East-West J. of Mathematics: Vol. 8, No 1 (2006) pp. 49-59

ON UNIFIED PRESENTATIONS OF THE MULTIVARIABLE VOIGT FUNCTIONS

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Abstract

Voigt functions and their various generalizations have been studied by H.M. Srivastava and several others through a series of papers. Recently Pathan *et al.* [10] have given a unified multivariable presentation of the Voigt functions. In the present paper we give a generalization and unification of these results and study certain topics not covered therein. We first give a multivariable presentation of generalized Voigt functions involving a product of Fox-Wright generalized hypergeometric functions and then obtain its explicit representations. Next we derive a new special case involving the products of generalized Bessel functions $J^{\mu}_{\nu,\lambda}(x)$. Then, we obtain partly bilateral and partly unilateral representation of the generalized Voigt functions of several variables. Most of the results involving a single variable Voigt function obtained earlier follow as special cases of the results presented here.

1 Introduction

The Voigt functions are quite useful in a wide variety of problems of physics notably astrophysical spectroscopy, transfer of radiation in heated atmosphere and in the theory of neutron reactions [1]. Furthermore, the function

$$K(x,y)+\omega L(x,y), \quad \omega=\sqrt{-1}$$

Key words: Voigt function, generalized Bessel function, Fox-Wright function, multivariable H function.
2000 AMS Mathematics Subject Classification: 33C70, 33E20, 33C60.

is to a numerical factor identical to the so-called 'plasma dispersion function', which is tabulated by Fried and Conte [7] and several representations (integrals and series) of the Voigt functions have been given by a number of workers, for example Exton [3], Reiche [12], Fettis [5], Srivastava and Miller [17], Srivastava *et al.* [20] etc.

Recently, Pathan *et al.*[10] have introduced and studied the multivariable Voigt functions of the first kind, in the form

$$K[x_1, \cdots, x_n, y] = (\pi)^{-n/2} \int_0^\infty t^{(1-n)/2} \exp\left(-yt - \frac{1}{4}t^2\right) \prod_{j=1}^n (\cos(x_j t))dt \tag{1.1}$$

and

$$L[x_1, \cdots, x_n, y] = (\pi)^{-n/2} \int_0^\infty t^{(1-n)/2} \exp\left(-yt - \frac{1}{4}t^2\right) \prod_{j=1}^n (\sin(x_j t))dt \tag{1.2}$$

$$(y \in R^+ \quad \text{and} \quad x_1, \cdots, x_n \in R)$$

so that

$$K[x_1, \cdots, x_n, y] \pm \omega L[x_1, \cdots, x_n, y]$$
$$= (\pi)^{-n/2} \int_0^\infty t^{(1-n)/2} \exp\left(-yt - \frac{1}{4}t^2\right) \left\{ \prod_{j=1}^n (\cos(x_j t)) \pm \omega \prod_{j=1}^n (\sin(x_j t)) \right\} dt \tag{1.3}$$

They have also defined the generalized Voigt functions of multivariable as follows

$$V_{\mu,\nu_1,\cdots,\nu_n}(x_1, \cdots, x_n, y) = \prod_{i=1}^n \left(\frac{x_i}{2}\right)^{1/2} \int_0^\infty t^\mu \exp\left(-yt - \frac{1}{4}t^2\right) \prod_{j=1}^n (J_{\nu_j}(x_j t))\, dt, \tag{1.4}$$

$$\left[y > 0;\ x_1, \cdots, x_n \in R \text{ and } \mathrm{Re}\left(\mu + \sum_{j=1}^n \nu_j\right) > -1 \right]$$

We easily have

$$K[x_1, \cdots, x_n, y] = V_{\left(\frac{1}{2}, -\frac{1}{2}, \cdots, -\frac{1}{2}\right)}[x_1, \cdots, x_n, y] \tag{1.5}$$

and

$$L[x_1, \cdots, x_n, y] = V_{\left(\frac{1}{2}, \frac{1}{2}, \cdots, \frac{1}{2}\right)}[x_1, \cdots, x_n, y] \tag{1.6}$$

The present paper is devoted to a multivariable presentation of generalized Voigt functions involving a product of generalized hypergeometric functions of

Fox [6] and Wright [21] and then obtain its explicit representations in terms of familiar special functions of several variables. The representation of Voigt functions given in Section 2 will be seen to be extremely useful, in that most properties of Fox-Wright hypergeometric series carry over simply for this representation and provide connections with various well known representations and even new representations for special cases of these functions involving the products of generalized Bessel functions $J^{\mu}_{\nu,\lambda}(x)$ [9], Mittag-Leffler function E_α and its such generalizations as $E_{\alpha,\beta}$ studied in the literature [16]. Further representations and generating functions which are partly unilateral and partly bilateral are obtained. A unified presentation of multivariable Voigt functions in terms of multivariable Fox H-function is given in Section 3. By appropriately specializing these variables, a number of (known or new) results are shown to follow as applications of our main results.

2 Unified multivariable Voigt functions

In this section we study a unified multivariable generalized Voigt functions involving the product of functions ${}_p\Psi_q$. We define:

$$
\begin{aligned}
V_{\eta,p^\star,q^\star}(x^\star,y,z) &= \prod_{i=1}^{n}\left(\frac{x_i}{2}\right)^{1/2}\int_0^\infty t^\eta \exp(-yt-zt^2) \\
&\quad \prod_{j=1}^{n} {}_{p_j}\Psi_{q_j}\left[\begin{array}{c|c} (a_i^{(j)},\alpha_i^{(j)})_{1,p_j} & \\ & -x_j t^2 \\ (b_i^{(j)},\beta_i^{(j)})_{1,q_j} & \end{array}\right] dt \qquad (2.1)
\end{aligned}
$$

$$[y,z>0;\; x_1,\cdots,x_n\in R;\; \mathrm{Re}(\eta)>-1]$$

where for convenience we have used the symbol $x^\star$ to denote the array of symbols $x_1,\cdots,x_n$ with a similar meaning to $p^\star$ and $q^\star$, ${}_p\Psi_q$ occuring in the right hand side of (2.1) stands for Fox-Wright generalized hypergeometric function [6],[21] defined and represented as follows:

$$
{}_p\Psi_q(z) = {}_p\Psi_q\left[\begin{array}{ccc} (a_1,\alpha_1),\cdots,(a_p,\alpha_p) & ; & \\ & & z \\ (b_1,\beta_1),\cdots,(b_q,\beta_q) & ; & \end{array}\right] = \sum_{m=0}^{\infty}\frac{\prod_{j=1}^{p}\Gamma(a_j+\alpha_j m)}{\prod_{j=1}^{q}\Gamma(b_j+\beta_j m)}\frac{z^m}{m!} \qquad (2.2)
$$

$$[\alpha_j>0\;(j=1,\cdots,p);\;\; \beta_j>0\;(j=1,\cdots,q)]$$

Expanding the exponential function $\exp(-yt)$ involved in the right hand side of (2.1), making use of (2.2) and integrating the resulting multiple series term by term (which is justified under the conditions stated), we obtain after some simplifications

$$V_{\eta,p^\star,q^\star}(x^\star,y,z)=\frac{z^{-(\eta+1)/2}}{2}\prod_{j=1}^{n}\left(\frac{x_j}{2}\right)^{1/2}\sum_{r=0}^{\infty}\frac{(-y/\sqrt{z})^r}{r!}$$

$$\prod_{j=1}^{n}\left[\sum_{m_j=0}^{\infty}\Phi(m_j)\,\frac{(-x_j/z)^{m_j}}{m_j}\,\Gamma\left(\frac{1}{2}(\eta+r+2\sum_{j=1}^{n}m_j+1\right)\right] \tag{2.3}$$

where

$$\Phi(m_j)\;=\;\left[\prod_{i=1}^{p_j}\Gamma(a_i^{(j)}+\alpha_i^{(j)}m_j)\right]\left[\prod_{i=1}^{q_j}\Gamma(b_i^{(j)}+\beta_i^{(j)}m_j)\right]^{-1}$$

On separating the r-series into its even and odd terms, we obtain

$$V_{\eta,p^\star,q^\star}(x^\star,y,z)=\frac{z^{-(\eta+1)/2}}{2}\prod_{j=1}^{n}\left(\frac{x_j}{2}\right)^{1/2}\left[\sum_{r=0}^{\infty}\frac{(y^2/4z)^r}{r!\,(1/2)_r}\prod_{j=1}^{n}\left\{\sum_{m_j=0}^{\infty}\phi(m_j)\frac{(x_j/z)^{m_j}}{m_j}\right.\right.$$

$$\left.\times\;\Gamma\left(\frac{1}{2}(\eta+2r+2\sum m_j+1\right)\right\}-\frac{y}{\sqrt{z}}\sum_{r=0}^{\infty}\frac{(y^2/4z)^r}{r!\,(3/2)_r}$$

$$\left.\times\prod_{j=1}^{n}\left\{\sum_{m_j=0}^{\infty}\phi(m_j)\frac{(x_j/z)^{m_j}}{m_j!}\Gamma\left(\frac{1}{2}(\eta+2r+2\sum m_j+2\right)\right\}\right] \tag{2.4}$$

2.1 Special Cases

(i) On substituting $p_j=0,\ q_j=2,\ b_1^{(j)}=\lambda_j+1,\ \beta_1^{(j)}=1,\ b_2^{(j)}=\nu_j+\lambda_j+1,\ \beta_2^{(j)}=\mu_j$, replacing η by $\eta+\sum_{j=1}^{n}\nu_j+2\sum_{j=1}^{n}\lambda_j$ and x_j by $\frac{x_j^2}{4}$ $(j=1,\cdots,n)$ respectively and multiplying by $\prod_{j=1}^{n}\left(\frac{x_j}{2}\right)^{\nu_j+2\lambda_j-\frac{1}{2}}$, (2.1) reduces to the following generalized Voigt functions of several variables involving product of $J_{\nu,\lambda}^{\mu}$ which is also new, sufficiently general and of interest by itself:

$$V_{\eta,\nu^\star,\lambda^\star}^{\mu^\star}(x,y,z)=\left(\frac{x_1}{2}\right)^{1/2}\cdots\left(\frac{x_n}{2}\right)^{1/2}\int_0^\infty t^\eta\exp(-yt-zt^2)\prod_{j=1}^{n}(J_{\nu_j,\lambda_j}^{\mu_j}(x_jt))\,dt \tag{2.5}$$

$$\left[y,z,\mu_j>0,\ x_j\in R\ (j=1,\cdots,n);\ \mathrm{Re}\left(\eta+\sum_{j=1}^{n}\nu_j+2\sum_{j=1}^{n}\lambda_j\right)>-1\right]$$

where $J_{\nu,\lambda}^{\mu}(z)$ is a generalization of Bessel function defined by Pathak [9] as follows

$$J_{\nu,\lambda}^{\mu}(z) \;=\; \sum_{m=0}^{\infty} \frac{(-1)^m\,(z/2)^{(\nu+2\lambda+2m}}{\Gamma(\lambda+m+1)\,\Gamma(\nu+\lambda+\mu m+1)}$$

On making similar substitutions in results (2.3) and (2.4), we can easily obtain the corresponding results for generalized Voigt functions of several variables defined by (2.5).

(ii) For $n = 1$, (2.5) reduces to a known representation of Voigt function of one variable given by Srivastava *et al.* [20, p.53(1.27)].

(iii) For $n = 1$ and $z = 1/4$, (2.5) reduces to a known generalization of Voigt functions of one variable given by Siddiqui [13, p.265(8)].

(iv) For $\lambda_j = 0$, $\mu_j = 1$ $(j = 1, \cdots, n)$ and $z = 1/4$, (2.5) give rise to (1.4) which is the multivariable Voigt function introduced by Pathan [10, p.3(2.4)].

2.2 Partly bilateral and partly unilateral representations

We start by recalling an interesting (partly bilateral and partly unilateral) generating function due to Exton [4, p.147] in the following modified form due to Pathan and Yasmeen [11, p.2(1.2)]

$$\exp(s+t-xt/s) = \sum_{m=-\infty}^{\infty} \sum_{p=m^\star}^{\infty} \frac{s^m\,t^p}{m!\,p!}\,{}_1F_1(-p;\,m+1;x) \tag{2.6}$$

where

$$m^\star = \max\{0,-m\} \quad (m \in \mathbb{Z} := \{0, \pm 1, \pm 2, \cdots, \}) \tag{2.7}$$

and ${}_1F_1$ is confluent hypergeometric function [16].

Exton's generating function (2.6) has since been extended by a number of workers including Srivastava, Pathan and Bin-Saad [19], Pathan and Yasmeen [11], Srivastava, Pathan and Kamarujjama [20], Gupta *et al* [8] and Pathan *et al* [10]. Recently, Srivastava, Pathan and Bin-Saad [19] derived a general theorem involving bilateral series and further extended (2.6) in the following form

Let $\{\Omega(m,n,k)\}$ be a suitably bounded triple sequence of complex numbers. Also let $m^\star$ be defined by (2.7). Then

$$\sum_{m,p,k=0}^{\infty} \Omega(m,p,k)\frac{s^m t^p}{m!p!}\frac{(-xt/s)^k}{k!} = \sum_{m=-\infty}^{\infty} \sum_{p=m^\star}^{\infty} \frac{s^m t^p}{m!\,p!} \sum_{k=0}^{p} \binom{p}{k} \Omega(m+k,p-k,k)\frac{(-x)^k}{(m+1)_k} \tag{2.8}$$

provided that each member of (2.8) exists.

In the above equation we choose $\Omega(m,n,k) = 1$, replace s,t and x by $s\xi^2$, $t\xi^2$and $x\xi^2$ respectively, multiply its both sides by $\xi^\eta \exp(-\nu\xi - w\xi^2) \prod_{j=1}^{n} {}_{p_j}\Psi_{q_j}(-u_j\xi^2)$ and integrate with respect to ξ from 0 to ∞. On comparing the resulting equation with (2.1), we easily arrive at the following representation of generalized Voigt function

$$V_{\eta,p^\star,q^\star}(u^\star, v, w-s-t+xt/s) = \prod_{j=1}^{n}\left(\frac{u_j}{2}\right)^{1/2} \sum_{m=-\infty}^{\infty}\sum_{p=m^\star}^{\infty} \frac{s^m\, t^p}{m!\, p!} \sum_{k=0}^{p}\binom{p}{k}\frac{(-x)^k}{(m+1)_k}$$

$$\times \int_0^\infty \xi^{\eta+2m+2p+2k} \exp(-\nu\xi - w\xi^2) \prod_{j=1}^{n} {}_{p_j}\Psi_{q_j}(-u_j\xi^2)\, d\xi \tag{2.9}$$

$$= \sum_{m=-\infty}^{\infty}\sum_{p=m^\star}^{\infty} \frac{s^m\, t^p}{m!\, p!} \sum_{k=0}^{p}\binom{p}{k}\frac{(-x)^k}{(m+1)_k} V_{\eta+2m+2p+2k,p^\star,q^\star}(u^\star, v, w) \tag{2.10}$$

$$(\nu, w > 0,\ u_1, \cdots, u_n \in \mathbb{R},\ \mathrm{Re}(\eta) > -1, w - s - t + xt/s > 0)$$

On summing the k-series in (2.9), we have

$$V_{\eta,p^\star,q^\star}(u^\star, v, w-s-t+xt/s) = \prod_{j=1}^{n}\left(\frac{u_j}{2}\right)^{\frac{1}{2}} \sum_{m=-\infty}^{\infty}\sum_{p=m^\star}^{\infty} \frac{s^m\, t^p}{m!\, p!} \int_0^\infty \xi^{\eta+2m+2p} \exp(-\nu\xi - w\xi^2)$$

$$\times \prod_{j=1}^{n} {}_{p_j}\Psi_{q_j}(-u_j\xi^2)\, {}_1F_1[-p;\, m+1;\, x\xi^2]\, d\xi \tag{2.11}$$

$$(\nu, w > 0,\ u_1, \cdots, u_n \in \mathbb{R},\ \mathrm{Re}(\eta) > -1, w - s - t + xt/s > 0)$$

Now, using the series representation (2.2), expanding the exponential function $\exp(-\nu\xi)$ and making use of a known result [2, p.337(9)], we obtain

$$V_{\eta,p^\star,q^\star}(u^\star, v, w-s-t+xt/s) = \frac{w^{-(\eta+1)/2}}{2} \prod_{j=1}^{n}\left(\frac{u_j}{2}\right)^{\frac{1}{2}} \sum_{m=-\infty}^{\infty}\sum_{p=m^\star}^{\infty} \frac{(s/w)^m\, (t/w)^p}{m!\, p!}$$

$$\times \sum_{r=0}^{\infty} \frac{(-v/\sqrt{w})^r}{r!} \prod_{j=1}^{n}\left[\sum_{m_j=0}^{\infty} \Phi(m_j)\frac{(-u_j/z)^{m_j}}{m_j!}\, \Gamma(\alpha)\, {}_2F_1\left[-p, \alpha;\, m+1;\, \frac{x}{w}\right]\right] \tag{2.12}$$

where $\alpha = \frac{1}{2}(\eta + r + 2m + 2p + 2\sum_{j=1}^{n} m_j + 1)$.

On separating the r-series into even and odd terms, we get

$$V_{\eta,p^\star,q^\star}(u^\star, v, w{-}s{-}t{+}xt/s)$$
$$= \frac{w^{-(\eta+1)/2}}{2} \prod_{j=1}^{n} \Big(\frac{u_j}{2}\Big)^{\frac{1}{2}} \sum_{m=-\infty}^{\infty} \sum_{p=m^\star}^{\infty} \frac{(s/w)^m\,(t/w)^p}{m!\,p!} \prod_{j=1}^{n} \Bigg[\sum_{m_j=0}^{\infty} \Phi(m_j) \frac{(-u_j/z)^{m_j}}{m_j!}$$
$$\Bigg\{\Gamma(\beta)\Psi_1\Big[\beta, -p; m+1, \frac{1}{2}; \frac{x}{w}\frac{v^2}{4w}\Big] - \frac{v}{\sqrt{w}}\Gamma(\beta)\Psi_1\Big[\beta+\frac{1}{2}, -p; m+1, \frac{3}{2}; \frac{x}{w}, \frac{v^2}{4w}\Big]\Bigg\}\Bigg] \tag{2.13}$$

where $\beta = \dfrac{1}{2}(\eta + 2m + 2p + 2\sum_{j=1}^{n} m_j + 1)$ and Ψ_1 is confluent hypergeometric function of two variables [16, p.59(1.6)(41)] given as:

$$\Psi_1[\alpha, \beta;\, \gamma, \gamma';\, x, y] = \sum_{m,n=0}^{\infty} \frac{(\alpha)_{m+n}\,(\beta)_m}{(\gamma)_n\,(\gamma')_n} \frac{x^m}{m!} \frac{y^n}{n!}, \qquad (|x| < 1;\ |y| < \infty). \tag{2.14}$$

On making suitable substitutions as mentioned in Section 2.1, we can obtain the partly bilateral and partly unilateral representation for the Voigt functions of several variables defined by equation (2.5) as follows:

$$V^{\mu^\star}_{\eta,\nu^\star,\lambda^\star}(u^\star, v, w{-}s{-}t{+}xt/s)$$
$$= \frac{w^{-(\eta+1)/2}}{2} \prod_{j=1}^{n} \Big(\frac{u_j}{2}\Big)^{1/2} \sum_{m=-\infty}^{\infty} \sum_{p=m^\star}^{\infty} \frac{(s/w)^m\,(t/w)^p}{m!\,p!} \sum_{r=0}^{\infty} \frac{(-v/\sqrt{w})^r}{r!}$$
$$\prod_{j=1}^{n} \Bigg[\sum_{m_j=0}^{\infty} \frac{(-1)^{m_j}(-u_j/2\sqrt{w})^{\nu_j+2\lambda_j+2m_j}}{\Gamma(\lambda_j+m_j+1)\Gamma(\nu_j+\lambda_j+\mu_j m_j+1)} \Gamma(\delta)\,{}_2F_1\Big[-p, \delta;\, m+1;\, \frac{x}{w}\Big]\Bigg] \tag{2.15}$$

$\Big[v, w, \mu_j > 0,\ u_j \in \mathbb{R}\,(j = 1, \cdots, n);\ \mathrm{Re}\Big(\eta + \sum_{j=1}^{n} \nu_j + 2\sum_{j=1}^{n} \lambda_j\Big) > -1;$
$w - s - t + \frac{xt}{s} > 0\Big]$ where $\delta = \dfrac{1}{2}(\eta{+}r{+}2m{+}2p{+}\sum_{j=1}^{n}\nu_j{+}2\sum_{j=1}^{n}\lambda_j{+}2\sum_{j=1}^{n} m_j{+}1)$.

On separating the r-series into even and odd terms, we obtain

$$V^{\mu^\star}_{\eta,\nu^\star,\lambda^\star}(u^\star, v, w{-}s{-}t{+}xt/s)$$
$$= \frac{w^{-(\eta+1)/2}}{2} \prod_{j=1}^{n} \Big(\frac{u_j}{2}\Big)^{1/2} \sum_{m=-\infty}^{\infty} \sum_{p=m^\star}^{\infty} \frac{(s/w)^m\,(t/w)^p}{m!\,p!}$$
$$\prod_{j=1}^{n}\Bigg(\sum_{m_j=0}^{\infty} \frac{(-1)^{m_j}(u_j/2\sqrt{w})^{\nu_j+2\lambda_j+2m_j}}{\Gamma(\lambda_j+m_j+1)\Gamma(\nu_j+\lambda_j+\mu_j m_j+1)} \Big\{\Gamma(\sigma)\Psi_1\Big[\sigma, -p; m+1, \frac{1}{2}; \frac{x}{w}, \frac{v^2}{4w}\Big]$$

$$- \frac{v}{\sqrt{w}}\Gamma(\sigma+\frac{1}{2})\Psi_1\left[\sigma+\frac{1}{2},-p;m+1,\frac{3}{2};\frac{x}{w},\frac{v^2}{4w}\right]\Bigg\}\Bigg) \tag{2.16}$$

where $\sigma = \frac{1}{2}(\eta + 2m + 2p + \sum_{j=1}^{n}\nu_j + 2\sum_{j=1}^{n}\lambda_j + 2\sum_{j=1}^{n}m_j + 1)$.

For $n = 1$, the results (2.15) and (2.16) reduce to known results given by Srivastava *et al.* [20, p.59(3.5)(3.6)].

3. Multivariable Voigt functions in terms of multivariable *H*-functions

In an attempt to further provide a unified presentation of multivariable Voigt functions we give the following definitions in terms of multivariable H-function:

$$V_{\mu,\eta,\nu,\lambda^\star}(x^\star,y,z) \;=\; (x_1)^\mu\cdots(x_s)^\mu\int^\infty t^\eta\exp(-zt^\nu)\,H[x_1t^{\lambda_1},\cdots,x_st^{\lambda_s}]\,dt \tag{3.1}$$

$[z,\mu,\nu,\lambda_i>0,\ (i=1,\cdots,s);\ x_1,\cdots,x_s\in\mathbb{R};$

$$\mathrm{Re}(\eta)+\textstyle\sum_{i=1}^{s}\lambda_i\min\limits_{1\le j\le m_j}\mathrm{Re}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)>-1]$$

where the multivariable H-function occuring in the right hand side of (3.1) is introduced by Srivastava and Panda [18] and is defined as follows [15, p.251(C1)-(C4)]:

$$H[x_1,\cdots,x_s]=H^{0,0:M_1N_1;\cdots;M_sN_s}_{P,Q:P_1,Q_1;\cdots;P_s,Q_s}\left[\begin{array}{l|llll} x_1 & (a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(s)})_{1,P} & : & (c_j^{(1)},\gamma_j^{(1)})_{1,P_1} & ;\cdots & ;(c_j^{(s)},\gamma_j^{(s)})_{1,P_s} \\ \vdots & & & & & \\ x_s & (b_j;\beta_j^{(1)},\cdots,\beta_j^{(s)})_{1,Q} & : & (d_j^{(1)},\delta_j^{(1)})_{1,Q_1} & ;\cdots & ;(d_j^{(s)},\delta_j^{(s)})_{1,Q_s}\end{array}\right]$$

$$=\frac{1}{(2\pi\omega)^s}\int_{L_1}\cdots\int_{L_s}\prod_{i=1}^{s}\{\phi_i(\xi_i)(x_i)^{\xi_i}\}\Psi(\xi_1,\cdots,\xi_s)\,d\xi_1\cdots d\xi_s \tag{3.2}$$

where $\omega=\sqrt{-1}$.

For the convergence, existence conditions and other details of the multivariable H-function, we refer to Srivastava *et al.* [15].

If we set $zt^\nu=\tau$ and evaluate the resulting integral in terms of multivariable H-function by appealing to Mellin-Barnes contour integral representation using (3.2), we arrive at the following:

$$V_{\mu,\eta,\nu,\lambda^\star}(x^\star, y, z) = \frac{z^{-(\eta+1)/\nu}}{\nu}(x_1)^\mu, \cdots, (x_s)^\mu H^{0,1\;\;:I}_{P+1,Q:J}\left[\begin{array}{c|c} x_1 z^{-\lambda_1/\nu} & C:E \\ \vdots & \\ x_s z^{-\lambda_s/\nu} & D:F \end{array}\right] \tag{3.3}$$

where $I = M_1, N_1; \cdots ; M_s, N_s$, $J = P_1, Q_1; \cdots ; P_s, Q_s$,
$C = \left(1 - \frac{1}{\nu} - \frac{\eta}{\nu}; \frac{\lambda_1}{\nu}, \cdots, \frac{\lambda_s}{\nu}\right), (a_j; \alpha_j^{(1)}, \cdots, \alpha_j^{(s)})_{1,P}, D = (b_j; \beta_j^{(1)}, \cdots, \beta_j^{(s)})_{1,Q}$,
$E = (c_j^{(1)}, \gamma_j^{(1)})_{1,P_1}; \cdots ; (c_j^{(s)}, \gamma_j^{(s)})_{1,P_s}$, $F = (d_j^{(1)}, \delta_j^{(1)})_{1,Q_1}; \cdots ; (d_j^{(s)}, \delta_j^{(s)})_{1,Q_s}$

For $P = Q = 0$, the definition (3.1) and the result (3.3) reduce to the following:

$$S_{\mu,\eta,\nu,\lambda^\star}(x^\star, y, z) = (x_1)^\mu \cdots (x_s)^\mu \int_0^\infty t^\eta \exp(-zt^\nu) \prod_{i=1}^{s} \left\{ H^{M_i,N_i}_{P_i,Q_i}\left[x_i t^{\lambda_i} \left| \begin{array}{c} (c_j^{(i)}, \gamma_j^{(i)})_{1,P_i} \\ \\ (d_j^{(i)}, \delta_j^{(i)})_{1,Q_i} \end{array}\right.\right]\right\} \tag{3.4}$$

and

$$S_{\mu,\eta,\nu,\lambda^\star}(x^\star, y, z] = \frac{z^{-(\eta+1)/\nu}}{\nu}(x_1)^\mu, \cdots, (x_s)^\mu$$

$$H^{0,1;I}_{1,0;J}\left[\begin{array}{c|l} x_1 z^{-\lambda_1/\nu} & \left(1 - \frac{1}{\nu} - \frac{\eta}{\nu}; \frac{\lambda_1}{\nu}, \cdots, \frac{\lambda_s}{\nu}\right);\ E \\ \vdots & \\ x_s z^{-\lambda_s/\nu} & \qquad\qquad \underline{\qquad}\qquad\qquad ;\ F \end{array}\right] \tag{3.5}$$

whereE, F, I, J are same as defined with result (3.3).

If we take $s = 2$, $z = 1/4$, $\nu = \lambda = 2$, $x_1 = \frac{x^2}{4}$, $x_2 = \frac{y^2}{4}$, $\eta = \zeta + \nu$, $2\mu = \nu + \frac{1}{2}$, (3.1),(3.3),(3.4) and (3.5) reduce to known results given by Srivastava and Chen [14, p.71(41),(43),(42) and (44)].

If we take $s = 2$, $m_1 = 1$, $n_1 = p$, $b_1^{(1)} = 0$, $\beta_1^{(1)} = 1$, $m_2 = 1$, $n_2 = 0$, $p_2 = 0$, $q_2 = 1$, $b_1^{(2)} = 0$, $\beta_1^{(2)} = 1$, $x_2 = y$, $\lambda_2 = 1$, (3.4) reduces to the definition given by Gupta *et al.* [8, p.300(2)].

The results similar to those obtained in Section 2 can also be obtained for the multivariable Voigt functions defined by equation (3.1). We however, omit the details.

Acknowledgement

The second author is thankful to University Grants Commission, New Delhi for providing necessary financial assistance to carry out present work. The authors are thankful to Dr. K.C. Gupta for his useful suggestions and generous help during the preparation of the paper.

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