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ON (σ,τ)-$\star$-DERIVATION AND COMMUTATIVITY OF $\star$-PRIME RINGS

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Abstract

In this paper we study the notion of (σ,τ)-$\star$-derivation and prove the following result: Let R be a $\star$-prime ring with characteristic different from two and $Z(R)$ be the center of R. If R admits a non-zero (σ,τ)-$\star$-derivation d of R, with associated automorphisms σ and τ of R, such that σ, τ and d commute with $\star$ satisfying $[d(U), d(U)]_{\sigma,\tau} = \{0\}$, then R is commutative, where U is an ideal of R such that $U^\star = U$.

1 Introduction

Throughout, R will denote an associative ring with center $Z(R)$. An additive mapping $d : R \to R$ is said to be a derivation of R if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$. For a fixed $a \in R$, the mapping $I_a : R \to R$ given by $I_a(x) = [a, x] = ax - xa$ is a derivation which is said to be an inner derivation. Recall that R is said to be prime if $aRb = \{0\}$ implies $a = 0$ or $b = 0$. A ring R is said to be 2-torsion free, if $2x = 0$ implies $x = 0$.

For any two endomorphisms σ and τ of R, we call an additive mapping $d : R \to R$ a (σ,τ)-derivation of R if $d(xy) = d(x)\sigma(y) + \tau(x)d(y)$ for all $x, y \in R$. Of course, a $(1,1)$-derivation is a derivation on R, where 1 is the

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identity mapping on R. We set $[x,y]_{\sigma,\tau} = x\sigma(y) - \tau(y)x$. In particular $[x,y]_{1,1} = [x,y] = xy - yx$, is the usual Lie product.

An additive mapping $x \mapsto x^\star$ on a ring R is called an involution if $(x^\star)^\star = x$ and $(xy)^\star = y^\star x^\star$ hold for all $x, y \in R$. A ring equipped with an involution is called a ring with involution or $\star$-ring. A ring R equipped with an involution $\star$ is said to be $\star$-prime if $aRb = aRb^\star = \{0\}$ (or, equivalently $aRb = a^\star Rb = \{0\}$) implies $a = 0$ or $b = 0$. It is important to note that, a prime ring is $\star$-prime, but the converse is in general not true. An example due to Shulaing [13] justifies this fact. If R° denotes the opposite ring of a prime ring R, then $S = R \times R^\circ$ equipped with the exchange involution $\star_{ex}$ defined by $\star_{ex}(x,y) = (y,x)$ is $\star_{ex}$-prime, but not a prime ring because of the fact that $(1,0)S(0,1) = 0$. In all that follows, $Sa_\star(R)$ will denote the set of symmetric and skew symmetric elements of R, i.e., $Sa_\star(R) = \{x \in R | x^\star = \pm x\}$. An ideal U of R is said to be a $\star$-ideal of R if $U^\star = U$. It can also be noted that an ideal of a ring R may not be $\star$-ideal of R. As an example, let $R = \mathbb{Z} \times \mathbb{Z}$, and consider an involution $\star$ on R such that $(a,b)^\star = (b,a)$ for all $(a,b) \in R$. The subset $U = \mathbb{Z} \times \{0\}$ of R is an ideal of R but it is not a $\star$-ideal of R, because $U^\star = \{0\} \times \mathbb{Z} \neq U$.

Let R be a ring with involution $\star$. An additive mapping $d : R \to R$ is said to be a $\star$-derivation if $d(xy) = d(x)y^\star + xd(y)$ holds for all $x, y \in R$. The concept of $\star$-derivation was introduced by Brešar and Vukman in [8]. In [1], Shakir and Fošner introduced (σ,τ)-$\star$-derivation as follows: Let σ and τ be two endomorphism of R. An additive mapping $d : R \to R$ is said to be (σ,τ)-$\star$-derivation if $d(xy) = d(x)\sigma(y^\star) + \tau(x)d(y)$, holds for all $x, y \in R$. In [8], Brešar and Vukman studied some algebraic properties of $\star$-derivations.

Recently many authors have studied commutativity of prime and semiprime rings with involution admitting suitably constrained derivations (for reference see [2, 12, 16, 20] etc). A lot of work have been done by L. Okhtite and his co-authors on rings with involution (see for reference [17, 18, 19], where further references can be found).

In [15], Lee and Lee proved that if a prime ring of characteristic different from 2 admits a derivation d such that $[d(R), d(R)] \subseteq Z(R)$, then R is commutative. On the other hand in [11] for $a \in R$, Herstein proved that if $[a, d(R)] = \{0\}$, then $a \in Z(R)$. Further in the year 1992, Aydin together with Kaya [7] extended the theorems mentioned above by replacing derivation by (σ,τ)-derivation and in some of those, R by a non-zero ideal of R. Recently, in [4] we investigated the commutativity of $\star$-prime ring R equipped with an involution $\star$ admitting a (σ,τ)-derivation d satisfying $[d(U), d(U)]_{\sigma,\tau} = \{0\}$, where U is a nonzero $\star$-ideal of R. In this paper we prove the above mentioned theorem in case of (σ,τ)-$\star$-derivation. In fact, it is shown that if a $\star$-prime ring admits a nonzero (σ,τ)-$\star$-derivation d satisfying $[d(U), d(U)]_{\sigma,\tau} = \{0\}$, then R is commutative.

2 The Results

In the remaining part of the paper, R will represent a $\star$-prime ring which admits a nonzero (σ,τ)-$\star$-derivation d with automorphisms σ and τ such that $\star$ commutes with d,σ and τ. We shall use the following relations frequently without specific mention:

$$[xy,z]_{\sigma,\tau}=x[y,z]_{\sigma,\tau}+[x,\tau(z)]y=x[y,\sigma(z)]+[x,z]_{\sigma,\tau}y,$$

$$[x,yz]_{\sigma,\tau}=\tau(y)[x,z]_{\sigma,\tau}+[x,y]_{\sigma,\tau}\sigma(z),$$

and

$$[x,[y,z]]_{\sigma,\tau}+[[x,z]_{\sigma,\tau},y]_{\sigma,\tau}-[[x,y]_{\sigma,\tau},z]_{\sigma,\tau}=0.$$

Remark 2.1. *We find that if R is a $\star$-prime ring with characteristic different from 2, then R is a 2-torsion free. In fact, if $2x=0$ for all $x\in R$, then $xr(2s)=0$ for all $r,s\in R$. But since char $R\neq 2$, there exists a nonzero $l\in R$ such that $2l\neq 0$ and hence by the above $xR2l=\{0\}$. This also gives that $xR(2l)^\star=\{0\}$ and $\star$-primeness of R yields that $x=0$, i.e., R is 2-torsion free.*

The main result of the present paper states as follows:

Theorem 2.2. Let R be a $\star$- prime ring with characteristic different from two and σ,τ be automorphisms of R, and U a $\star$-ideal of R. If R admits a nonzero (σ,τ)-$\star$-derivation $d:R\to R$ such that $[d(U),d(U)]_{\sigma,\tau}=\{0\}$, then R is commutative.

We facilitate our discussion with the following lemmas which are required for developing the proof of our main result.

Since every $\star$-prime ring is semiprime and every $\star$-right ideal is right ideal. Hence Lemma 1.1.5 of [9] can be rewritten in case of $\star$-prime ring as follows:

Lemma 2.3. Let R be a $\star$-prime ring and U a non-zero $\star$-right ideal of R. Then $Z(U)\subseteq Z(R)$.

Corollary 2.4. Let R be a $\star$-prime ring and U a non-zero $\star$-right ideal of R. If U is commutative then R is commutative.

Proof. Since U, is commutative, by the Lemma 2.3, we have $U=Z(U)\subseteq Z(R)$. If for any $x,y\in R$, $a\in U$ we have $ax\in U$ and hence $ax\in Z(R)$ and hence $(ax)y=y(ax)=ayx$. This further yields $U(xy-yx)=\{0\}$. Since U is a non-zero $\star$-right ideal of R, we have $UR(xy-yx)=\{0\}=U^\star R(xy-yx)$. Also, since $U\neq\{0\}$ right ideal, $\star$-primeness of R gives $xy-yx=0$, for all $x,y\in R$. Hence R is commutative. □

Lemma 2.5. Let R be a $\star$-prime ring and U a non-zero $\star$-right ideal of R. Suppose that $a \in R$ centralizes U. Then $a \in Z(R)$.

Proof. Since a centralizes U, for all $u \in U$ and $x \in R$, $aux = uxa$. But $au = ua$, therefore $uax = uxa$, i.e., $u[a,x] = 0$. On replacing u by uy for any $y \in R$, we get $uR[a,x] = \{0\}$ for all $u \in U$, $x \in R$. Also, since U is $\star$-right ideal, we get $u^\star R[a,x] = \{0\}$. Again since $U \neq \{0\}$, $\star$-primeness of R yields that $[a,x] = 0$ for all $x \in R$. Therefore, $a \in Z(R)$. □

Lemma 2.6. Let R be a $\star$-prime ring and U a $\star$-right ideal of R. Suppose d is a (σ,τ)-$\star$-derivation of R satisfying $d(U) = \{0\}$, then $d = 0$.

Proof. For all $u \in U$ and $x \in R$, $0 = d(ux) = d(u)\sigma(x^\star) + \tau(u)d(x) = \tau(u)d(x)$. On replacing x by xy for any $y \in R$, we get $\tau(u)d(x)\sigma(y^\star) + \tau(u)\tau(x)d(y) = 0$, or, $\tau(u)\tau(x)d(y) = 0$, i.e., $\tau(u)Rd(y) = \{0\}$ for all $u \in U$ and $y \in R$. Also since U is a $\star$-right ideal, we get $\tau(u)^\star Rd(y) = \{0\}$. Also, $\star$-primeness of R yields that $\tau(u) = 0$ for all $u \in U$ or $d = 0$. Since $U \neq \{0\}$, we get $d = 0$. □

Lemma 2.7. Let R be a $\star$-prime ring, U a non-zero $\star$-ideal of R and $a \in R$. Suppose d is a (σ,τ)-$\star$-derivation of R satisfying $ad(U) = \{0\}$ (or, $d(U)a = \{0\}$), then $a = 0$ or $d = 0$.

Proof. For $u \in U$, $x \in R$, $0 = ad(ux) = ad(u)\sigma(x^\star) + a\tau(u)d(x)$. By assumption, we have $a\tau(u)d(x) = 0$, for all $x \in R$. On replacing u by uy for any $y \in R$, we obtain $a\tau(u)Rd(x) = \{0\}$ for all $u \in U$, $x \in R$. Also, $a\tau(u)Rd(x)^\star = \{0\}$. Since R is $\star$-prime, we find that either $a\tau(u) = 0$ or $d(x) = 0$. If $a\tau(u) = 0$ for all $u \in U$, then or $\tau^{-1}(a)U = \{0\}$. Now since U is $\star$-ideal, we can write $\tau^{-1}(a)U^\star = \{0\}$. This implies that $\tau^{-1}(a)RU = \{0\} = \tau^{-1}(a)RU^\star$. By the $\star$-primeness of R, we obtain $\tau^{-1}(a) = 0$, since $U \neq \{0\}$. In conclusion, we get either $a = 0$ or $d = 0$. Similarly, $d(U)a = \{0\}$ implies $a = 0$ or $d = 0$. □

Lemma 2.8. Let d be a non-zero (σ,τ)-$\star$-derivation of $\star$-prime ring R and U a $\star$-right ideal of R. If $d(U) \subseteq Z(R)$, then R is commutative.

Proof. Since $d(U) \subseteq Z(R)$, we have $[d(U), R] = \{0\}$. For $u, v \in U$ and $x \in R$,

$$[x, d(uv)] = [x, d(u)\sigma(v^\star) + \tau(u)d(v)] = d(u)[x, \sigma(v^\star)] + d(v)[x, \tau(u)] = 0. \quad (1)$$

Replacing x by $x\sigma(v^\star)$, $v \in U$ in (1), we have

$$\begin{aligned} 0 &= d(u)[x\sigma(v^\star), \sigma(v^\star)] + d(v)[x\sigma(v^\star), \tau(u)] \\ &= d(u)[x, \sigma(v^\star)]\sigma(v^\star) + d(v)(x[\sigma(v^\star), \tau(u)] + [x, \tau(u)]\sigma(v^\star)). \end{aligned}$$

By using (1), we get

$$d(v)R[\sigma(v^\star), \tau(u)] = \{0\}, \text{ for all } u, v \in U. \quad (2)$$

Let $v \in U \cap Sa_\star(R)$. From (2), it follows that

$$d(v)^\star R[\sigma(v^\star), \tau(u)] = \{0\}, \text{ for all } u \in U. \quad (3)$$

By (2) and (3), the $\star$-primeness of R yields that $d(v)=0$ or $[\sigma(v^\star),\tau(u)]=0$ for all $u\in U$. Let $w\in U$, since $w-w^\star\in U\cap Sa_\star(R)$, then

$$d(w-w^\star)=0 \text{ or } [\sigma(w-w^\star)^\star,\tau(u)]=0.$$

Assume that $d(w-w^\star)=0$. Then $d(w)=d(w^\star)$. Replacing v by $w^\star$ in (2) and since U is $\star$-right ideal, we get $d(w^\star)R[\sigma(w^\star)^\star,\tau(u)]=\{0\}$ for all $u\in U$. Consequently,

$$d(w)R[\sigma(w^\star),\tau(u)]^\star=\{0\}, \text{ for all } u,w\in U. \tag{4}$$

Also by (2), we get $d(w)R[\sigma(w^\star),\tau(u)]=\{0\}$, on using $\star$-primeness of R together with (4), we find that for each $w\in U$ either $d(w)=0$ or $[\sigma(w)^\star,\tau(u)]=0$, for all $u\in U$. Now suppose the remaining case that $[\sigma(v)^\star,\tau(u)]=0$, for all $u\in U$. Then we have $[\sigma(w-w^\star)^\star,\tau(u)]=0=[\sigma(w-w^\star),\tau(u)]$, or $[\sigma(w),\tau(u)]=[\sigma(w^\star),\tau(u)]$. Replacing v by $w^\star$ in (2), we get $d(w^\star)R[\sigma(w^\star)^\star,\tau(u)]=\{0\}$ for all $u\in U$. Consequently, $d(w^\star)R[\sigma(w),\tau(u)]=\{0\}$. This yields that

$$\text{or, } d(w^\star)R[\sigma(w)^\star,\tau(u)]=\{0\}, \text{ for all } u,w\in U. \tag{5}$$

Since $d(w)R[\sigma(w^\star),\tau(u)]=\{0\}$, by (2), the $\star$-primeness of R together with (5) assure that for each $w\in U$ either $d(w)=0$ or $[\sigma(w^\star),\tau(u)]=0$, for all $u\in U$. In conclusion, for each fixed $w\in U$, we have

$$\text{either } d(w)=0 \text{ or } [\sigma(w^\star),\tau(u)]=0 \text{ for all } u\in U.$$

Now, define

$$K=\{w\in U\mid d(w)=0\} \text{ and } L=\{w\in U\mid [\sigma(w^\star),\tau(u)]=0 \text{ for all } u\in U\}.$$

Clearly both K and L are additive subgroups of U whose union is U. But a group cannot be a set theoretic union of two of it's proper subgroups and hence either $K=U$ or $L=U$. If $K=U$, then $d(U)=\{0\}$ and hence by Lemma 2.6, $d=0$, a contradiction, therefore now assume that $L=U$, i.e.,

$$[\sigma(w^\star),\tau(u)]=0 \text{ for all } u,w\in U. \tag{6}$$

Replacing $w^\star$ by $w'\sigma^{-1}(\tau(v))$, $u\in U$, in (6) and using (6), we get $\sigma(w')\tau([v,u])=0$, for all $u,v,w'\in U$. On replacing w' by $w'x$ for any $x\in R$, we get $\sigma(w')R\tau([v,u])=\{0\}$, for all $u,v,w'\in U$. Also, since U is $\star$-right ideal, we get $\sigma(w')^\star R\tau([v,u])=\{0\}$, for all $u,v,w'\in U$. Since R is $\star$-prime, we find that $\sigma(w')=0$ or $\tau[v,u]=0$ for all $u,v,w'\in U$. Since $U\neq\{0\}$, we have U is commutative. In view of Corollary 2.4, we obtain the commutativity of R. □

We are now well equipped to prove our main theorem:

Proof of Theorem 2.2. First we will show that for any $a \in Sa_\star(R)$ such that $[d(U),a]_{\sigma,\tau} = \{0\}$, then $a \in Z(R)$. For any $v \in U$, using the hypothesis, we have

$$\begin{aligned} 0 &= [d(uv^\star),a]_{\sigma,\tau} \\ &= [d(u)\sigma(v)+\tau(u)d(v^\star),a]_{\sigma,\tau} \\ &= d(u)\sigma(v)\sigma(a)+\tau(u)d(v^\star)\sigma(a)-\tau(a)d(u)\sigma(v)-\tau(a)\tau(u)d(v^\star). \end{aligned}$$

In view of the hypothesis the above relation yields

$$d(u)\sigma([v,a])+\tau([u,a])d(v^\star)=0 \text{ for all } u,v\in U. \tag{7}$$

Replace u by au in (7) and use (7) to get

$$\begin{aligned} 0 &= d(au)\sigma([v,a])+\tau([au,a])(d(v^\star) \\ &= \{d(a)\sigma(u^\star)+\tau(a)d(u)\}\sigma([v,a])+\tau(a)\tau([u,a])d(v^\star). \end{aligned}$$

We have $d(a)\sigma(u^\star)\sigma([v,a]) = 0$, for all $u,v \in U$. Replace $u^\star$ by xw for any $x \in R,\ w \in U$ we find that $d(a)R\sigma(w)\sigma([v,a]) = \{0\}$, for all $w,v \in U$. Since $a \in Sa_\star(R)$, the above expression can be rewritten as $d(a)^\star R\sigma(w)\sigma([v,a]) = \{0\}$, for all $u,v \in U$. On using $\star$-primeness of R, we obtain that for all $u,v \in U$

$$\sigma(w)\sigma([v,a])=0 \text{ or } d(a)=0. \tag{8}$$

Let us suppose that $d(a) = 0$. Then for all $u \in U$,

$$\begin{aligned} d([u,a^\star]) &= d(ua^\star-a^\star u) \\ &= d(u)\sigma(a)+\tau(u)d(a^\star)-d(a^\star)\sigma(u^\star)-\tau(a^\star)d(u) \\ &= d(u)\sigma(a)-\tau(a^\star)d(u)-\tau(a)d(u)+\tau(a)d(u) \\ &= [d(u),a]_{\sigma,\tau}+\tau(a-a^\star)d(u) \\ &= \tau(a-a^\star)d(u). \end{aligned}$$

Hence the above yields that

$$d([u,a^\star])-\tau(a-a^\star)d(u)=0. \tag{9}$$

On replacing u by uv, $v \in U$, in (9) and on using the same, we get

$$\tau([u,a^\star])d(v)+d(u)\sigma([v,a^\star])^\star+\tau(u)d([v,a^\star])-\tau(a-a^\star)\tau(u)d(v)=0.$$

By using (9), for all $u,v,w \in U$ we have

$$\begin{aligned} 0 &= \tau([u,a^\star])d(v)+d(u)\sigma([v,a^\star])^\star \\ &\quad +\tau(u)\tau(a-a^\star)d(v)-\tau(a-a^\star)\tau(u)d(v) \\ &= \tau([u,a^\star])d(v)+d(u)\sigma([v,a^\star])^\star+\tau([u,a-a^\star])d(v) \\ &= \tau([u,a])d(v)+d(u)\sigma([v,a^\star])^\star. \end{aligned}$$

Again by using (7), we have

$$\begin{aligned} 0 &= -d(u)\sigma([v^\star, a]) + d(u)\sigma([v, a^\star])^\star \\ &= 2d(u)\sigma([a, v^\star]). \end{aligned}$$

Since *char* $R \neq 2$, we get $d(u)\sigma([a, v^\star]) = 0$ for all $u, v \in U$. Replacing $v^\star$ by w in the above relation, we get $d(u)\sigma([a, w]) = 0$ for all $u, w \in U$. Substituting w by ww' for any $w' \in U$, reduces the above relation to $d(u)U\sigma([a, w']) = \{0\}$ for all $u, v, w \in U$, or $\sigma^{-1}(d(u))U[a, w'] = \{0\}$ for all $u, v, w \in U$. Therefore,

$$\sigma^{-1}(d(u))RU[a, w'] = \{0\} \text{ for all } u, v, w \in U.$$

Since U is a $\star$-ideal, using $\star$-primeness of R, we get either $\sigma^{-1}(d(u)) = 0$ for all $u \in U$ or $U[a, w'] = \{0\}$ for all $w' \in U$. Since $d(U) \neq \{0\}$, we have $U[a, w'] = \{0\} = UR[a, w']$. Since U is a nonzero $\star$-ideal, using $\star$-primeness of R, we get $[a, w'] = 0$, for all $w' \in U$. This reduces to $[U, a] = \{0\}$. In view of Lemma 2.5, we find that $a \in Z(R)$. In view of (8) consider the remaining part $\sigma(w)\sigma([v, a]) = 0$ for all $w, v \in U$, i.e., $w[v, a] = 0$ for all $w, v \in U$. On replacing w by wx for any $x \in R$, the above equation reduces to $wR[v, a] = \{0\}$, for all $w, v \in U$. Also, U being a $\star$-ideal, we get $w^\star R[v, a] = \{0\}$. Using the $\star$-primeness of R we find that either $[v, a] = \{0\}$ or $U = \{0\}$. Since $U = \{0\}$ is not possible, it reduces to $[U, a] = \{0\}$. Hence again in view of Lemma 2.5, we find that $a \in Z(R)$, and by our hypothesis we obtain that $d(U) \subseteq Z(R)$. So by Lemma 2.8, R is commutative. □

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