}x_2^{3}x_3^{15}x_4^{7}x_5^{16}$ |
| 785. $x_1^{3}x_2^{3}x_3^{15}x_4^{16}x_5^{7}$ | 786. $x_1^{3}x_2^{7}x_3^{3}x_4^{15}x_5^{16}$ | 787. $x_1^{3}x_2^{7}x_3^{15}x_4^{3}x_5^{16}$ | 788. $x_1^{3}x_2^{7}x_3^{15}x_4^{16}x_5^{3}$ |
| 789. $x_1^{3}x_2^{15}x_3^{3}x_4^{7}x_5^{16}$ | 790. $x_1^{3}x_2^{15}x_3^{3}x_4^{16}x_5^{7}$ | 791. $x_1^{3}x_2^{15}x_3^{7}x_4^{3}x_5^{16}$ | 792. $x_1^{3}x_2^{15}x_3^{7}x_4^{16}x_5^{3}$ |
| 793. $x_1^{7}x_2^{3}x_3^{3}x_4^{15}x_5^{16}$ | 794. $x_1^{7}x_2^{3}x_3^{15}x_4^{3}x_5^{16}$ | 795. $x_1^{7}x_2^{3}x_3^{15}x_4^{16}x_5^{3}$ | 796. $x_1^{7}x_2^{15}x_3^{3}x_4^{3}x_5^{16}$ |
| 797. $x_1^{7}x_2^{15}x_3^{3}x_4^{16}x_5^{3}$ | 798. $x_1^{15}x_2^{3}x_3^{3}x_4^{7}x_5^{16}$ | 799. $x_1^{15}x_2^{3}x_3^{3}x_4^{16}x_5^{7}$ | 800. $x_1^{15}x_2^{3}x_3^{7}x_4^{3}x_5^{16}$ |
| 801. $x_1^{15}x_2^{3}x_3^{7}x_4^{16}x_5^{3}$ | 802. $x_1^{15}x_2^{7}x_3^{3}x_4^{3}x_5^{16}$ | 803. $x_1^{15}x_2^{7}x_3^{3}x_4^{16}x_5^{3}$ | 804. $x_1^{3}x_2^{7}x_3^{11}x_4^{4}x_5^{19}$ |
| 805. $x_1^{3}x_2^{7}x_3^{11}x_4^{19}x_5^{4}$ | 806. $x_1^{7}x_2^{3}x_3^{11}x_4^{4}x_5^{19}$ | 807. $x_1^{7}x_2^{3}x_3^{11}x_4^{19}x_5^{4}$ | 808. $x_1^{7}x_2^{11}x_3^{3}x_4^{4}x_5^{19}$ |
| 809. $x_1^{7}x_2^{11}x_3^{3}x_4^{19}x_5^{4}$ | 810. $x_1^{7}x_2^{11}x_3^{19}x_4^{3}x_5^{4}$ | 811. $x_1^{3}x_2^{5}x_3^{6}x_4^{11}x_5^{19}$ | 812. $x_1^{3}x_2^{5}x_3^{11}x_4^{6}x_5^{19}$ |
| 813. $x_1^{3}x_2^{5}x_3^{11}x_4^{19}x_5^{6}$ | 814. $x_1^{3}x_2^{5}x_3^{7}x_4^{10}x_5^{19}$ | 815. $x_1^{3}x_2^{5}x_3^{10}x_4^{7}x_5^{19}$ | 816. $x_1^{3}x_2^{5}x_3^{10}x_4^{19}x_5^{7}$ |
| 817. $x_1^{3}x_2^{7}x_3^{5}x_4^{10}x_5^{19}$ | 818. $x_1^{7}x_2^{3}x_3^{5}x_4^{10}x_5^{19}$ | 819. $x_1^{3}x_2^{5}x_3^{7}x_4^{11}x_5^{18}$ | 820. $x_1^{3}x_2^{5}x_3^{11}x_4^{7}x_5^{18}$ |
| 821. $x_1^{3}x_2^{5}x_3^{11}x_4^{18}x_5^{7}$ | 822. $x_1^{3}x_2^{7}x_3^{5}x_4^{11}x_5^{18}$ | 823. $x_1^{3}x_2^{7}x_3^{11}x_4^{5}x_5^{18}$ | 824. $x_1^{7}x_2^{3}x_3^{5}x_4^{11}x_5^{18}$ |
| 825. $x_1^{7}x_2^{3}x_3^{11}x_4^{5}x_5^{18}$ | 826. $x_1^{7}x_2^{11}x_3^{3}x_4^{5}x_5^{18}$ | 827. $x_1^{7}x_2^{11}x_3^{5}x_4^{3}x_5^{18}$ | 828. $x_1^{7}x_2^{11}x_3^{5}x_4^{18}x_5^{3}$ |
| 829. $x_1^{3}x_2^{7}x_3^{9}x_4^{6}x_5^{19}$ | 830. $x_1^{3}x_2^{7}x_3^{9}x_4^{19}x_5^{6}$ | 831. $x_1^{7}x_2^{3}x_3^{9}x_4^{6}x_5^{19}$ | 832. $x_1^{7}x_2^{3}x_3^{9}x_4^{19}x_5^{6}$ |
| 833. $x_1^{7}x_2^{9}x_3^{3}x_4^{6}x_5^{19}$ | 834. $x_1^{3}x_2^{7}x_3^{11}x_4^{17}x_5^{6}$ | 835. $x_1^{7}x_2^{3}x_3^{11}x_4^{17}x_5^{6}$ | 836. $x_1^{7}x_2^{11}x_3^{3}x_4^{17}x_5^{6}$ |
| 837. $x_1^{7}x_2^{11}x_3^{17}x_4^{3}x_5^{6}$ | 838. $x_1^{7}x_2^{11}x_3^{17}x_4^{6}x_5^{3}$ | 839. $x_1^{3}x_2^{7}x_3^{7}x_4^{8}x_5^{19}$ | 840. $x_1^{3}x_2^{7}x_3^{8}x_4^{7}x_5^{19}$ |
| 841. $x_1^{7}x_2^{3}x_3^{7}x_4^{8}x_5^{19}$ | 842. $x_1^{7}x_2^{3}x_3^{8}x_4^{7}x_5^{19}$ | 843. $x_1^{7}x_2^{7}x_3^{3}x_4^{8}x_5^{19}$ | 844. $x_1^{7}x_2^{7}x_3^{8}x_4^{3}x_5^{19}$ |
| 845. $x_1^{7}x_2^{7}x_3^{8}x_4^{19}x_5^{3}$ | 846. $x_1^{3}x_2^{7}x_3^{7}x_4^{9}x_5^{18}$ | 847. $x_1^{3}x_2^{7}x_3^{9}x_4^{7}x_5^{18}$ | 848. $x_1^{3}x_2^{7}x_3^{9}x_4^{18}x_5^{7}$ |
| 849. $x_1^{7}x_2^{3}x_3^{7}x_4^{9}x_5^{18}$ | 850. $x_1^{7}x_2^{3}x_3^{9}x_4^{7}x_5^{18}$ | 851. $x_1^{7}x_2^{3}x_3^{9}x_4^{18}x_5^{7}$ | 852. $x_1^{7}x_2^{7}x_3^{3}x_4^{9}x_5^{18}$ |
| 853. $x_1^{7}x_2^{7}x_3^{9}x_4^{3}x_5^{18}$ | 854. $x_1^{7}x_2^{7}x_3^{9}x_4^{18}x_5^{3}$ | 855. $x_1^{7}x_2^{9}x_3^{3}x_4^{7}x_5^{18}$ | 856. $x_1^{7}x_2^{9}x_3^{7}x_4^{3}x_5^{18}$ |
| 857. $x_1^{7}x_2^{9}x_3^{7}x_4^{18}x_5^{3}$ | 858. $x_1^{3}x_2^{7}x_3^{7}x_4^{11}x_5^{16}$ | 859. $x_1^{3}x_2^{7}x_3^{11}x_4^{7}x_5^{16}$ | 860. $x_1^{3}x_2^{7}x_3^{11}x_4^{16}x_5^{7}$ |
| 861. $x_1^{7}x_2^{3}x_3^{7}x_4^{11}x_5^{16}$ | 862. $x_1^{7}x_2^{3}x_3^{11}x_4^{7}x_5^{16}$ | 863. $x_1^{7}x_2^{3}x_3^{11}x_4^{16}x_5^{7}$ | 864. $x_1^{7}x_2^{7}x_3^{3}x_4^{11}x_5^{16}$ |
| 865. $x_1^{7}x_2^{7}x_3^{11}x_4^{3}x_5^{16}$ | 866. $x_1^{7}x_2^{7}x_3^{11}x_4^{16}x_5^{3}$ | 867. $x_1^{7}x_2^{11}x_3^{3}x_4^{7}x_5^{16}$ | 868. $x_1^{7}x_2^{11}x_3^{3}x_4^{16}x_5^{7}$ |
| 869. $x_1^{7}x_2^{11}x_3^{7}x_4^{3}x_5^{16}$ | 870. $x_1^{7}x_2^{11}x_3^{7}x_4^{16}x_5^{3}$ | | |

$B_5^+((4)|^2|(2)|(3)) = \{\bar{b}_t : 1 \leqslant t \leqslant 90\}$, where

| | | | |
|---|---|---|---|
| 1. $x_1x_2^{6}x_3^{11}x_4^{11}x_5^{15}$ | 2. $x_1x_2^{6}x_3^{11}x_4^{15}x_5^{11}$ | 3. $x_1x_2^{6}x_3^{15}x_4^{11}x_5^{11}$ | 4. $x_1x_2^{7}x_3^{10}x_4^{11}x_5^{15}$ |
| 5. $x_1x_2^{7}x_3^{10}x_4^{15}x_5^{11}$ | 6. $x_1x_2^{7}x_3^{11}x_4^{10}x_5^{15}$ | 7. $x_1x_2^{7}x_3^{11}x_4^{11}x_5^{14}$ | 8. $x_1x_2^{7}x_3^{11}x_4^{14}x_5^{11}$ |
| 9. $x_1x_2^{7}x_3^{11}x_4^{15}x_5^{10}$ | 10. $x_1x_2^{7}x_3^{14}x_4^{11}x_5^{11}$ | 11. $x_1x_2^{7}x_3^{15}x_4^{10}x_5^{11}$ | 12. $x_1x_2^{7}x_3^{15}x_4^{11}x_5^{10}$ |
| 13. $x_1x_2^{14}x_3^{7}x_4^{11}x_5^{11}$ | 14. $x_1x_2^{15}x_3^{6}x_4^{11}x_5^{11}$ | 15. $x_1x_2^{15}x_3^{7}x_4^{10}x_5^{11}$ | 16. $x_1x_2^{15}x_3^{7}x_4^{11}x_5^{10}$ |
| 17. $x_1^{3}x_2^{5}x_3^{10}x_4^{11}x_5^{15}$ | 18. $x_1^{3}x_2^{5}x_3^{10}x_4^{15}x_5^{11}$ | 19. $x_1^{3}x_2^{5}x_3^{11}x_4^{10}x_5^{15}$ | 20. $x_1^{3}x_2^{5}x_3^{11}x_4^{11}x_5^{14}$ |
| 21. $x_1^{3}x_2^{5}x_3^{11}x_4^{14}x_5^{11}$ | 22. $x_1^{3}x_2^{5}x_3^{11}x_4^{15}x_5^{10}$ | 23. $x_1^{3}x_2^{5}x_3^{14}x_4^{11}x_5^{11}$ | 24. $x_1^{3}x_2^{5}x_3^{15}x_4^{10}x_5^{11}$ |
| 25. $x_1^{3}x_2^{5}x_3^{15}x_4^{11}x_5^{10}$ | 26. $x_1^{3}x_2^{7}x_3^{9}x_4^{10}x_5^{15}$ | 27. $x_1^{3}x_2^{7}x_3^{9}x_4^{11}x_5^{14}$ | 28. $x_1^{3}x_2^{7}x_3^{9}x_4^{14}x_5^{11}$ |
| 29. $x_1^{3}x_2^{7}x_3^{9}x_4^{15}x_5^{10}$ | 30. $x_1^{3}x_2^{7}x_3^{11}x_4^{9}x_5^{14}$ | 31. $x_1^{3}x_2^{7}x_3^{11}x_4^{13}x_5^{10}$ | 32. $x_1^{3}x_2^{7}x_3^{13}x_4^{10}x_5^{11}$ |
| 33. $x_1^{3}x_2^{7}x_3^{13}x_4^{11}x_5^{10}$ | 34. $x_1^{3}x_2^{7}x_3^{15}x_4^{9}x_5^{10}$ | 35. $x_1^{3}x_2^{13}x_3^{6}x_4^{11}x_5^{11}$ | 36. $x_1^{3}x_2^{13}x_3^{7}x_4^{10}x_5^{11}$ |
| 37. $x_1^{3}x_2^{13}x_3^{7}x_4^{11}x_5^{10}$ | 38. $x_1^{3}x_2^{15}x_3^{5}x_4^{10}x_5^{11}$ | 39. $x_1^{3}x_2^{15}x_3^{5}x_4^{11}x_5^{10}$ | 40. $x_1^{3}x_2^{15}x_3^{7}x_4^{9}x_5^{10}$ |
| 41. $x_1^{7}x_2x_3^{10}x_4^{11}x_5^{15}$ | 42. $x_1^{7}x_2x_3^{10}x_4^{15}x_5^{11}$ | 43. $x_1^{7}x_2x_3^{11}x_4^{10}x_5^{15}$ | 44. $x_1^{7}x_2x_3^{11}x_4^{11}x_5^{14}$ |
| 45. $x_1^{7}x_2x_3^{11}x_4^{14}x_5^{11}$ | 46. $x_1^{7}x_2x_3^{11}x_4^{15}x_5^{10}$ | 47. $x_1^{7}x_2x_3^{14}x_4^{11}x_5^{11}$ | 48. $x_1^{7}x_2x_3^{15}x_4^{10}x_5^{11}$ |
| 49. $x_1^{7}x_2x_3^{15}x_4^{11}x_5^{10}$ | 50. $x_1^{7}x_2^{3}x_3^{9}x_4^{10}x_5^{15}$ | 51. $x_1^{7}x_2^{3}x_3^{9}x_4^{11}x_5^{14}$ | 52. $x_1^{7}x_2^{3}x_3^{9}x_4^{14}x_5^{11}$ |
| 53. $x_1^{7}x_2^{3}x_3^{9}x_4^{15}x_5^{10}$ | 54. $x_1^{7}x_2^{3}x_3^{11}x_4^{9}x_5^{14}$ | 55. $x_1^{7}x_2^{3}x_3^{11}x_4^{13}x_5^{10}$ | 56. $x_1^{7}x_2^{3}x_3^{13}x_4^{10}x_5^{11}$ |
| 57. $x_1^{7}x_2^{3}x_3^{13}x_4^{11}x_5^{10}$ | 58. $x_1^{7}x_2^{3}x_3^{15}x_4^{9}x_5^{10}$ | 59. $x_1^{7}x_2^{7}x_3^{8}x_4^{11}x_5^{11}$ | 60. $x_1^{7}x_2^{7}x_3^{9}x_4^{10}x_5^{11}$ |
| 61. $x_1^{7}x_2^{7}x_3^{9}x_4^{11}x_5^{10}$ | 62. $x_1^{7}x_2^{7}x_3^{11}x_4^{8}x_5^{11}$ | 63. $x_1^{7}x_2^{7}x_3^{11}x_4^{9}x_5^{10}$ | 64. $x_1^{7}x_2^{7}x_3^{11}x_4^{11}x_5^{8}$ |
| 65. $x_1^{7}x_2^{11}x_3x_4^{10}x_5^{15}$ | 66. $x_1^{7}x_2^{11}x_3x_4^{11}x_5^{14}$ | 67. $x_1^{7}x_2^{11}x_3x_4^{14}x_5^{11}$ | 68. $x_1^{7}x_2^{11}x_3x_4^{15}x_5^{10}$ |
| 69. $x_1^{7}x_2^{11}x_3^{3}x_4^{9}x_5^{14}$ | 70. $x_1^{7}x_2^{11}x_3^{3}x_4^{13}x_5^{10}$ | 71. $x_1^{7}x_2^{11}x_3^{5}x_4^{10}x_5^{11}$ | 72. $x_1^{7}x_2^{11}x_3^{5}x_4^{11}x_5^{10}$ |
| 73. $x_1^{7}x_2^{11}x_3^{7}x_4^{9}x_5^{10}$ | 74. $x_1^{7}x_2^{11}x_3^{11}x_4x_5^{14}$ | 75. $x_1^{7}x_2^{11}x_3^{11}x_4^{7}x_5^{8}$ | 76. $x_1^{7}x_2^{11}x_3^{15}x_4x_5^{10}$ |
| 77. $x_1^{7}x_2^{15}x_3x_4^{10}x_5^{11}$ | 78. $x_1^{7}x_2^{15}x_3x_4^{11}x_5^{10}$ | 79. $x_1^{7}x_2^{15}x_3^{3}x_4^{9}x_5^{10}$ | 80. $x_1^{7}x_2^{15}x_3^{11}x_4x_5^{10}$ |
| 81. $x_1^{15}x_2x_3^{6}x_4^{11}x_5^{11}$ | 82. $x_1^{15}x_2x_3^{7}x_4^{10}x_5^{11}$ | 83. $x_1^{15}x_2x_3^{7}x_4^{11}x_5^{10}$ | 84. $x_1^{15}x_2^{3}x_3^{5}x_4^{10}x_5^{11}$ |
| 85. $x_1^{15}x_2^{3}x_3^{5}x_4^{11}x_5^{10}$ | 86. $x_1^{15}x_2^{3}x_3^{7}x_4^{9}x_5^{10}$ | 87. $x_1^{15}x_2^{7}x_3x_4^{10}x_5^{11}$ | 88. $x_1^{15}x_2^{7}x_3x_4^{11}x_5^{10}$ |
| 89. $x_1^{15}x_2^{7}x_3^{3}x_4^{9}x_5^{10}$ | 90. $x_1^{15}x_2^{7}x_3^{11}x_4x_5^{10}$ | | |

$B_5^+((4)|^3|(2)) = \{\tilde{b}_t : 1 \leqslant t \leqslant 146\}$, where

| | | | |
|---|---|---|---|
| 1. $x_1x_2^{6}x_3^{7}x_4^{15}x_5^{15}$ | 2. $x_1x_2^{6}x_3^{15}x_4^{7}x_5^{15}$ | 3. $x_1x_2^{6}x_3^{15}x_4^{15}x_5^{7}$ | 4. $x_1x_2^{7}x_3^{6}x_4^{15}x_5^{15}$ |
| 5. $x_1x_2^{7}x_3^{7}x_4^{14}x_5^{15}$ | 6. $x_1x_2^{7}x_3^{7}x_4^{15}x_5^{14}$ | 7. $x_1x_2^{7}x_3^{14}x_4^{7}x_5^{15}$ | 8. $x_1x_2^{7}x_3^{14}x_4^{15}x_5^{7}$ |
| 9. $x_1x_2^{7}x_3^{15}x_4^{6}x_5^{15}$ | 10. $x_1x_2^{7}x_3^{15}x_4^{7}x_5^{14}$ | 11. $x_1x_2^{7}x_3^{15}x_4^{14}x_5^{7}$ | 12. $x_1x_2^{7}x_3^{15}x_4^{15}x_5^{6}$ |
| 13. $x_1x_2^{14}x_3^{7}x_4^{7}x_5^{15}$ | 14. $x_1x_2^{14}x_3^{7}x_4^{15}x_5^{7}$ | 15. $x_1x_2^{14}x_3^{15}x_4^{7}x_5^{7}$ | 16. $x_1x_2^{15}x_3^{6}x_4^{7}x_5^{15}$ |
| 17. $x_1x_2^{15}x_3^{6}x_4^{15}x_5^{7}$ | 18. $x_1x_2^{15}x_3^{7}x_4^{6}x_5^{15}$ | 19. $x_1x_2^{15}x_3^{7}x_4^{7}x_5^{14}$ | 20. $x_1x_2^{15}x_3^{7}x_4^{14}x_5^{7}$ |
| 21. $x_1x_2^{15}x_3^{7}x_4^{15}x_5^{6}$ | 22. $x_1x_2^{15}x_3^{14}x_4^{7}x_5^{7}$ | 23. $x_1x_2^{15}x_3^{15}x_4^{6}x_5^{7}$ | 24. $x_1x_2^{15}x_3^{15}x_4^{7}x_5^{6}$ |
| 25. $x_1^{3}x_2^{5}x_3^{6}x_4^{15}x_5^{15}$ | 26. $x_1^{3}x_2^{5}x_3^{7}x_4^{14}x_5^{15}$ | 27. $x_1^{3}x_2^{5}x_3^{7}x_4^{15}x_5^{14}$ | 28. $x_1^{3}x_2^{5}x_3^{14}x_4^{7}x_5^{15}$ |
| 29. $x_1^{3}x_2^{5}x_3^{14}x_4^{15}x_5^{7}$ | 30. $x_1^{3}x_2^{5}x_3^{15}x_4^{6}x_5^{15}$ | 31. $x_1^{3}x_2^{5}x_3^{15}x_4^{7}x_5^{14}$ | 32. $x_1^{3}x_2^{5}x_3^{15}x_4^{14}x_5^{7}$ |
| 33. $x_1^{3}x_2^{5}x_3^{15}x_4^{15}x_5^{6}$ | 34. $x_1^{3}x_2^{7}x_3^{5}x_4^{14}x_5^{15}$ | 35. $x_1^{3}x_2^{7}x_3^{5}x_4^{15}x_5^{14}$ | 36. $x_1^{3}x_2^{7}x_3^{7}x_4^{13}x_5^{14}$ |
| 37. $x_1^{3}x_2^{7}x_3^{13}x_4^{6}x_5^{15}$ | 38. $x_1^{3}x_2^{7}x_3^{13}x_4^{7}x_5^{14}$ | 39. $x_1^{3}x_2^{7}x_3^{13}x_4^{14}x_5^{7}$ | 40. $x_1^{3}x_2^{7}x_3^{13}x_4^{15}x_5^{6}$ |
| 41. $x_1^{3}x_2^{7}x_3^{15}x_4^{5}x_5^{14}$ | 42. $x_1^{3}x_2^{7}x_3^{15}x_4^{13}x_5^{6}$ | 43. $x_1^{3}x_2^{13}x_3^{6}x_4^{7}x_5^{15}$ | 44. $x_1^{3}x_2^{13}x_3^{6}x_4^{15}x_5^{7}$ |
| 45. $x_1^{3}x_2^{13}x_3^{7}x_4^{6}x_5^{15}$ | 46. $x_1^{3}x_2^{13}x_3^{7}x_4^{7}x_5^{14}$ | 47. $x_1^{3}x_2^{13}x_3^{7}x_4^{14}x_5^{7}$ | 48. $x_1^{3}x_2^{13}x_3^{7}x_4^{15}x_5^{6}$ |
| 49. $x_1^{3}x_2^{13}x_3^{14}x_4^{7}x_5^{7}$ | 50. $x_1^{3}x_2^{13}x_3^{15}x_4^{6}x_5^{7}$ | 51. $x_1^{3}x_2^{13}x_3^{15}x_4^{7}x_5^{6}$ | 52. $x_1^{3}x_2^{15}x_3^{5}x_4^{6}x_5^{15}$ |
| 53. $x_1^{3}x_2^{15}x_3^{5}x_4^{7}x_5^{14}$ | 54. $x_1^{3}x_2^{15}x_3^{5}x_4^{14}x_5^{7}$ | 55. $x_1^{3}x_2^{15}x_3^{5}x_4^{15}x_5^{6}$ | 56. $x_1^{3}x_2^{15}x_3^{7}x_4^{5}x_5^{14}$ |
| 57. $x_1^{3}x_2^{15}x_3^{7}x_4^{13}x_5^{6}$ | 58. $x_1^{3}x_2^{15}x_3^{13}x_4^{6}x_5^{7}$ | 59. $x_1^{3}x_2^{15}x_3^{13}x_4^{7}x_5^{6}$ | 60. $x_1^{3}x_2^{15}x_3^{15}x_4^{5}x_5^{6}$ |
| 61. $x_1^{7}x_2x_3^{6}x_4^{15}x_5^{15}$ | 62. $x_1^{7}x_2x_3^{7}x_4^{14}x_5^{15}$ | 63. $x_1^{7}x_2x_3^{7}x_4^{15}x_5^{14}$ | 64. $x_1^{7}x_2x_3^{14}x_4^{7}x_5^{15}$ |

| | | | |
|---|---|---|---|
| 65. $x_1^7x_2x_3^{14}x_4^{15}x_5^7$ | 66. $x_1^7x_2x_3^{15}x_4^6x_5^{15}$ | 67. $x_1^7x_2x_3^{15}x_4^7x_5^{14}$ | 68. $x_1^7x_2x_3^{15}x_4^{14}x_5^7$ |
| 69. $x_1^7x_2x_3^{15}x_4^{15}x_5^6$ | 70. $x_1^7x_2^3x_3^5x_4^{14}x_5^{15}$ | 71. $x_1^7x_2^3x_3^5x_4^{15}x_5^{14}$ | 72. $x_1^7x_2^3x_3^7x_4^{13}x_5^{14}$ |
| 73. $x_1^7x_2^3x_3^{13}x_4^6x_5^{15}$ | 74. $x_1^7x_2^3x_3^{13}x_4^7x_5^{14}$ | 75. $x_1^7x_2^3x_3^{13}x_4^{14}x_5^7$ | 76. $x_1^7x_2^3x_3^{13}x_4^{15}x_5^6$ |
| 77. $x_1^7x_2^3x_3^{15}x_4^5x_5^{14}$ | 78. $x_1^7x_2^3x_3^{15}x_4^{13}x_5^6$ | 79. $x_1^7x_2^7x_3x_4^{14}x_5^{15}$ | 80. $x_1^7x_2^7x_3x_4^{15}x_5^{14}$ |
| 81. $x_1^7x_2^7x_3^3x_4^{13}x_5^{14}$ | 82. $x_1^7x_2^7x_3^7x_4^8x_5^{15}$ | 83. $x_1^7x_2^7x_3^7x_4^9x_5^{14}$ | 84. $x_1^7x_2^7x_3^7x_4^{15}x_5^8$ |
| 85. $x_1^7x_2^7x_3^9x_4^6x_5^{15}$ | 86. $x_1^7x_2^7x_3^9x_4^7x_5^{14}$ | 87. $x_1^7x_2^7x_3^9x_4^{14}x_5^7$ | 88. $x_1^7x_2^7x_3^9x_4^{15}x_5^6$ |
| 89. $x_1^7x_2^7x_3^{11}x_4^5x_5^{14}$ | 90. $x_1^7x_2^7x_3^{11}x_4^{13}x_5^6$ | 91. $x_1^7x_2^7x_3^{15}x_4x_5^{14}$ | 92. $x_1^7x_2^7x_3^{15}x_4^7x_5^8$ |
| 93. $x_1^7x_2^7x_3^{15}x_4^9x_5^6$ | 94. $x_1^7x_2^{11}x_3^5x_4^6x_5^{15}$ | 95. $x_1^7x_2^{11}x_3^5x_4^7x_5^{14}$ | 96. $x_1^7x_2^{11}x_3^5x_4^{14}x_5^7$ |
| 97. $x_1^7x_2^{11}x_3^5x_4^{15}x_5^6$ | 98. $x_1^7x_2^{11}x_3^7x_4^5x_5^{14}$ | 99. $x_1^7x_2^{11}x_3^7x_4^{13}x_5^6$ | 100. $x_1^7x_2^{11}x_3^{13}x_4^6x_5^7$ |
| 101. $x_1^7x_2^{11}x_3^{13}x_4^7x_5^6$ | 102. $x_1^7x_2^{11}x_3^{15}x_4^5x_5^6$ | 103. $x_1^7x_2^{15}x_3x_4^6x_5^{15}$ | 104. $x_1^7x_2^{15}x_3x_4^7x_5^{14}$ |
| 105. $x_1^7x_2^{15}x_3x_4^{14}x_5^7$ | 106. $x_1^7x_2^{15}x_3x_4^{15}x_5^6$ | 107. $x_1^7x_2^{15}x_3^3x_4^5x_5^{14}$ | 108. $x_1^7x_2^{15}x_3^3x_4^{13}x_5^6$ |
| 109. $x_1^7x_2^{15}x_3^7x_4x_5^{14}$ | 110. $x_1^7x_2^{15}x_3^7x_4^7x_5^8$ | 111. $x_1^7x_2^{15}x_3^7x_4^9x_5^6$ | 112. $x_1^7x_2^{15}x_3^{11}x_4^5x_5^6$ |
| 113. $x_1^7x_2^{15}x_3^{15}x_4x_5^6$ | 114. $x_1^{15}x_2x_3^6x_4^7x_5^{15}$ | 115. $x_1^{15}x_2x_3^6x_4^{15}x_5^7$ | 116. $x_1^{15}x_2x_3^7x_4^6x_5^{15}$ |
| 117. $x_1^{15}x_2x_3^7x_4^7x_5^{14}$ | 118. $x_1^{15}x_2x_3^7x_4^{14}x_5^7$ | 119. $x_1^{15}x_2x_3^7x_4^{15}x_5^6$ | 120. $x_1^{15}x_2x_3^{14}x_4^7x_5^7$ |
| 121. $x_1^{15}x_2x_3^{15}x_4^6x_5^7$ | 122. $x_1^{15}x_2x_3^{15}x_4^7x_5^6$ | 123. $x_1^{15}x_2^3x_3^5x_4^6x_5^{15}$ | 124. $x_1^{15}x_2^3x_3^5x_4^7x_5^{14}$ |
| 125. $x_1^{15}x_2^3x_3^5x_4^{14}x_5^7$ | 126. $x_1^{15}x_2^3x_3^5x_4^{15}x_5^6$ | 127. $x_1^{15}x_2^3x_3^7x_4^5x_5^{14}$ | 128. $x_1^{15}x_2^3x_3^7x_4^{13}x_5^6$ |
| 129. $x_1^{15}x_2^3x_3^{13}x_4^6x_5^7$ | 130. $x_1^{15}x_2^3x_3^{13}x_4^7x_5^6$ | 131. $x_1^{15}x_2^3x_3^{15}x_4^5x_5^6$ | 132. $x_1^{15}x_2^7x_3x_4^6x_5^{15}$ |
| 133. $x_1^{15}x_2^7x_3x_4^7x_5^{14}$ | 134. $x_1^{15}x_2^7x_3x_4^{14}x_5^7$ | 135. $x_1^{15}x_2^7x_3x_4^{15}x_5^6$ | 136. $x_1^{15}x_2^7x_3^3x_4^5x_5^{14}$ |
| 137. $x_1^{15}x_2^7x_3^3x_4^{13}x_5^6$ | 138. $x_1^{15}x_2^7x_3^7x_4x_5^{14}$ | 139. $x_1^{15}x_2^7x_3^7x_4^7x_5^8$ | 140. $x_1^{15}x_2^7x_3^7x_4^9x_5^6$ |
| 141. $x_1^{15}x_2^7x_3^{11}x_4^5x_5^6$ | 142. $x_1^{15}x_2^7x_3^{15}x_4x_5^6$ | 143. $x_1^{15}x_2^{15}x_3x_4^6x_5^7$ | 144. $x_1^{15}x_2^{15}x_3x_4^7x_5^6$ |
| 145. $x_1^{15}x_2^{15}x_3^3x_4^5x_5^6$ | 146. $x_1^{15}x_2^{15}x_3^7x_4x_5^6$ | | |

## 6.3 The case $d = 5$

$\widetilde{B}_5^+((4)|^3(2)|(1)|^2) = \{x_i^{63}\theta_{J_i}(w) \mid w \in B_4^+(29), 1 \leqslant i \leqslant 5\} \bigcup \{c_t : 1 \leqslant t \leqslant 1005\}$, where $B_4^+(29) = \{\widetilde{w}_u : 1 \leqslant u \leqslant 33\}$ with $\widetilde{w}_u$ the monomials determined as follows:

| | | | | |
|---|---|---|---|---|
| 1. $x_1x_2^6x_3^7x_4^{15}$ | 2. $x_1x_2^6x_3^{15}x_4^7$ | 3. $x_1x_2^7x_3^6x_4^{15}$ | 4. $x_1x_2^7x_3^7x_4^{14}$ | 5. $x_1x_2^7x_3^{14}x_4^7$ |
| 6. $x_1x_2^7x_3^{15}x_4^6$ | 7. $x_1x_2^{14}x_3^7x_4^7$ | 8. $x_1x_2^{15}x_3^6x_4^7$ | 9. $x_1x_2^{15}x_3^7x_4^6$ | 10. $x_1^3x_2^5x_3^6x_4^{15}$ |
| 11. $x_1^3x_2^5x_3^7x_4^{14}$ | 12. $x_1^3x_2^5x_3^{14}x_4^7$ | 13. $x_1^3x_2^5x_3^{15}x_4^6$ | 14. $x_1^3x_2^7x_3^5x_4^{14}$ | 15. $x_1^3x_2^7x_3^{13}x_4^6$ |
| 16. $x_1^3x_2^{13}x_3^6x_4^7$ | 17. $x_1^3x_2^{13}x_3^7x_4^6$ | 18. $x_1^3x_2^{15}x_3^5x_4^6$ | 19. $x_1^7x_2x_3^6x_4^{15}$ | 20. $x_1^7x_2x_3^7x_4^{14}$ |
| 21. $x_1^7x_2x_3^{14}x_4^7$ | 22. $x_1^7x_2x_3^{15}x_4^6$ | 23. $x_1^7x_2^3x_3^5x_4^{14}$ | 24. $x_1^7x_2^3x_3^{13}x_4^6$ | 25. $x_1^7x_2^7x_3x_4^{14}$ |
| 26. $x_1^7x_2^7x_3^7x_4^8$ | 27. $x_1^7x_2^7x_3^9x_4^6$ | 28. $x_1^7x_2^{11}x_3^5x_4^6$ | 29. $x_1^7x_2^{15}x_3x_4^6$ | 30. $x_1^{15}x_2x_3^6x_4^7$ |
| 31. $x_1^{15}x_2x_3^7x_4^6$ | 32. $x_1^{15}x_2^3x_3^5x_4^6$ | 33. $x_1^{15}x_2^7x_3x_4^6$ | | |

The monomials $c_t$ are determined as follows:

| | | |
|---|---|---|
| 1. $x_1x_2^6x_3^7x_4^{31}x_5^{47}$ | 2. $x_1x_2^6x_3^{15}x_4^{15}x_5^{55}$ | 3. $x_1x_2^6x_3^{15}x_4^{23}x_5^{47}$ |
| 4. $x_1x_2^6x_3^{15}x_4^{31}x_5^{39}$ | 5. $x_1x_2^6x_3^{15}x_4^{55}x_5^{15}$ | 6. $x_1x_2^6x_3^{31}x_4^7x_5^{47}$ |
| 7. $x_1x_2^6x_3^{31}x_4^{15}x_5^{39}$ | 8. $x_1x_2^6x_3^{31}x_4^{39}x_5^{15}$ | 9. $x_1x_2^6x_3^{31}x_4^{47}x_5^7$ |
| 10. $x_1x_2^7x_3^6x_4^{31}x_5^{47}$ | 11. $x_1x_2^7x_3^7x_4^{15}x_5^{62}$ | 12. $x_1x_2^7x_3^7x_4^{30}x_5^{47}$ |
| 13. $x_1x_2^7x_3^7x_4^{31}x_5^{46}$ | 14. $x_1x_2^7x_3^7x_4^{62}x_5^{15}$ | 15. $x_1x_2^7x_3^{14}x_4^{15}x_5^{55}$ |

16. $x_1x_2^{7}x_3^{14}x_4^{23}x_5^{47}$
17. $x_1x_2^{7}x_3^{14}x_4^{31}x_5^{39}$
18. $x_1x_2^{7}x_3^{14}x_4^{55}x_5^{15}$
19. $x_1x_2^{7}x_3^{15}x_4^{7}x_5^{62}$
20. $x_1x_2^{7}x_3^{15}x_4^{14}x_5^{55}$
21. $x_1x_2^{7}x_3^{15}x_4^{15}x_5^{54}$
22. $x_1x_2^{7}x_3^{15}x_4^{22}x_5^{47}$
23. $x_1x_2^{7}x_3^{15}x_4^{23}x_5^{46}$
24. $x_1x_2^{7}x_3^{15}x_4^{30}x_5^{39}$
25. $x_1x_2^{7}x_3^{15}x_4^{31}x_5^{38}$
26. $x_1x_2^{7}x_3^{15}x_4^{54}x_5^{15}$
27. $x_1x_2^{7}x_3^{15}x_4^{55}x_5^{14}$
28. $x_1x_2^{7}x_3^{15}x_4^{62}x_5^{7}$
29. $x_1x_2^{7}x_3^{30}x_4^{7}x_5^{47}$
30. $x_1x_2^{7}x_3^{30}x_4^{15}x_5^{39}$
31. $x_1x_2^{7}x_3^{30}x_4^{39}x_5^{15}$
32. $x_1x_2^{7}x_3^{30}x_4^{47}x_5^{7}$
33. $x_1x_2^{7}x_3^{31}x_4^{6}x_5^{47}$
34. $x_1x_2^{7}x_3^{31}x_4^{7}x_5^{46}$
35. $x_1x_2^{7}x_3^{31}x_4^{14}x_5^{39}$
36. $x_1x_2^{7}x_3^{31}x_4^{15}x_5^{38}$
37. $x_1x_2^{7}x_3^{31}x_4^{38}x_5^{15}$
38. $x_1x_2^{7}x_3^{31}x_4^{39}x_5^{14}$
39. $x_1x_2^{7}x_3^{31}x_4^{46}x_5^{7}$
40. $x_1x_2^{7}x_3^{31}x_4^{47}x_5^{6}$
41. $x_1x_2^{7}x_3^{62}x_4^{7}x_5^{15}$
42. $x_1x_2^{7}x_3^{62}x_4^{15}x_5^{7}$
43. $x_1x_2^{14}x_3^{7}x_4^{15}x_5^{55}$
44. $x_1x_2^{14}x_3^{7}x_4^{23}x_5^{47}$
45. $x_1x_2^{14}x_3^{7}x_4^{31}x_5^{39}$
46. $x_1x_2^{14}x_3^{7}x_4^{55}x_5^{15}$
47. $x_1x_2^{14}x_3^{15}x_4^{7}x_5^{55}$
48. $x_1x_2^{14}x_3^{15}x_4^{23}x_5^{39}$
49. $x_1x_2^{14}x_3^{15}x_4^{55}x_5^{7}$
50. $x_1x_2^{14}x_3^{23}x_4^{7}x_5^{47}$
51. $x_1x_2^{14}x_3^{23}x_4^{15}x_5^{39}$
52. $x_1x_2^{14}x_3^{23}x_4^{39}x_5^{15}$
53. $x_1x_2^{14}x_3^{23}x_4^{47}x_5^{7}$
54. $x_1x_2^{14}x_3^{31}x_4^{7}x_5^{39}$
55. $x_1x_2^{14}x_3^{31}x_4^{39}x_5^{7}$
56. $x_1x_2^{14}x_3^{55}x_4^{7}x_5^{15}$
57. $x_1x_2^{14}x_3^{55}x_4^{15}x_5^{7}$
58. $x_1x_2^{15}x_3^{6}x_4^{15}x_5^{55}$
59. $x_1x_2^{15}x_3^{6}x_4^{23}x_5^{47}$
60. $x_1x_2^{15}x_3^{6}x_4^{31}x_5^{39}$
61. $x_1x_2^{15}x_3^{6}x_4^{55}x_5^{15}$
62. $x_1x_2^{15}x_3^{7}x_4^{7}x_5^{62}$
63. $x_1x_2^{15}x_3^{7}x_4^{14}x_5^{55}$
64. $x_1x_2^{15}x_3^{7}x_4^{15}x_5^{54}$
65. $x_1x_2^{15}x_3^{7}x_4^{22}x_5^{47}$
66. $x_1x_2^{15}x_3^{7}x_4^{23}x_5^{46}$
67. $x_1x_2^{15}x_3^{7}x_4^{30}x_5^{39}$
68. $x_1x_2^{15}x_3^{7}x_4^{31}x_5^{38}$
69. $x_1x_2^{15}x_3^{7}x_4^{54}x_5^{15}$
70. $x_1x_2^{15}x_3^{7}x_4^{55}x_5^{14}$
71. $x_1x_2^{15}x_3^{7}x_4^{62}x_5^{7}$
72. $x_1x_2^{15}x_3^{14}x_4^{7}x_5^{55}$
73. $x_1x_2^{15}x_3^{14}x_4^{23}x_5^{39}$
74. $x_1x_2^{15}x_3^{14}x_4^{55}x_5^{7}$
75. $x_1x_2^{15}x_3^{15}x_4^{6}x_5^{55}$
76. $x_1x_2^{15}x_3^{15}x_4^{7}x_5^{54}$
77. $x_1x_2^{15}x_3^{15}x_4^{22}x_5^{39}$
78. $x_1x_2^{15}x_3^{15}x_4^{23}x_5^{38}$
79. $x_1x_2^{15}x_3^{15}x_4^{54}x_5^{7}$
80. $x_1x_2^{15}x_3^{15}x_4^{55}x_5^{6}$
81. $x_1x_2^{15}x_3^{22}x_4^{7}x_5^{47}$
82. $x_1x_2^{15}x_3^{22}x_4^{15}x_5^{39}$
83. $x_1x_2^{15}x_3^{22}x_4^{39}x_5^{15}$
84. $x_1x_2^{15}x_3^{22}x_4^{47}x_5^{7}$
85. $x_1x_2^{15}x_3^{23}x_4^{6}x_5^{47}$
86. $x_1x_2^{15}x_3^{23}x_4^{7}x_5^{46}$
87. $x_1x_2^{15}x_3^{23}x_4^{14}x_5^{39}$
88. $x_1x_2^{15}x_3^{23}x_4^{15}x_5^{38}$
89. $x_1x_2^{15}x_3^{23}x_4^{38}x_5^{15}$
90. $x_1x_2^{15}x_3^{23}x_4^{39}x_5^{14}$
91. $x_1x_2^{15}x_3^{23}x_4^{46}x_5^{7}$
92. $x_1x_2^{15}x_3^{23}x_4^{47}x_5^{6}$
93. $x_1x_2^{15}x_3^{30}x_4^{7}x_5^{39}$
94. $x_1x_2^{15}x_3^{30}x_4^{39}x_5^{7}$
95. $x_1x_2^{15}x_3^{31}x_4^{6}x_5^{39}$
96. $x_1x_2^{15}x_3^{31}x_4^{7}x_5^{38}$
97. $x_1x_2^{15}x_3^{31}x_4^{38}x_5^{7}$
98. $x_1x_2^{15}x_3^{31}x_4^{39}x_5^{6}$
99. $x_1x_2^{15}x_3^{54}x_4^{7}x_5^{15}$
100. $x_1x_2^{15}x_3^{54}x_4^{15}x_5^{7}$
101. $x_1x_2^{15}x_3^{55}x_4^{6}x_5^{15}$
102. $x_1x_2^{15}x_3^{55}x_4^{7}x_5^{14}$
103. $x_1x_2^{15}x_3^{55}x_4^{14}x_5^{7}$
104. $x_1x_2^{15}x_3^{55}x_4^{15}x_5^{6}$
105. $x_1x_2^{15}x_3^{62}x_4^{7}x_5^{7}$
106. $x_1x_2^{30}x_3^{7}x_4^{7}x_5^{47}$
107. $x_1x_2^{30}x_3^{7}x_4^{15}x_5^{39}$
108. $x_1x_2^{30}x_3^{7}x_4^{39}x_5^{15}$
109. $x_1x_2^{30}x_3^{7}x_4^{47}x_5^{7}$
110. $x_1x_2^{30}x_3^{15}x_4^{7}x_5^{39}$
111. $x_1x_2^{30}x_3^{15}x_4^{39}x_5^{7}$
112. $x_1x_2^{30}x_3^{39}x_4^{7}x_5^{15}$
113. $x_1x_2^{30}x_3^{39}x_4^{15}x_5^{7}$
114. $x_1x_2^{30}x_3^{47}x_4^{7}x_5^{7}$
115. $x_1x_2^{31}x_3^{6}x_4^{7}x_5^{47}$
116. $x_1x_2^{31}x_3^{6}x_4^{15}x_5^{39}$
117. $x_1x_2^{31}x_3^{6}x_4^{39}x_5^{15}$
118. $x_1x_2^{31}x_3^{6}x_4^{47}x_5^{7}$
119. $x_1x_2^{31}x_3^{7}x_4^{6}x_5^{47}$
120. $x_1x_2^{31}x_3^{7}x_4^{7}x_5^{46}$
121. $x_1x_2^{31}x_3^{7}x_4^{14}x_5^{39}$
122. $x_1x_2^{31}x_3^{7}x_4^{15}x_5^{38}$
123. $x_1x_2^{31}x_3^{7}x_4^{38}x_5^{15}$
124. $x_1x_2^{31}x_3^{7}x_4^{39}x_5^{14}$
125. $x_1x_2^{31}x_3^{7}x_4^{46}x_5^{7}$
126. $x_1x_2^{31}x_3^{7}x_4^{47}x_5^{6}$
127. $x_1x_2^{31}x_3^{14}x_4^{7}x_5^{39}$
128. $x_1x_2^{31}x_3^{14}x_4^{39}x_5^{7}$
129. $x_1x_2^{31}x_3^{15}x_4^{6}x_5^{39}$
130. $x_1x_2^{31}x_3^{15}x_4^{7}x_5^{38}$
131. $x_1x_2^{31}x_3^{15}x_4^{38}x_5^{7}$
132. $x_1x_2^{31}x_3^{15}x_4^{39}x_5^{6}$
133. $x_1x_2^{31}x_3^{38}x_4^{7}x_5^{15}$
134. $x_1x_2^{31}x_3^{38}x_4^{15}x_5^{7}$
135. $x_1x_2^{31}x_3^{39}x_4^{6}x_5^{15}$
136. $x_1x_2^{31}x_3^{39}x_4^{7}x_5^{14}$
137. $x_1x_2^{31}x_3^{39}x_4^{14}x_5^{7}$
138. $x_1x_2^{31}x_3^{39}x_4^{15}x_5^{6}$
139. $x_1x_2^{31}x_3^{46}x_4^{7}x_5^{7}$
140. $x_1x_2^{31}x_3^{47}x_4^{6}x_5^{7}$
141. $x_1x_2^{31}x_3^{47}x_4^{7}x_5^{6}$
142. $x_1x_2^{62}x_3^{7}x_4^{7}x_5^{15}$
143. $x_1x_2^{62}x_3^{7}x_4^{15}x_5^{7}$
144. $x_1x_2^{62}x_3^{15}x_4^{7}x_5^{7}$

145. $x_1^{3}x_2^{5}x_3^{6}x_4^{31}x_5^{47}$ 146. $x_1^{3}x_2^{5}x_3^{7}x_4^{15}x_5^{62}$ 147. $x_1^{3}x_2^{5}x_3^{7}x_4^{30}x_5^{47}$
148. $x_1^{3}x_2^{5}x_3^{7}x_4^{31}x_5^{46}$ 149. $x_1^{3}x_2^{5}x_3^{7}x_4^{62}x_5^{15}$ 150. $x_1^{3}x_2^{5}x_3^{14}x_4^{15}x_5^{55}$
151. $x_1^{3}x_2^{5}x_3^{14}x_4^{23}x_5^{47}$ 152. $x_1^{3}x_2^{5}x_3^{14}x_4^{31}x_5^{39}$ 153. $x_1^{3}x_2^{5}x_3^{14}x_4^{55}x_5^{15}$
154. $x_1^{3}x_2^{5}x_3^{15}x_4^{7}x_5^{62}$ 155. $x_1^{3}x_2^{5}x_3^{15}x_4^{14}x_5^{55}$ 156. $x_1^{3}x_2^{5}x_3^{15}x_4^{15}x_5^{54}$
157. $x_1^{3}x_2^{5}x_3^{15}x_4^{22}x_5^{47}$ 158. $x_1^{3}x_2^{5}x_3^{15}x_4^{23}x_5^{46}$ 159. $x_1^{3}x_2^{5}x_3^{15}x_4^{30}x_5^{39}$
160. $x_1^{3}x_2^{5}x_3^{15}x_4^{31}x_5^{38}$ 161. $x_1^{3}x_2^{5}x_3^{15}x_4^{54}x_5^{15}$ 162. $x_1^{3}x_2^{5}x_3^{15}x_4^{55}x_5^{14}$
163. $x_1^{3}x_2^{5}x_3^{15}x_4^{62}x_5^{7}$ 164. $x_1^{3}x_2^{5}x_3^{30}x_4^{7}x_5^{47}$ 165. $x_1^{3}x_2^{5}x_3^{30}x_4^{15}x_5^{39}$
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271. $x_1^{3}x_2^{15}x_3^{5}x_4^{31}x_5^{38}$ 272. $x_1^{3}x_2^{15}x_3^{5}x_4^{54}x_5^{15}$ 273. $x_1^{3}x_2^{15}x_3^{5}x_4^{55}x_5^{14}$

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391. $x_1^{7}x_2x_3^{15}x_4^{22}x_5^{47}$
392. $x_1^{7}x_2x_3^{15}x_4^{23}x_5^{46}$
393. $x_1^{7}x_2x_3^{15}x_4^{30}x_5^{39}$
394. $x_1^{7}x_2x_3^{15}x_4^{31}x_5^{38}$
395. $x_1^{7}x_2x_3^{15}x_4^{54}x_5^{15}$
396. $x_1^{7}x_2x_3^{15}x_4^{55}x_5^{14}$
397. $x_1^{7}x_2x_3^{15}x_4^{62}x_5^{7}$
398. $x_1^{7}x_2x_3^{30}x_4^{7}x_5^{47}$
399. $x_1^{7}x_2x_3^{30}x_4^{15}x_5^{39}$
400. $x_1^{7}x_2x_3^{30}x_4^{39}x_5^{15}$
401. $x_1^{7}x_2x_3^{30}x_4^{47}x_5^{7}$
402. $x_1^{7}x_2x_3^{31}x_4^{6}x_5^{47}$

| | | |
|---|---|---|
| 403. $x_1^7x_2x_3^{31}x_4^7x_5^{46}$ | 404. $x_1^7x_2x_3^{31}x_4^{14}x_5^{39}$ | 405. $x_1^7x_2x_3^{31}x_4^{15}x_5^{38}$ |
| 406. $x_1^7x_2x_3^{31}x_4^{38}x_5^{15}$ | 407. $x_1^7x_2x_3^{31}x_4^{39}x_5^{14}$ | 408. $x_1^7x_2x_3^{31}x_4^{46}x_5^7$ |
| 409. $x_1^7x_2x_3^{31}x_4^{47}x_5^6$ | 410. $x_1^7x_2x_3^{62}x_4^7x_5^{15}$ | 411. $x_1^7x_2x_3^{62}x_4^{15}x_5^7$ |
| 412. $x_1^7x_2^3x_3^5x_4^{15}x_5^{62}$ | 413. $x_1^7x_2^3x_3^5x_4^{30}x_5^{47}$ | 414. $x_1^7x_2^3x_3^5x_4^{31}x_5^{46}$ |
| 415. $x_1^7x_2^3x_3^5x_4^{62}x_5^{15}$ | 416. $x_1^7x_2^3x_3^7x_4^{13}x_5^{62}$ | 417. $x_1^7x_2^3x_3^7x_4^{29}x_5^{46}$ |
| 418. $x_1^7x_2^3x_3^7x_4^{61}x_5^{14}$ | 419. $x_1^7x_2^3x_3^{13}x_4^7x_5^{62}$ | 420. $x_1^7x_2^3x_3^{13}x_4^{14}x_5^{55}$ |
| 421. $x_1^7x_2^3x_3^{13}x_4^{15}x_5^{54}$ | 422. $x_1^7x_2^3x_3^{13}x_4^{22}x_5^{47}$ | 423. $x_1^7x_2^3x_3^{13}x_4^{23}x_5^{46}$ |
| 424. $x_1^7x_2^3x_3^{13}x_4^{30}x_5^{39}$ | 425. $x_1^7x_2^3x_3^{13}x_4^{31}x_5^{38}$ | 426. $x_1^7x_2^3x_3^{13}x_4^{54}x_5^{15}$ |
| 427. $x_1^7x_2^3x_3^{13}x_4^{55}x_5^{14}$ | 428. $x_1^7x_2^3x_3^{13}x_4^{62}x_5^7$ | 429. $x_1^7x_2^3x_3^{15}x_4^5x_5^{62}$ |
| 430. $x_1^7x_2^3x_3^{15}x_4^{13}x_5^{54}$ | 431. $x_1^7x_2^3x_3^{15}x_4^{21}x_5^{46}$ | 432. $x_1^7x_2^3x_3^{15}x_4^{29}x_5^{38}$ |
| 433. $x_1^7x_2^3x_3^{15}x_4^{53}x_5^{14}$ | 434. $x_1^7x_2^3x_3^{15}x_4^{61}x_5^6$ | 435. $x_1^7x_2^3x_3^{29}x_4^6x_5^{47}$ |
| 436. $x_1^7x_2^3x_3^{29}x_4^7x_5^{46}$ | 437. $x_1^7x_2^3x_3^{29}x_4^{14}x_5^{39}$ | 438. $x_1^7x_2^3x_3^{29}x_4^{15}x_5^{38}$ |
| 439. $x_1^7x_2^3x_3^{29}x_4^{38}x_5^{15}$ | 440. $x_1^7x_2^3x_3^{29}x_4^{39}x_5^{14}$ | 441. $x_1^7x_2^3x_3^{29}x_4^{46}x_5^7$ |
| 442. $x_1^7x_2^3x_3^{29}x_4^{47}x_5^6$ | 443. $x_1^7x_2^3x_3^{31}x_4^5x_5^{46}$ | 444. $x_1^7x_2^3x_3^{31}x_4^{13}x_5^{38}$ |
| 445. $x_1^7x_2^3x_3^{31}x_4^{37}x_5^{14}$ | 446. $x_1^7x_2^3x_3^{31}x_4^{45}x_5^6$ | 447. $x_1^7x_2^3x_3^{61}x_4^6x_5^{15}$ |
| 448. $x_1^7x_2^3x_3^{61}x_4^7x_5^{14}$ | 449. $x_1^7x_2^3x_3^{61}x_4^{14}x_5^7$ | 450. $x_1^7x_2^3x_3^{61}x_4^{15}x_5^6$ |
| 451. $x_1^7x_2^7x_3x_4^{15}x_5^{62}$ | 452. $x_1^7x_2^7x_3x_4^{30}x_5^{47}$ | 453. $x_1^7x_2^7x_3x_4^{31}x_5^{46}$ |
| 454. $x_1^7x_2^7x_3x_4^{62}x_5^{15}$ | 455. $x_1^7x_2^7x_3^3x_4^{13}x_5^{62}$ | 456. $x_1^7x_2^7x_3^3x_4^{29}x_5^{46}$ |
| 457. $x_1^7x_2^7x_3^3x_4^{61}x_5^{14}$ | 458. $x_1^7x_2^7x_3^7x_4^9x_5^{62}$ | 459. $x_1^7x_2^7x_3^7x_4^{15}x_5^{56}$ |
| 460. $x_1^7x_2^7x_3^7x_4^{24}x_5^{47}$ | 461. $x_1^7x_2^7x_3^7x_4^{25}x_5^{46}$ | 462. $x_1^7x_2^7x_3^7x_4^{31}x_5^{40}$ |
| 463. $x_1^7x_2^7x_3^7x_4^{56}x_5^{15}$ | 464. $x_1^7x_2^7x_3^7x_4^{57}x_5^{14}$ | 465. $x_1^7x_2^7x_3^9x_4^7x_5^{62}$ |
| 466. $x_1^7x_2^7x_3^9x_4^{14}x_5^{55}$ | 467. $x_1^7x_2^7x_3^9x_4^{15}x_5^{54}$ | 468. $x_1^7x_2^7x_3^9x_4^{22}x_5^{47}$ |
| 469. $x_1^7x_2^7x_3^9x_4^{23}x_5^{46}$ | 470. $x_1^7x_2^7x_3^9x_4^{30}x_5^{39}$ | 471. $x_1^7x_2^7x_3^9x_4^{31}x_5^{38}$ |
| 472. $x_1^7x_2^7x_3^9x_4^{54}x_5^{15}$ | 473. $x_1^7x_2^7x_3^9x_4^{55}x_5^{14}$ | 474. $x_1^7x_2^7x_3^9x_4^{62}x_5^7$ |
| 475. $x_1^7x_2^7x_3^{11}x_4^5x_5^{62}$ | 476. $x_1^7x_2^7x_3^{11}x_4^{13}x_5^{54}$ | 477. $x_1^7x_2^7x_3^{11}x_4^{21}x_5^{46}$ |
| 478. $x_1^7x_2^7x_3^{11}x_4^{29}x_5^{38}$ | 479. $x_1^7x_2^7x_3^{11}x_4^{53}x_5^{14}$ | 480. $x_1^7x_2^7x_3^{11}x_4^{61}x_5^6$ |
| 481. $x_1^7x_2^7x_3^{15}x_4x_5^{62}$ | 482. $x_1^7x_2^7x_3^{15}x_4^7x_5^{56}$ | 483. $x_1^7x_2^7x_3^{15}x_4^9x_5^{54}$ |
| 484. $x_1^7x_2^7x_3^{15}x_4^{15}x_5^{48}$ | 485. $x_1^7x_2^7x_3^{15}x_4^{17}x_5^{46}$ | 486. $x_1^7x_2^7x_3^{15}x_4^{23}x_5^{40}$ |
| 487. $x_1^7x_2^7x_3^{15}x_4^{25}x_5^{38}$ | 488. $x_1^7x_2^7x_3^{15}x_4^{49}x_5^{14}$ | 489. $x_1^7x_2^7x_3^{15}x_4^{55}x_5^8$ |
| 490. $x_1^7x_2^7x_3^{15}x_4^{57}x_5^6$ | 491. $x_1^7x_2^7x_3^{25}x_4^6x_5^{47}$ | 492. $x_1^7x_2^7x_3^{25}x_4^7x_5^{46}$ |
| 493. $x_1^7x_2^7x_3^{25}x_4^{14}x_5^{39}$ | 494. $x_1^7x_2^7x_3^{25}x_4^{15}x_5^{38}$ | 495. $x_1^7x_2^7x_3^{25}x_4^{38}x_5^{15}$ |
| 496. $x_1^7x_2^7x_3^{25}x_4^{39}x_5^{14}$ | 497. $x_1^7x_2^7x_3^{25}x_4^{46}x_5^7$ | 498. $x_1^7x_2^7x_3^{25}x_4^{47}x_5^6$ |
| 499. $x_1^7x_2^7x_3^{27}x_4^5x_5^{46}$ | 500. $x_1^7x_2^7x_3^{27}x_4^{13}x_5^{38}$ | 501. $x_1^7x_2^7x_3^{27}x_4^{37}x_5^{14}$ |
| 502. $x_1^7x_2^7x_3^{27}x_4^{45}x_5^6$ | 503. $x_1^7x_2^7x_3^{31}x_4x_5^{46}$ | 504. $x_1^7x_2^7x_3^{31}x_4^7x_5^{40}$ |
| 505. $x_1^7x_2^7x_3^{31}x_4^9x_5^{38}$ | 506. $x_1^7x_2^7x_3^{31}x_4^{33}x_5^{14}$ | 507. $x_1^7x_2^7x_3^{31}x_4^{41}x_5^6$ |
| 508. $x_1^7x_2^7x_3^{57}x_4^6x_5^{15}$ | 509. $x_1^7x_2^7x_3^{57}x_4^7x_5^{14}$ | 510. $x_1^7x_2^7x_3^{57}x_4^{14}x_5^7$ |
| 511. $x_1^7x_2^7x_3^{57}x_4^{15}x_5^6$ | 512. $x_1^7x_2^7x_3^{59}x_4^5x_5^{14}$ | 513. $x_1^7x_2^7x_3^{59}x_4^{13}x_5^6$ |
| 514. $x_1^7x_2^{11}x_3^5x_4^7x_5^{62}$ | 515. $x_1^7x_2^{11}x_3^5x_4^{14}x_5^{55}$ | 516. $x_1^7x_2^{11}x_3^5x_4^{15}x_5^{54}$ |
| 517. $x_1^7x_2^{11}x_3^5x_4^{22}x_5^{47}$ | 518. $x_1^7x_2^{11}x_3^5x_4^{23}x_5^{46}$ | 519. $x_1^7x_2^{11}x_3^5x_4^{30}x_5^{39}$ |
| 520. $x_1^7x_2^{11}x_3^5x_4^{31}x_5^{38}$ | 521. $x_1^7x_2^{11}x_3^5x_4^{54}x_5^{15}$ | 522. $x_1^7x_2^{11}x_3^5x_4^{55}x_5^{14}$ |
| 523. $x_1^7x_2^{11}x_3^5x_4^{62}x_5^7$ | 524. $x_1^7x_2^{11}x_3^7x_4^5x_5^{62}$ | 525. $x_1^7x_2^{11}x_3^7x_4^{13}x_5^{54}$ |
| 526. $x_1^7x_2^{11}x_3^7x_4^{21}x_5^{46}$ | 527. $x_1^7x_2^{11}x_3^7x_4^{29}x_5^{38}$ | 528. $x_1^7x_2^{11}x_3^7x_4^{53}x_5^{14}$ |
| 529. $x_1^7x_2^{11}x_3^7x_4^{61}x_5^6$ | 530. $x_1^7x_2^{11}x_3^{13}x_4^6x_5^{55}$ | 531. $x_1^7x_2^{11}x_3^{13}x_4^7x_5^{54}$ |

| | | |
|---|---|---|
| 532. $x_1^7x_2^{11}x_3^{13}x_4^{22}x_5^{39}$ | 533. $x_1^7x_2^{11}x_3^{13}x_4^{23}x_5^{38}$ | 534. $x_1^7x_2^{11}x_3^{13}x_4^{54}x_5^{7}$ |
| 535. $x_1^7x_2^{11}x_3^{13}x_4^{55}x_5^{6}$ | 536. $x_1^7x_2^{11}x_3^{15}x_4^{5}x_5^{54}$ | 537. $x_1^7x_2^{11}x_3^{15}x_4^{21}x_5^{38}$ |
| 538. $x_1^7x_2^{11}x_3^{15}x_4^{53}x_5^{6}$ | 539. $x_1^7x_2^{11}x_3^{21}x_4^{6}x_5^{47}$ | 540. $x_1^7x_2^{11}x_3^{21}x_4^{7}x_5^{46}$ |
| 541. $x_1^7x_2^{11}x_3^{21}x_4^{14}x_5^{39}$ | 542. $x_1^7x_2^{11}x_3^{21}x_4^{15}x_5^{38}$ | 543. $x_1^7x_2^{11}x_3^{21}x_4^{38}x_5^{15}$ |
| 544. $x_1^7x_2^{11}x_3^{21}x_4^{39}x_5^{14}$ | 545. $x_1^7x_2^{11}x_3^{21}x_4^{46}x_5^{7}$ | 546. $x_1^7x_2^{11}x_3^{21}x_4^{47}x_5^{6}$ |
| 547. $x_1^7x_2^{11}x_3^{23}x_4^{5}x_5^{46}$ | 548. $x_1^7x_2^{11}x_3^{23}x_4^{13}x_5^{38}$ | 549. $x_1^7x_2^{11}x_3^{23}x_4^{37}x_5^{14}$ |
| 550. $x_1^7x_2^{11}x_3^{23}x_4^{45}x_5^{6}$ | 551. $x_1^7x_2^{11}x_3^{29}x_4^{6}x_5^{39}$ | 552. $x_1^7x_2^{11}x_3^{29}x_4^{7}x_5^{38}$ |
| 553. $x_1^7x_2^{11}x_3^{29}x_4^{38}x_5^{7}$ | 554. $x_1^7x_2^{11}x_3^{29}x_4^{39}x_5^{6}$ | 555. $x_1^7x_2^{11}x_3^{31}x_4^{5}x_5^{38}$ |
| 556. $x_1^7x_2^{11}x_3^{31}x_4^{37}x_5^{6}$ | 557. $x_1^7x_2^{11}x_3^{53}x_4^{6}x_5^{15}$ | 558. $x_1^7x_2^{11}x_3^{53}x_4^{7}x_5^{14}$ |
| 559. $x_1^7x_2^{11}x_3^{53}x_4^{14}x_5^{7}$ | 560. $x_1^7x_2^{11}x_3^{53}x_4^{15}x_5^{6}$ | 561. $x_1^7x_2^{11}x_3^{55}x_4^{5}x_5^{14}$ |
| 562. $x_1^7x_2^{11}x_3^{55}x_4^{13}x_5^{6}$ | 563. $x_1^7x_2^{11}x_3^{61}x_4^{6}x_5^{7}$ | 564. $x_1^7x_2^{11}x_3^{61}x_4^{7}x_5^{6}$ |
| 565. $x_1^7x_2^{15}x_3x_4^{7}x_5^{62}$ | 566. $x_1^7x_2^{15}x_3x_4^{14}x_5^{55}$ | 567. $x_1^7x_2^{15}x_3x_4^{15}x_5^{54}$ |
| 568. $x_1^7x_2^{15}x_3x_4^{22}x_5^{47}$ | 569. $x_1^7x_2^{15}x_3x_4^{23}x_5^{46}$ | 570. $x_1^7x_2^{15}x_3x_4^{30}x_5^{39}$ |
| 571. $x_1^7x_2^{15}x_3x_4^{31}x_5^{38}$ | 572. $x_1^7x_2^{15}x_3x_4^{54}x_5^{15}$ | 573. $x_1^7x_2^{15}x_3x_4^{55}x_5^{14}$ |
| 574. $x_1^7x_2^{15}x_3x_4^{62}x_5^{7}$ | 575. $x_1^7x_2^{15}x_3^{3}x_4^{5}x_5^{62}$ | 576. $x_1^7x_2^{15}x_3^{3}x_4^{13}x_5^{54}$ |
| 577. $x_1^7x_2^{15}x_3^{3}x_4^{21}x_5^{46}$ | 578. $x_1^7x_2^{15}x_3^{3}x_4^{29}x_5^{38}$ | 579. $x_1^7x_2^{15}x_3^{3}x_4^{53}x_5^{14}$ |
| 580. $x_1^7x_2^{15}x_3^{3}x_4^{61}x_5^{6}$ | 581. $x_1^7x_2^{15}x_3^{7}x_4x_5^{62}$ | 582. $x_1^7x_2^{15}x_3^{7}x_4^{7}x_5^{56}$ |
| 583. $x_1^7x_2^{15}x_3^{7}x_4^{9}x_5^{54}$ | 584. $x_1^7x_2^{15}x_3^{7}x_4^{15}x_5^{48}$ | 585. $x_1^7x_2^{15}x_3^{7}x_4^{17}x_5^{46}$ |
| 586. $x_1^7x_2^{15}x_3^{7}x_4^{23}x_5^{40}$ | 587. $x_1^7x_2^{15}x_3^{7}x_4^{25}x_5^{38}$ | 588. $x_1^7x_2^{15}x_3^{7}x_4^{49}x_5^{14}$ |
| 589. $x_1^7x_2^{15}x_3^{7}x_4^{55}x_5^{8}$ | 590. $x_1^7x_2^{15}x_3^{7}x_4^{57}x_5^{6}$ | 591. $x_1^7x_2^{15}x_3^{11}x_4^{5}x_5^{54}$ |
| 592. $x_1^7x_2^{15}x_3^{11}x_4^{21}x_5^{38}$ | 593. $x_1^7x_2^{15}x_3^{11}x_4^{53}x_5^{6}$ | 594. $x_1^7x_2^{15}x_3^{15}x_4x_5^{54}$ |
| 595. $x_1^7x_2^{15}x_3^{15}x_4^{7}x_5^{48}$ | 596. $x_1^7x_2^{15}x_3^{15}x_4^{16}x_5^{39}$ | 597. $x_1^7x_2^{15}x_3^{15}x_4^{17}x_5^{38}$ |
| 598. $x_1^7x_2^{15}x_3^{15}x_4^{23}x_5^{32}$ | 599. $x_1^7x_2^{15}x_3^{15}x_4^{48}x_5^{7}$ | 600. $x_1^7x_2^{15}x_3^{15}x_4^{49}x_5^{6}$ |
| 601. $x_1^7x_2^{15}x_3^{19}x_4^{5}x_5^{46}$ | 602. $x_1^7x_2^{15}x_3^{19}x_4^{13}x_5^{38}$ | 603. $x_1^7x_2^{15}x_3^{19}x_4^{37}x_5^{14}$ |
| 604. $x_1^7x_2^{15}x_3^{19}x_4^{45}x_5^{6}$ | 605. $x_1^7x_2^{15}x_3^{23}x_4x_5^{46}$ | 606. $x_1^7x_2^{15}x_3^{23}x_4^{7}x_5^{40}$ |
| 607. $x_1^7x_2^{15}x_3^{23}x_4^{9}x_5^{38}$ | 608. $x_1^7x_2^{15}x_3^{23}x_4^{33}x_5^{14}$ | 609. $x_1^7x_2^{15}x_3^{23}x_4^{41}x_5^{6}$ |
| 610. $x_1^7x_2^{15}x_3^{27}x_4^{5}x_5^{38}$ | 611. $x_1^7x_2^{15}x_3^{27}x_4^{37}x_5^{6}$ | 612. $x_1^7x_2^{15}x_3^{31}x_4x_5^{38}$ |
| 613. $x_1^7x_2^{15}x_3^{51}x_4^{5}x_5^{14}$ | 614. $x_1^7x_2^{15}x_3^{51}x_4^{13}x_5^{6}$ | 615. $x_1^7x_2^{15}x_3^{55}x_4x_5^{14}$ |
| 616. $x_1^7x_2^{15}x_3^{55}x_4^{7}x_5^{8}$ | 617. $x_1^7x_2^{15}x_3^{55}x_4^{9}x_5^{6}$ | 618. $x_1^7x_2^{15}x_3^{59}x_4^{5}x_5^{6}$ |
| 619. $x_1^7x_2^{27}x_3^{5}x_4^{6}x_5^{47}$ | 620. $x_1^7x_2^{27}x_3^{5}x_4^{7}x_5^{46}$ | 621. $x_1^7x_2^{27}x_3^{5}x_4^{14}x_5^{39}$ |
| 622. $x_1^7x_2^{27}x_3^{5}x_4^{15}x_5^{38}$ | 623. $x_1^7x_2^{27}x_3^{5}x_4^{38}x_5^{15}$ | 624. $x_1^7x_2^{27}x_3^{5}x_4^{39}x_5^{14}$ |
| 625. $x_1^7x_2^{27}x_3^{5}x_4^{46}x_5^{7}$ | 626. $x_1^7x_2^{27}x_3^{5}x_4^{47}x_5^{6}$ | 627. $x_1^7x_2^{27}x_3^{7}x_4^{5}x_5^{46}$ |
| 628. $x_1^7x_2^{27}x_3^{7}x_4^{13}x_5^{38}$ | 629. $x_1^7x_2^{27}x_3^{7}x_4^{37}x_5^{14}$ | 630. $x_1^7x_2^{27}x_3^{7}x_4^{45}x_5^{6}$ |
| 631. $x_1^7x_2^{27}x_3^{13}x_4^{6}x_5^{39}$ | 632. $x_1^7x_2^{27}x_3^{13}x_4^{7}x_5^{38}$ | 633. $x_1^7x_2^{27}x_3^{13}x_4^{38}x_5^{7}$ |
| 634. $x_1^7x_2^{27}x_3^{13}x_4^{39}x_5^{6}$ | 635. $x_1^7x_2^{27}x_3^{15}x_4^{5}x_5^{38}$ | 636. $x_1^7x_2^{27}x_3^{15}x_4^{37}x_5^{6}$ |
| 637. $x_1^7x_2^{27}x_3^{37}x_4^{6}x_5^{15}$ | 638. $x_1^7x_2^{27}x_3^{37}x_4^{7}x_5^{14}$ | 639. $x_1^7x_2^{27}x_3^{37}x_4^{14}x_5^{7}$ |
| 640. $x_1^7x_2^{27}x_3^{37}x_4^{15}x_5^{6}$ | 641. $x_1^7x_2^{27}x_3^{39}x_4^{5}x_5^{14}$ | 642. $x_1^7x_2^{27}x_3^{39}x_4^{13}x_5^{6}$ |
| 643. $x_1^7x_2^{27}x_3^{45}x_4^{6}x_5^{7}$ | 644. $x_1^7x_2^{27}x_3^{45}x_4^{7}x_5^{6}$ | 645. $x_1^7x_2^{27}x_3^{47}x_4^{5}x_5^{6}$ |
| 646. $x_1^7x_2^{31}x_3x_4^{6}x_5^{47}$ | 647. $x_1^7x_2^{31}x_3x_4^{7}x_5^{46}$ | 648. $x_1^7x_2^{31}x_3x_4^{14}x_5^{39}$ |
| 649. $x_1^7x_2^{31}x_3x_4^{15}x_5^{38}$ | 650. $x_1^7x_2^{31}x_3x_4^{38}x_5^{15}$ | 651. $x_1^7x_2^{31}x_3x_4^{39}x_5^{14}$ |
| 652. $x_1^7x_2^{31}x_3x_4^{46}x_5^{7}$ | 653. $x_1^7x_2^{31}x_3x_4^{47}x_5^{6}$ | 654. $x_1^7x_2^{31}x_3^{3}x_4^{5}x_5^{46}$ |
| 655. $x_1^7x_2^{31}x_3^{3}x_4^{13}x_5^{38}$ | 656. $x_1^7x_2^{31}x_3^{3}x_4^{37}x_5^{14}$ | 657. $x_1^7x_2^{31}x_3^{3}x_4^{45}x_5^{6}$ |
| 658. $x_1^7x_2^{31}x_3^{7}x_4x_5^{46}$ | 659. $x_1^7x_2^{31}x_3^{7}x_4^{7}x_5^{40}$ | 660. $x_1^7x_2^{31}x_3^{7}x_4^{9}x_5^{38}$ |

661. $x_1^{7}x_2^{31}x_3^{7}x_4^{33}x_5^{14}$ 662. $x_1^{7}x_2^{31}x_3^{7}x_4^{41}x_5^{6}$ 663. $x_1^{7}x_2^{31}x_3^{11}x_4^{5}x_5^{38}$
664. $x_1^{7}x_2^{31}x_3^{11}x_4^{37}x_5^{6}$ 665. $x_1^{7}x_2^{31}x_3^{15}x_4x_5^{38}$ 666. $x_1^{7}x_2^{31}x_3^{35}x_4^{5}x_5^{14}$
667. $x_1^{7}x_2^{31}x_3^{35}x_4^{13}x_5^{6}$ 668. $x_1^{7}x_2^{31}x_3^{39}x_4x_5^{14}$ 669. $x_1^{7}x_2^{31}x_3^{43}x_4^{5}x_5^{6}$
670. $x_1^{7}x_2^{31}x_3^{47}x_4x_5^{6}$ 671. $x_1^{7}x_2^{59}x_3^{5}x_4^{6}x_5^{15}$ 672. $x_1^{7}x_2^{59}x_3^{5}x_4^{7}x_5^{14}$
673. $x_1^{7}x_2^{59}x_3^{5}x_4^{14}x_5^{7}$ 674. $x_1^{7}x_2^{59}x_3^{5}x_4^{15}x_5^{6}$ 675. $x_1^{7}x_2^{59}x_3^{7}x_4^{5}x_5^{14}$
676. $x_1^{7}x_2^{59}x_3^{7}x_4^{13}x_5^{6}$ 677. $x_1^{7}x_2^{59}x_3^{13}x_4^{6}x_5^{7}$ 678. $x_1^{7}x_2^{59}x_3^{13}x_4^{7}x_5^{6}$
679. $x_1^{7}x_2^{59}x_3^{15}x_4^{5}x_5^{6}$ 680. $x_1^{15}x_2x_3^{6}x_4^{15}x_5^{55}$ 681. $x_1^{15}x_2x_3^{6}x_4^{23}x_5^{47}$
682. $x_1^{15}x_2x_3^{6}x_4^{31}x_5^{39}$ 683. $x_1^{15}x_2x_3^{6}x_4^{55}x_5^{15}$ 684. $x_1^{15}x_2x_3^{7}x_4^{7}x_5^{62}$
685. $x_1^{15}x_2x_3^{7}x_4^{14}x_5^{55}$ 686. $x_1^{15}x_2x_3^{7}x_4^{15}x_5^{54}$ 687. $x_1^{15}x_2x_3^{7}x_4^{22}x_5^{47}$
688. $x_1^{15}x_2x_3^{7}x_4^{23}x_5^{46}$ 689. $x_1^{15}x_2x_3^{7}x_4^{30}x_5^{39}$ 690. $x_1^{15}x_2x_3^{7}x_4^{31}x_5^{38}$
691. $x_1^{15}x_2x_3^{7}x_4^{54}x_5^{15}$ 692. $x_1^{15}x_2x_3^{7}x_4^{55}x_5^{14}$ 693. $x_1^{15}x_2x_3^{7}x_4^{62}x_5^{7}$
694. $x_1^{15}x_2x_3^{14}x_4^{7}x_5^{55}$ 695. $x_1^{15}x_2x_3^{14}x_4^{23}x_5^{39}$ 696. $x_1^{15}x_2x_3^{14}x_4^{55}x_5^{7}$
697. $x_1^{15}x_2x_3^{15}x_4^{6}x_5^{55}$ 698. $x_1^{15}x_2x_3^{15}x_4^{7}x_5^{54}$ 699. $x_1^{15}x_2x_3^{15}x_4^{22}x_5^{39}$
700. $x_1^{15}x_2x_3^{15}x_4^{23}x_5^{38}$ 701. $x_1^{15}x_2x_3^{15}x_4^{54}x_5^{7}$ 702. $x_1^{15}x_2x_3^{15}x_4^{55}x_5^{6}$
703. $x_1^{15}x_2x_3^{22}x_4^{7}x_5^{47}$ 704. $x_1^{15}x_2x_3^{22}x_4^{15}x_5^{39}$ 705. $x_1^{15}x_2x_3^{22}x_4^{39}x_5^{15}$
706. $x_1^{15}x_2x_3^{22}x_4^{47}x_5^{7}$ 707. $x_1^{15}x_2x_3^{23}x_4^{6}x_5^{47}$ 708. $x_1^{15}x_2x_3^{23}x_4^{7}x_5^{46}$
709. $x_1^{15}x_2x_3^{23}x_4^{14}x_5^{39}$ 710. $x_1^{15}x_2x_3^{23}x_4^{15}x_5^{38}$ 711. $x_1^{15}x_2x_3^{23}x_4^{38}x_5^{15}$
712. $x_1^{15}x_2x_3^{23}x_4^{39}x_5^{14}$ 713. $x_1^{15}x_2x_3^{23}x_4^{46}x_5^{7}$ 714. $x_1^{15}x_2x_3^{23}x_4^{47}x_5^{6}$
715. $x_1^{15}x_2x_3^{30}x_4^{7}x_5^{39}$ 716. $x_1^{15}x_2x_3^{30}x_4^{39}x_5^{7}$ 717. $x_1^{15}x_2x_3^{31}x_4^{6}x_5^{39}$
718. $x_1^{15}x_2x_3^{31}x_4^{7}x_5^{38}$ 719. $x_1^{15}x_2x_3^{31}x_4^{38}x_5^{7}$ 720. $x_1^{15}x_2x_3^{31}x_4^{39}x_5^{6}$
721. $x_1^{15}x_2x_3^{54}x_4^{7}x_5^{15}$ 722. $x_1^{15}x_2x_3^{54}x_4^{15}x_5^{7}$ 723. $x_1^{15}x_2x_3^{55}x_4^{6}x_5^{15}$
724. $x_1^{15}x_2x_3^{55}x_4^{7}x_5^{14}$ 725. $x_1^{15}x_2x_3^{55}x_4^{14}x_5^{7}$ 726. $x_1^{15}x_2x_3^{55}x_4^{15}x_5^{6}$
727. $x_1^{15}x_2x_3^{62}x_4^{7}x_5^{7}$ 728. $x_1^{15}x_2^{3}x_3^{5}x_4^{7}x_5^{62}$ 729. $x_1^{15}x_2^{3}x_3^{5}x_4^{14}x_5^{55}$
730. $x_1^{15}x_2^{3}x_3^{5}x_4^{15}x_5^{54}$ 731. $x_1^{15}x_2^{3}x_3^{5}x_4^{22}x_5^{47}$ 732. $x_1^{15}x_2^{3}x_3^{5}x_4^{23}x_5^{46}$
733. $x_1^{15}x_2^{3}x_3^{5}x_4^{30}x_5^{39}$ 734. $x_1^{15}x_2^{3}x_3^{5}x_4^{31}x_5^{38}$ 735. $x_1^{15}x_2^{3}x_3^{5}x_4^{54}x_5^{15}$
736. $x_1^{15}x_2^{3}x_3^{5}x_4^{55}x_5^{14}$ 737. $x_1^{15}x_2^{3}x_3^{5}x_4^{62}x_5^{7}$ 738. $x_1^{15}x_2^{3}x_3^{7}x_4^{5}x_5^{62}$
739. $x_1^{15}x_2^{3}x_3^{7}x_4^{13}x_5^{54}$ 740. $x_1^{15}x_2^{3}x_3^{7}x_4^{21}x_5^{46}$ 741. $x_1^{15}x_2^{3}x_3^{7}x_4^{29}x_5^{38}$
742. $x_1^{15}x_2^{3}x_3^{7}x_4^{53}x_5^{14}$ 743. $x_1^{15}x_2^{3}x_3^{7}x_4^{61}x_5^{6}$ 744. $x_1^{15}x_2^{3}x_3^{13}x_4^{6}x_5^{55}$
745. $x_1^{15}x_2^{3}x_3^{13}x_4^{7}x_5^{54}$ 746. $x_1^{15}x_2^{3}x_3^{13}x_4^{22}x_5^{39}$ 747. $x_1^{15}x_2^{3}x_3^{13}x_4^{23}x_5^{38}$
748. $x_1^{15}x_2^{3}x_3^{13}x_4^{54}x_5^{7}$ 749. $x_1^{15}x_2^{3}x_3^{13}x_4^{55}x_5^{6}$ 750. $x_1^{15}x_2^{3}x_3^{15}x_4^{5}x_5^{54}$
751. $x_1^{15}x_2^{3}x_3^{15}x_4^{21}x_5^{38}$ 752. $x_1^{15}x_2^{3}x_3^{15}x_4^{53}x_5^{6}$ 753. $x_1^{15}x_2^{3}x_3^{21}x_4^{6}x_5^{47}$
754. $x_1^{15}x_2^{3}x_3^{21}x_4^{7}x_5^{46}$ 755. $x_1^{15}x_2^{3}x_3^{21}x_4^{14}x_5^{39}$ 756. $x_1^{15}x_2^{3}x_3^{21}x_4^{15}x_5^{38}$
757. $x_1^{15}x_2^{3}x_3^{21}x_4^{38}x_5^{15}$ 758. $x_1^{15}x_2^{3}x_3^{21}x_4^{39}x_5^{14}$ 759. $x_1^{15}x_2^{3}x_3^{21}x_4^{46}x_5^{7}$
760. $x_1^{15}x_2^{3}x_3^{21}x_4^{47}x_5^{6}$ 761. $x_1^{15}x_2^{3}x_3^{23}x_4^{5}x_5^{46}$ 762. $x_1^{15}x_2^{3}x_3^{23}x_4^{13}x_5^{38}$
763. $x_1^{15}x_2^{3}x_3^{23}x_4^{37}x_5^{14}$ 764. $x_1^{15}x_2^{3}x_3^{23}x_4^{45}x_5^{6}$ 765. $x_1^{15}x_2^{3}x_3^{29}x_4^{6}x_5^{39}$
766. $x_1^{15}x_2^{3}x_3^{29}x_4^{7}x_5^{38}$ 767. $x_1^{15}x_2^{3}x_3^{29}x_4^{38}x_5^{7}$ 768. $x_1^{15}x_2^{3}x_3^{29}x_4^{39}x_5^{6}$
769. $x_1^{15}x_2^{3}x_3^{31}x_4^{5}x_5^{38}$ 770. $x_1^{15}x_2^{3}x_3^{31}x_4^{37}x_5^{6}$ 771. $x_1^{15}x_2^{3}x_3^{53}x_4^{6}x_5^{15}$
772. $x_1^{15}x_2^{3}x_3^{53}x_4^{7}x_5^{14}$ 773. $x_1^{15}x_2^{3}x_3^{53}x_4^{14}x_5^{7}$ 774. $x_1^{15}x_2^{3}x_3^{53}x_4^{15}x_5^{6}$
775. $x_1^{15}x_2^{3}x_3^{55}x_4^{5}x_5^{14}$ 776. $x_1^{15}x_2^{3}x_3^{55}x_4^{13}x_5^{6}$ 777. $x_1^{15}x_2^{3}x_3^{61}x_4^{6}x_5^{7}$
778. $x_1^{15}x_2^{3}x_3^{61}x_4^{7}x_5^{6}$ 779. $x_1^{15}x_2^{7}x_3x_4^{7}x_5^{62}$ 780. $x_1^{15}x_2^{7}x_3x_4^{14}x_5^{55}$
781. $x_1^{15}x_2^{7}x_3x_4^{15}x_5^{54}$ 782. $x_1^{15}x_2^{7}x_3x_4^{22}x_5^{47}$ 783. $x_1^{15}x_2^{7}x_3x_4^{23}x_5^{46}$
784. $x_1^{15}x_2^{7}x_3x_4^{30}x_5^{39}$ 785. $x_1^{15}x_2^{7}x_3x_4^{31}x_5^{38}$ 786. $x_1^{15}x_2^{7}x_3x_4^{54}x_5^{15}$
787. $x_1^{15}x_2^{7}x_3x_4^{55}x_5^{14}$ 788. $x_1^{15}x_2^{7}x_3x_4^{62}x_5^{7}$ 789. $x_1^{15}x_2^{7}x_3^{3}x_4^{5}x_5^{62}$

790. $x_1^{15}x_2^{7}x_3^{3}x_4^{13}x_5^{54}$ 791. $x_1^{15}x_2^{7}x_3^{3}x_4^{21}x_5^{46}$ 792. $x_1^{15}x_2^{7}x_3^{3}x_4^{29}x_5^{38}$
793. $x_1^{15}x_2^{7}x_3^{3}x_4^{53}x_5^{14}$ 794. $x_1^{15}x_2^{7}x_3^{3}x_4^{61}x_5^{6}$ 795. $x_1^{15}x_2^{7}x_3^{7}x_4x_5^{62}$
796. $x_1^{15}x_2^{7}x_3^{7}x_4^{7}x_5^{56}$ 797. $x_1^{15}x_2^{7}x_3^{7}x_4^{9}x_5^{54}$ 798. $x_1^{15}x_2^{7}x_3^{7}x_4^{15}x_5^{48}$
799. $x_1^{15}x_2^{7}x_3^{7}x_4^{17}x_5^{46}$ 800. $x_1^{15}x_2^{7}x_3^{7}x_4^{23}x_5^{40}$ 801. $x_1^{15}x_2^{7}x_3^{7}x_4^{25}x_5^{38}$
802. $x_1^{15}x_2^{7}x_3^{7}x_4^{49}x_5^{14}$ 803. $x_1^{15}x_2^{7}x_3^{7}x_4^{55}x_5^{8}$ 804. $x_1^{15}x_2^{7}x_3^{7}x_4^{57}x_5^{6}$
805. $x_1^{15}x_2^{7}x_3^{11}x_4^{5}x_5^{54}$ 806. $x_1^{15}x_2^{7}x_3^{11}x_4^{21}x_5^{38}$ 807. $x_1^{15}x_2^{7}x_3^{11}x_4^{53}x_5^{6}$
808. $x_1^{15}x_2^{7}x_3^{15}x_4x_5^{54}$ 809. $x_1^{15}x_2^{7}x_3^{15}x_4^{7}x_5^{48}$ 810. $x_1^{15}x_2^{7}x_3^{15}x_4^{16}x_5^{39}$
811. $x_1^{15}x_2^{7}x_3^{15}x_4^{17}x_5^{38}$ 812. $x_1^{15}x_2^{7}x_3^{15}x_4^{23}x_5^{32}$ 813. $x_1^{15}x_2^{7}x_3^{15}x_4^{48}x_5^{7}$
814. $x_1^{15}x_2^{7}x_3^{15}x_4^{49}x_5^{6}$ 815. $x_1^{15}x_2^{7}x_3^{19}x_4^{5}x_5^{46}$ 816. $x_1^{15}x_2^{7}x_3^{19}x_4^{13}x_5^{38}$
817. $x_1^{15}x_2^{7}x_3^{19}x_4^{37}x_5^{14}$ 818. $x_1^{15}x_2^{7}x_3^{19}x_4^{45}x_5^{6}$ 819. $x_1^{15}x_2^{7}x_3^{23}x_4x_5^{46}$
820. $x_1^{15}x_2^{7}x_3^{23}x_4^{7}x_5^{40}$ 821. $x_1^{15}x_2^{7}x_3^{23}x_4^{9}x_5^{38}$ 822. $x_1^{15}x_2^{7}x_3^{23}x_4^{33}x_5^{14}$
823. $x_1^{15}x_2^{7}x_3^{23}x_4^{41}x_5^{6}$ 824. $x_1^{15}x_2^{7}x_3^{27}x_4^{5}x_5^{38}$ 825. $x_1^{15}x_2^{7}x_3^{27}x_4^{37}x_5^{6}$
826. $x_1^{15}x_2^{7}x_3^{31}x_4x_5^{38}$ 827. $x_1^{15}x_2^{7}x_3^{51}x_4^{5}x_5^{14}$ 828. $x_1^{15}x_2^{7}x_3^{51}x_4^{13}x_5^{6}$
829. $x_1^{15}x_2^{7}x_3^{55}x_4x_5^{14}$ 830. $x_1^{15}x_2^{7}x_3^{55}x_4^{7}x_5^{8}$ 831. $x_1^{15}x_2^{7}x_3^{55}x_4^{9}x_5^{6}$
832. $x_1^{15}x_2^{7}x_3^{59}x_4^{5}x_5^{6}$ 833. $x_1^{15}x_2^{15}x_3x_4^{6}x_5^{55}$ 834. $x_1^{15}x_2^{15}x_3x_4^{7}x_5^{54}$
835. $x_1^{15}x_2^{15}x_3x_4^{22}x_5^{39}$ 836. $x_1^{15}x_2^{15}x_3x_4^{23}x_5^{38}$ 837. $x_1^{15}x_2^{15}x_3x_4^{54}x_5^{7}$
838. $x_1^{15}x_2^{15}x_3x_4^{55}x_5^{6}$ 839. $x_1^{15}x_2^{15}x_3^{3}x_4^{5}x_5^{54}$ 840. $x_1^{15}x_2^{15}x_3^{3}x_4^{21}x_5^{38}$
841. $x_1^{15}x_2^{15}x_3^{3}x_4^{53}x_5^{6}$ 842. $x_1^{15}x_2^{15}x_3^{7}x_4x_5^{54}$ 843. $x_1^{15}x_2^{15}x_3^{7}x_4^{7}x_5^{48}$
844. $x_1^{15}x_2^{15}x_3^{7}x_4^{16}x_5^{39}$ 845. $x_1^{15}x_2^{15}x_3^{7}x_4^{17}x_5^{38}$ 846. $x_1^{15}x_2^{15}x_3^{7}x_4^{23}x_5^{32}$
847. $x_1^{15}x_2^{15}x_3^{7}x_4^{48}x_5^{7}$ 848. $x_1^{15}x_2^{15}x_3^{7}x_4^{49}x_5^{6}$ 849. $x_1^{15}x_2^{15}x_3^{17}x_4^{6}x_5^{39}$
850. $x_1^{15}x_2^{15}x_3^{17}x_4^{7}x_5^{38}$ 851. $x_1^{15}x_2^{15}x_3^{17}x_4^{38}x_5^{7}$ 852. $x_1^{15}x_2^{15}x_3^{17}x_4^{39}x_5^{6}$
853. $x_1^{15}x_2^{15}x_3^{19}x_4^{5}x_5^{38}$ 854. $x_1^{15}x_2^{15}x_3^{19}x_4^{37}x_5^{6}$ 855. $x_1^{15}x_2^{15}x_3^{23}x_4x_5^{38}$
856. $x_1^{15}x_2^{15}x_3^{23}x_4^{7}x_5^{32}$ 857. $x_1^{15}x_2^{15}x_3^{23}x_4^{33}x_5^{6}$ 858. $x_1^{15}x_2^{15}x_3^{49}x_4^{6}x_5^{7}$
859. $x_1^{15}x_2^{15}x_3^{49}x_4^{7}x_5^{6}$ 860. $x_1^{15}x_2^{15}x_3^{51}x_4^{5}x_5^{6}$ 861. $x_1^{15}x_2^{15}x_3^{55}x_4x_5^{6}$
862. $x_1^{15}x_2^{19}x_3^{7}x_4^{13}x_5^{38}$ 863. $x_1^{15}x_2^{23}x_3x_4^{6}x_5^{47}$ 864. $x_1^{15}x_2^{23}x_3x_4^{7}x_5^{46}$
865. $x_1^{15}x_2^{23}x_3x_4^{14}x_5^{39}$ 866. $x_1^{15}x_2^{23}x_3x_4^{15}x_5^{38}$ 867. $x_1^{15}x_2^{23}x_3x_4^{38}x_5^{15}$
868. $x_1^{15}x_2^{23}x_3x_4^{39}x_5^{14}$ 869. $x_1^{15}x_2^{23}x_3x_4^{46}x_5^{7}$ 870. $x_1^{15}x_2^{23}x_3x_4^{47}x_5^{6}$
871. $x_1^{15}x_2^{23}x_3^{3}x_4^{5}x_5^{46}$ 872. $x_1^{15}x_2^{23}x_3^{3}x_4^{13}x_5^{38}$ 873. $x_1^{15}x_2^{23}x_3^{3}x_4^{37}x_5^{14}$
874. $x_1^{15}x_2^{23}x_3^{3}x_4^{45}x_5^{6}$ 875. $x_1^{15}x_2^{23}x_3^{7}x_4x_5^{46}$ 876. $x_1^{15}x_2^{23}x_3^{7}x_4^{7}x_5^{40}$
877. $x_1^{15}x_2^{23}x_3^{7}x_4^{9}x_5^{38}$ 878. $x_1^{15}x_2^{23}x_3^{7}x_4^{33}x_5^{14}$ 879. $x_1^{15}x_2^{23}x_3^{7}x_4^{41}x_5^{6}$
880. $x_1^{15}x_2^{23}x_3^{11}x_4^{5}x_5^{38}$ 881. $x_1^{15}x_2^{23}x_3^{11}x_4^{37}x_5^{6}$ 882. $x_1^{15}x_2^{23}x_3^{15}x_4x_5^{38}$
883. $x_1^{15}x_2^{23}x_3^{35}x_4^{5}x_5^{14}$ 884. $x_1^{15}x_2^{23}x_3^{35}x_4^{13}x_5^{6}$ 885. $x_1^{15}x_2^{23}x_3^{39}x_4x_5^{14}$
886. $x_1^{15}x_2^{23}x_3^{43}x_4^{5}x_5^{6}$ 887. $x_1^{15}x_2^{23}x_3^{47}x_4x_5^{6}$ 888. $x_1^{15}x_2^{31}x_3x_4^{6}x_5^{39}$
889. $x_1^{15}x_2^{31}x_3x_4^{7}x_5^{38}$ 890. $x_1^{15}x_2^{31}x_3x_4^{38}x_5^{7}$ 891. $x_1^{15}x_2^{31}x_3x_4^{39}x_5^{6}$
892. $x_1^{15}x_2^{31}x_3^{3}x_4^{5}x_5^{38}$ 893. $x_1^{15}x_2^{31}x_3^{3}x_4^{37}x_5^{6}$ 894. $x_1^{15}x_2^{31}x_3^{7}x_4x_5^{38}$
895. $x_1^{15}x_2^{31}x_3^{39}x_4x_5^{6}$ 896. $x_1^{15}x_2^{55}x_3x_4^{6}x_5^{15}$ 897. $x_1^{15}x_2^{55}x_3x_4^{7}x_5^{14}$
898. $x_1^{15}x_2^{55}x_3x_4^{14}x_5^{7}$ 899. $x_1^{15}x_2^{55}x_3x_4^{15}x_5^{6}$ 900. $x_1^{15}x_2^{55}x_3^{3}x_4^{5}x_5^{14}$
901. $x_1^{15}x_2^{55}x_3^{3}x_4^{13}x_5^{6}$ 902. $x_1^{15}x_2^{55}x_3^{7}x_4x_5^{14}$ 903. $x_1^{15}x_2^{55}x_3^{7}x_4^{7}x_5^{8}$
904. $x_1^{15}x_2^{55}x_3^{7}x_4^{9}x_5^{6}$ 905. $x_1^{15}x_2^{55}x_3^{11}x_4^{5}x_5^{6}$ 906. $x_1^{15}x_2^{55}x_3^{15}x_4x_5^{6}$
907. $x_1^{31}x_2x_3^{6}x_4^{7}x_5^{47}$ 908. $x_1^{31}x_2x_3^{6}x_4^{15}x_5^{39}$ 909. $x_1^{31}x_2x_3^{6}x_4^{39}x_5^{15}$
910. $x_1^{31}x_2x_3^{6}x_4^{47}x_5^{7}$ 911. $x_1^{31}x_2x_3^{7}x_4^{6}x_5^{47}$ 912. $x_1^{31}x_2x_3^{7}x_4^{7}x_5^{46}$
913. $x_1^{31}x_2x_3^{7}x_4^{14}x_5^{39}$ 914. $x_1^{31}x_2x_3^{7}x_4^{15}x_5^{38}$ 915. $x_1^{31}x_2x_3^{7}x_4^{38}x_5^{15}$
916. $x_1^{31}x_2x_3^{7}x_4^{39}x_5^{14}$ 917. $x_1^{31}x_2x_3^{7}x_4^{46}x_5^{7}$ 918. $x_1^{31}x_2x_3^{7}x_4^{47}x_5^{6}$

919. $x_1^{31}x_2x_3^{14}x_4^{7}x_5^{39}$
920. $x_1^{31}x_2x_3^{14}x_4^{39}x_5^{7}$
921. $x_1^{31}x_2x_3^{15}x_4^{6}x_5^{39}$
922. $x_1^{31}x_2x_3^{15}x_4^{7}x_5^{38}$
923. $x_1^{31}x_2x_3^{15}x_4^{38}x_5^{7}$
924. $x_1^{31}x_2x_3^{15}x_4^{39}x_5^{6}$
925. $x_1^{31}x_2x_3^{38}x_4^{7}x_5^{15}$
926. $x_1^{31}x_2x_3^{38}x_4^{15}x_5^{7}$
927. $x_1^{31}x_2x_3^{39}x_4^{6}x_5^{15}$
928. $x_1^{31}x_2x_3^{39}x_4^{7}x_5^{14}$
929. $x_1^{31}x_2x_3^{39}x_4^{14}x_5^{7}$
930. $x_1^{31}x_2x_3^{39}x_4^{15}x_5^{6}$
931. $x_1^{31}x_2x_3^{46}x_4^{7}x_5^{7}$
932. $x_1^{31}x_2x_3^{47}x_4^{6}x_5^{7}$
933. $x_1^{31}x_2x_3^{47}x_4^{7}x_5^{6}$
934. $x_1^{31}x_2^{3}x_3^{5}x_4^{6}x_5^{47}$
935. $x_1^{31}x_2^{3}x_3^{5}x_4^{7}x_5^{46}$
936. $x_1^{31}x_2^{3}x_3^{5}x_4^{14}x_5^{39}$
937. $x_1^{31}x_2^{3}x_3^{5}x_4^{15}x_5^{38}$
938. $x_1^{31}x_2^{3}x_3^{5}x_4^{38}x_5^{15}$
939. $x_1^{31}x_2^{3}x_3^{5}x_4^{39}x_5^{14}$
940. $x_1^{31}x_2^{3}x_3^{5}x_4^{46}x_5^{7}$
941. $x_1^{31}x_2^{3}x_3^{5}x_4^{47}x_5^{6}$
942. $x_1^{31}x_2^{3}x_3^{7}x_4^{5}x_5^{46}$
943. $x_1^{31}x_2^{3}x_3^{7}x_4^{13}x_5^{38}$
944. $x_1^{31}x_2^{3}x_3^{7}x_4^{37}x_5^{14}$
945. $x_1^{31}x_2^{3}x_3^{7}x_4^{45}x_5^{6}$
946. $x_1^{31}x_2^{3}x_3^{13}x_4^{6}x_5^{39}$
947. $x_1^{31}x_2^{3}x_3^{13}x_4^{7}x_5^{38}$
948. $x_1^{31}x_2^{3}x_3^{13}x_4^{38}x_5^{7}$
949. $x_1^{31}x_2^{3}x_3^{13}x_4^{39}x_5^{6}$
950. $x_1^{31}x_2^{3}x_3^{15}x_4^{5}x_5^{38}$
951. $x_1^{31}x_2^{3}x_3^{15}x_4^{37}x_5^{6}$
952. $x_1^{31}x_2^{3}x_3^{37}x_4^{6}x_5^{15}$
953. $x_1^{31}x_2^{3}x_3^{37}x_4^{7}x_5^{14}$
954. $x_1^{31}x_2^{3}x_3^{37}x_4^{14}x_5^{7}$
955. $x_1^{31}x_2^{3}x_3^{37}x_4^{15}x_5^{6}$
956. $x_1^{31}x_2^{3}x_3^{39}x_4^{5}x_5^{14}$
957. $x_1^{31}x_2^{3}x_3^{39}x_4^{13}x_5^{6}$
958. $x_1^{31}x_2^{3}x_3^{45}x_4^{6}x_5^{7}$
959. $x_1^{31}x_2^{3}x_3^{45}x_4^{7}x_5^{6}$
960. $x_1^{31}x_2^{3}x_3^{47}x_4^{5}x_5^{6}$
961. $x_1^{31}x_2^{7}x_3x_4^{6}x_5^{47}$
962. $x_1^{31}x_2^{7}x_3x_4^{7}x_5^{46}$
963. $x_1^{31}x_2^{7}x_3x_4^{14}x_5^{39}$
964. $x_1^{31}x_2^{7}x_3x_4^{15}x_5^{38}$
965. $x_1^{31}x_2^{7}x_3x_4^{38}x_5^{15}$
966. $x_1^{31}x_2^{7}x_3x_4^{39}x_5^{14}$
967. $x_1^{31}x_2^{7}x_3x_4^{46}x_5^{7}$
968. $x_1^{31}x_2^{7}x_3x_4^{47}x_5^{6}$
969. $x_1^{31}x_2^{7}x_3^{3}x_4^{5}x_5^{46}$
970. $x_1^{31}x_2^{7}x_3^{3}x_4^{13}x_5^{38}$
971. $x_1^{31}x_2^{7}x_3^{3}x_4^{37}x_5^{14}$
972. $x_1^{31}x_2^{7}x_3^{3}x_4^{45}x_5^{6}$
973. $x_1^{31}x_2^{7}x_3^{7}x_4x_5^{46}$
974. $x_1^{31}x_2^{7}x_3^{7}x_4^{7}x_5^{40}$
975. $x_1^{31}x_2^{7}x_3^{7}x_4^{9}x_5^{38}$
976. $x_1^{31}x_2^{7}x_3^{7}x_4^{33}x_5^{14}$
977. $x_1^{31}x_2^{7}x_3^{7}x_4^{41}x_5^{6}$
978. $x_1^{31}x_2^{7}x_3^{11}x_4^{5}x_5^{38}$
979. $x_1^{31}x_2^{7}x_3^{11}x_4^{37}x_5^{6}$
980. $x_1^{31}x_2^{7}x_3^{15}x_4x_5^{38}$
981. $x_1^{31}x_2^{7}x_3^{35}x_4^{5}x_5^{14}$
982. $x_1^{31}x_2^{7}x_3^{35}x_4^{13}x_5^{6}$
983. $x_1^{31}x_2^{7}x_3^{39}x_4x_5^{14}$
984. $x_1^{31}x_2^{7}x_3^{43}x_4^{5}x_5^{6}$
985. $x_1^{31}x_2^{7}x_3^{47}x_4x_5^{6}$
986. $x_1^{31}x_2^{15}x_3x_4^{6}x_5^{39}$
987. $x_1^{31}x_2^{15}x_3x_4^{7}x_5^{38}$
988. $x_1^{31}x_2^{15}x_3x_4^{38}x_5^{7}$
989. $x_1^{31}x_2^{15}x_3x_4^{39}x_5^{6}$
990. $x_1^{31}x_2^{15}x_3^{3}x_4^{5}x_5^{38}$
991. $x_1^{31}x_2^{15}x_3^{3}x_4^{37}x_5^{6}$
992. $x_1^{31}x_2^{15}x_3^{7}x_4x_5^{38}$
993. $x_1^{31}x_2^{15}x_3^{39}x_4x_5^{6}$
994. $x_1^{31}x_2^{39}x_3x_4^{6}x_5^{15}$
995. $x_1^{31}x_2^{39}x_3x_4^{7}x_5^{14}$
996. $x_1^{31}x_2^{39}x_3x_4^{14}x_5^{7}$
997. $x_1^{31}x_2^{39}x_3x_4^{15}x_5^{6}$
998. $x_1^{31}x_2^{39}x_3^{3}x_4^{5}x_5^{14}$
999. $x_1^{31}x_2^{39}x_3^{3}x_4^{13}x_5^{6}$
1000. $x_1^{31}x_2^{39}x_3^{7}x_4x_5^{14}$
1001. $x_1^{31}x_2^{39}x_3^{15}x_4x_5^{6}$
1002. $x_1^{31}x_2^{47}x_3x_4^{6}x_5^{7}$
1003. $x_1^{31}x_2^{47}x_3x_4^{7}x_5^{6}$
1004. $x_1^{31}x_2^{47}x_3^{3}x_4^{5}x_5^{6}$
1005. $x_1^{31}x_2^{47}x_3^{7}x_4x_5^{6}$

$B_5^+((4)|^3|(2)|(3)) = \{\bar{c}_t : 1 \leqslant t \leqslant 104\}$, where

1. $x_1x_2^{14}x_3^{23}x_4^{23}x_5^{31}$
2. $x_1x_2^{14}x_3^{23}x_4^{31}x_5^{23}$
3. $x_1x_2^{14}x_3^{31}x_4^{23}x_5^{23}$
4. $x_1x_2^{15}x_3^{22}x_4^{23}x_5^{31}$
5. $x_1x_2^{15}x_3^{22}x_4^{31}x_5^{23}$
6. $x_1x_2^{15}x_3^{23}x_4^{22}x_5^{31}$
7. $x_1x_2^{15}x_3^{23}x_4^{23}x_5^{30}$
8. $x_1x_2^{15}x_3^{23}x_4^{30}x_5^{23}$
9. $x_1x_2^{15}x_3^{23}x_4^{31}x_5^{22}$
10. $x_1x_2^{15}x_3^{30}x_4^{23}x_5^{23}$
11. $x_1x_2^{15}x_3^{31}x_4^{22}x_5^{23}$
12. $x_1x_2^{15}x_3^{31}x_4^{23}x_5^{22}$
13. $x_1x_2^{30}x_3^{15}x_4^{23}x_5^{23}$
14. $x_1x_2^{31}x_3^{14}x_4^{23}x_5^{23}$
15. $x_1x_2^{31}x_3^{15}x_4^{22}x_5^{23}$
16. $x_1x_2^{31}x_3^{15}x_4^{23}x_5^{22}$
17. $x_1^{3}x_2^{13}x_3^{22}x_4^{23}x_5^{31}$
18. $x_1^{3}x_2^{13}x_3^{22}x_4^{31}x_5^{23}$
19. $x_1^{3}x_2^{13}x_3^{23}x_4^{22}x_5^{31}$
20. $x_1^{3}x_2^{13}x_3^{23}x_4^{23}x_5^{30}$
21. $x_1^{3}x_2^{13}x_3^{23}x_4^{30}x_5^{23}$
22. $x_1^{3}x_2^{13}x_3^{23}x_4^{31}x_5^{22}$
23. $x_1^{3}x_2^{13}x_3^{30}x_4^{23}x_5^{23}$
24. $x_1^{3}x_2^{13}x_3^{31}x_4^{22}x_5^{23}$
25. $x_1^{3}x_2^{13}x_3^{31}x_4^{23}x_5^{22}$
26. $x_1^{3}x_2^{15}x_3^{21}x_4^{22}x_5^{31}$
27. $x_1^{3}x_2^{15}x_3^{21}x_4^{23}x_5^{30}$
28. $x_1^{3}x_2^{15}x_3^{21}x_4^{30}x_5^{23}$
29. $x_1^{3}x_2^{15}x_3^{21}x_4^{31}x_5^{22}$
30. $x_1^{3}x_2^{15}x_3^{23}x_4^{21}x_5^{30}$
31. $x_1^{3}x_2^{15}x_3^{23}x_4^{29}x_5^{22}$
32. $x_1^{3}x_2^{15}x_3^{29}x_4^{22}x_5^{23}$
33. $x_1^{3}x_2^{15}x_3^{29}x_4^{23}x_5^{22}$
34. $x_1^{3}x_2^{15}x_3^{31}x_4^{21}x_5^{22}$
35. $x_1^{3}x_2^{29}x_3^{14}x_4^{23}x_5^{23}$
36. $x_1^{3}x_2^{29}x_3^{15}x_4^{22}x_5^{23}$

| | | |
|---|---|---|
| 37. $x_1^{3}x_2^{29}x_3^{15}x_4^{23}x_5^{22}$ | 38. $x_1^{3}x_2^{31}x_3^{13}x_4^{22}x_5^{23}$ | 39. $x_1^{3}x_2^{31}x_3^{13}x_4^{23}x_5^{22}$ |
| 40. $x_1^{3}x_2^{31}x_3^{15}x_4^{21}x_5^{22}$ | 41. $x_1^{7}x_2^{11}x_3^{21}x_4^{22}x_5^{31}$ | 42. $x_1^{7}x_2^{11}x_3^{21}x_4^{23}x_5^{30}$ |
| 43. $x_1^{7}x_2^{11}x_3^{21}x_4^{30}x_5^{23}$ | 44. $x_1^{7}x_2^{11}x_3^{21}x_4^{31}x_5^{22}$ | 45. $x_1^{7}x_2^{11}x_3^{23}x_4^{21}x_5^{30}$ |
| 46. $x_1^{7}x_2^{11}x_3^{23}x_4^{29}x_5^{22}$ | 47. $x_1^{7}x_2^{11}x_3^{29}x_4^{22}x_5^{23}$ | 48. $x_1^{7}x_2^{11}x_3^{29}x_4^{23}x_5^{22}$ |
| 49. $x_1^{7}x_2^{11}x_3^{31}x_4^{21}x_5^{22}$ | 50. $x_1^{7}x_2^{15}x_3^{19}x_4^{21}x_5^{30}$ | 51. $x_1^{7}x_2^{15}x_3^{19}x_4^{29}x_5^{22}$ |
| 52. $x_1^{7}x_2^{15}x_3^{27}x_4^{21}x_5^{22}$ | 53. $x_1^{7}x_2^{27}x_3^{13}x_4^{22}x_5^{23}$ | 54. $x_1^{7}x_2^{27}x_3^{13}x_4^{23}x_5^{22}$ |
| 55. $x_1^{7}x_2^{27}x_3^{15}x_4^{21}x_5^{22}$ | 56. $x_1^{7}x_2^{31}x_3^{11}x_4^{21}x_5^{22}$ | 57. $x_1^{15}x_2x_3^{22}x_4^{23}x_5^{31}$ |
| 58. $x_1^{15}x_2x_3^{22}x_4^{31}x_5^{23}$ | 59. $x_1^{15}x_2x_3^{23}x_4^{22}x_5^{31}$ | 60. $x_1^{15}x_2x_3^{23}x_4^{23}x_5^{30}$ |
| 61. $x_1^{15}x_2x_3^{23}x_4^{30}x_5^{23}$ | 62. $x_1^{15}x_2x_3^{23}x_4^{31}x_5^{22}$ | 63. $x_1^{15}x_2x_3^{30}x_4^{23}x_5^{23}$ |
| 64. $x_1^{15}x_2x_3^{31}x_4^{22}x_5^{23}$ | 65. $x_1^{15}x_2x_3^{31}x_4^{23}x_5^{22}$ | 66. $x_1^{15}x_2^{3}x_3^{21}x_4^{22}x_5^{31}$ |
| 67. $x_1^{15}x_2^{3}x_3^{21}x_4^{23}x_5^{30}$ | 68. $x_1^{15}x_2^{3}x_3^{21}x_4^{30}x_5^{23}$ | 69. $x_1^{15}x_2^{3}x_3^{21}x_4^{31}x_5^{22}$ |
| 70. $x_1^{15}x_2^{3}x_3^{23}x_4^{21}x_5^{30}$ | 71. $x_1^{15}x_2^{3}x_3^{23}x_4^{29}x_5^{22}$ | 72. $x_1^{15}x_2^{3}x_3^{29}x_4^{22}x_5^{23}$ |
| 73. $x_1^{15}x_2^{3}x_3^{29}x_4^{23}x_5^{22}$ | 74. $x_1^{15}x_2^{3}x_3^{31}x_4^{21}x_5^{22}$ | 75. $x_1^{15}x_2^{7}x_3^{19}x_4^{21}x_5^{30}$ |
| 76. $x_1^{15}x_2^{7}x_3^{19}x_4^{29}x_5^{22}$ | 77. $x_1^{15}x_2^{7}x_3^{27}x_4^{21}x_5^{22}$ | 78. $x_1^{15}x_2^{15}x_3^{17}x_4^{22}x_5^{23}$ |
| 79. $x_1^{15}x_2^{15}x_3^{17}x_4^{23}x_5^{22}$ | 80. $x_1^{15}x_2^{15}x_3^{19}x_4^{21}x_5^{22}$ | 81. $x_1^{15}x_2^{23}x_3x_4^{22}x_5^{31}$ |
| 82. $x_1^{15}x_2^{23}x_3x_4^{23}x_5^{30}$ | 83. $x_1^{15}x_2^{23}x_3x_4^{30}x_5^{23}$ | 84. $x_1^{15}x_2^{23}x_3x_4^{31}x_5^{22}$ |
| 85. $x_1^{15}x_2^{23}x_3^{3}x_4^{21}x_5^{30}$ | 86. $x_1^{15}x_2^{23}x_3^{3}x_4^{29}x_5^{22}$ | 87. $x_1^{15}x_2^{23}x_3^{11}x_4^{21}x_5^{22}$ |
| 88. $x_1^{15}x_2^{23}x_3^{23}x_4x_5^{30}$ | 89. $x_1^{15}x_2^{23}x_3^{31}x_4x_5^{22}$ | 90. $x_1^{15}x_2^{31}x_3x_4^{22}x_5^{23}$ |
| 91. $x_1^{15}x_2^{31}x_3x_4^{23}x_5^{22}$ | 92. $x_1^{15}x_2^{31}x_3^{3}x_4^{21}x_5^{22}$ | 93. $x_1^{15}x_2^{31}x_3^{23}x_4x_5^{22}$ |
| 94. $x_1^{31}x_2x_3^{14}x_4^{23}x_5^{23}$ | 95. $x_1^{31}x_2x_3^{15}x_4^{22}x_5^{23}$ | 96. $x_1^{31}x_2x_3^{15}x_4^{23}x_5^{22}$ |
| 97. $x_1^{31}x_2^{3}x_3^{13}x_4^{22}x_5^{23}$ | 98. $x_1^{31}x_2^{3}x_3^{13}x_4^{23}x_5^{22}$ | 99. $x_1^{31}x_2^{3}x_3^{15}x_4^{21}x_5^{22}$ |
| 100. $x_1^{31}x_2^{7}x_3^{11}x_4^{21}x_5^{22}$ | 101. $x_1^{31}x_2^{15}x_3x_4^{22}x_5^{23}$ | 102. $x_1^{31}x_2^{15}x_3x_4^{23}x_5^{22}$ |
| 103. $x_1^{31}x_2^{15}x_3^{3}x_4^{21}x_5^{22}$ | 104. $x_1^{31}x_2^{15}x_3^{23}x_4x_5^{22}$ | |

$B_5^+((4)|^4|(2)) = \{\tilde{c}_t : 1 \leqslant t \leqslant 156\}$, where

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| 4. $x_1x_2^{15}x_3^{14}x_4^{31}x_5^{31}$ | 5. $x_1x_2^{15}x_3^{15}x_4^{30}x_5^{31}$ | 6. $x_1x_2^{15}x_3^{15}x_4^{31}x_5^{30}$ |
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| 10. $x_1x_2^{15}x_3^{31}x_4^{15}x_5^{30}$ | 11. $x_1x_2^{15}x_3^{31}x_4^{30}x_5^{15}$ | 12. $x_1x_2^{15}x_3^{31}x_4^{31}x_5^{14}$ |
| 13. $x_1x_2^{30}x_3^{15}x_4^{15}x_5^{31}$ | 14. $x_1x_2^{30}x_3^{15}x_4^{31}x_5^{15}$ | 15. $x_1x_2^{30}x_3^{31}x_4^{15}x_5^{15}$ |
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| 19. $x_1x_2^{31}x_3^{15}x_4^{15}x_5^{30}$ | 20. $x_1x_2^{31}x_3^{15}x_4^{30}x_5^{15}$ | 21. $x_1x_2^{31}x_3^{15}x_4^{31}x_5^{14}$ |
| 22. $x_1x_2^{31}x_3^{30}x_4^{15}x_5^{15}$ | 23. $x_1x_2^{31}x_3^{31}x_4^{14}x_5^{15}$ | 24. $x_1x_2^{31}x_3^{31}x_4^{15}x_5^{14}$ |
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| 28. $x_1^{3}x_2^{13}x_3^{30}x_4^{15}x_5^{31}$ | 29. $x_1^{3}x_2^{13}x_3^{30}x_4^{31}x_5^{15}$ | 30. $x_1^{3}x_2^{13}x_3^{31}x_4^{14}x_5^{31}$ |
| 31. $x_1^{3}x_2^{13}x_3^{31}x_4^{15}x_5^{30}$ | 32. $x_1^{3}x_2^{13}x_3^{31}x_4^{30}x_5^{15}$ | 33. $x_1^{3}x_2^{13}x_3^{31}x_4^{31}x_5^{14}$ |
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# References

[1] D. P. Carlisle and R. M. W. Wood, *The boundedness conjecture for the action of the Steenrod algebra on polynomials*, in: N. Ray and G. Walker (ed.), Adams Memorial Symposium on Algebraic Topology 2, (Manchester, 1990), in: London Math. Soc. Lecture Notes Ser., Cambridge Univ. Press, Cambridge, vol. 176, 1992, pp. 203-216, MR1232207.

[2] M. C. Crabb and J. R. Hubbuck, *Representations of the homology of BV and the Steenrod algebra II*, Algebraic Topology: new trend in localization and periodicity,

Progr. Math. 136 (1996) 143-154, MR1397726.

[3] A. S. Janfada and R. M. W. Wood, *The hit problem for symmetric polynomials over the Steenrod algebra*, Math. Proc. Cambridge Philos. Soc.133 (2002) 295-303, MR1912402.

[4] M. Kameko, *Products of projective spaces as Steenrod modules*, PhD. Thesis, The Johns Hopkins University, ProQuest LLC, Ann Arbor, MI, 1990. 29 pp., MR2638633.

[5] M. F. Mothebe and L. Uys, *Some relations between admissible monomials for the polynomial algebra*, Int. J. Math. Math. Sci. 2015, Art. ID: 235806, 7 pp., MR3388909.

[6] T. N. Nam, *$\mathcal{A}$-générateurs génériquess pour l'algèbre polynomiale*, Adv. Math. 186 (2004) 334-362, MR2073910.

[7] F. P. Peterson, *Generators of $H^*(\mathbb{R}P^\infty \times \mathbb{R}P^\infty)$ as a module over the Steenrod algebra*, Abstracts Amer. Math. Soc. No. 833 (1987) 55-89.

[8] D. V. Phuc, *On the fifth Singer algebraic transfer in a generic family of internal degree characterized by $\mu(n) = 4$*, J. Pure Appl. Algebra 228 (2024) 107658.

[9] D. V. Phuc and N. Sum, *On the generators of the polynomial algebra as a module over the Steenrod algebra*, C. R. Acad. Sci. Paris, Ser. I 353 (2015) 1035-1040, MR3419856.

[10] D. V. Phuc and N. Sum, *On a minimal set of generators for the polynomial algebra of five variables as a module over the Steenrod algebra*, Acta Math. Vietnam. 42 (2017) 149-162, MR3595451.

[11] S. Priddy, *On characterizing summands in the classifying space of a group, I*, Amer. Jour. Math. 112 (1990) 737-748, MR1073007.

[12] J. Repka and P. Selick, *On the subalgebra of $H_*((\mathbb{R}P^\infty)^n;\mathbb{F}_2)$ annihilated by Steenrod operations*, J. Pure Appl. Algebra 127 (1998) 273-288, MR1617199.

[13] J. H. Silverman, *Hit polynomials and the canonical antiautomorphism of the Steenrod algebra*, Proc. Amer. Math. Soc. 123 (1995) 627-637, MR1254854.

[14] J. H. Silverman, *Hit polynomials and conjugation in the dual Steenrod algebra*, Math. Proc. Cambridge Philos. Soc. 123 (1998) 531-547, MR1607993.

[15] J. H. Silverman and W. M. Singer, *On the action of Steenrod squares on polynomial algebras II*, J. Pure Appl. Algebra 98 (1995) 95-103, MR1317001.

[16] W. M. Singer, *The transfer in homological algebra*, Math. Zeit. 202 (1989) 493-523, MR1022818.

[17] W. M. Singer, *On the action of the Steenrod squares on polynomial algebras*, Proc. Amer. Math. Soc. 111 (1991) 577-583, MR1045150.

[18] N. E. Steenrod and D. B. A. Epstein, *Cohomology operations*, Annals of Mathematics Studies 50, Princeton University Press, Princeton N.J (1962), MR0145525.

[19] N. Sum, *The negative answer to Kameko's conjecture on the hit problem*, Adv. Math. 225 (2010) 2365-2390, MR2680169.

[20] N. Sum, *On the Peterson hit problem*, Adv. Math. 274 (2015) 432-489, MR3318156.

[21] N. Sum, *The squaring operation and the Singer algebraic transfer*, Vietnam J. Math. 49 (2021), 1079-1096, MR4319539.

[22] N. Sum, *The squaring operation and the hit problem for the polynomial algebra in a type of generic degree*, J. Algebra 622 (2023), 165-196.

[23] N. Sum, *The admissible monomial bases for the polynomial algebra of five variables in some types of generic degrees*, Topology Appl. 349 (2024), 108909.

[24] N. Sum and N. K. Tin, *The hit problem for the polynomial algebra in some weight vectors*, Topology Appl. 290 (2021), 107579, 17 pp, MR4199843.

[25] N. K. Tin, *The admissible monomial basis for the polynomial algebra of five variables in degree eight*, Journal of Mathematical Sciences and Applications, 2 (2014), 21-24.

[26] N. K. Tin, *Tin, A note on the Peterson hit problem for the Steenrod algebra*, Proc. Japan Acad. Ser. A Math. Sci., 97 (2021), no. 4, 25-28, MR4255188.

[27] G. Walker and R. M. W. Wood, *Flag modules and the hit problem for the Steenrod algebra*, Math. Proc. Cambridge Philos. Soc. 147 (2009) 143-171, MR2507313.

[28] G. Walker and R. M. W. Wood, *Polynomials and the mod 2 Steenrod algebra, Vol. 1: The Peterson hit problem*, London Mathematical Society Lecture Note Series 441, Cambridge University Press, 2018, MR3729477.

[29] G. Walker and R. M. W. Wood, *Polynomials and the* mod 2 *Steenrod algebra. Vol. 2. Representations of* $\mathrm{GL}(n, \mathbb{F}_2)$, London Mathematical Society Lecture Note Series, 442. Cambridge University Press, 2018, MR3729478.

[30] R. M. W. Wood, *Steenrod squares of polynomials and the Peterson conjecture*, Math. Proc. Cambriges Phil. Soc. 105 (1989) 307-309, MR0974986.

[31] R. M. W. Wood, *Problems in the Steenrod algebra*, Bull. London Math. Soc. 30 (1998) 449-517, MR1643834.