East-West J. of Mathematics: Vol. 20, No 1 (2018) pp. 76-85

UNI-SOFT IDEALS OF SEMIRINGS

Debabrata Mandal

Department of Mathematics
Raja Peary Mohan College
Uttarpara, Hooghly-712258, India
e-mail: dmandaljumath@gmail.com

Abstract

The aim of this paper is to investigate the ideal theory of semirings based on uni-soft sets. Some basic operations on the ideals are considered and related properties are also studied. Finally, some characterizations of regular and intra-regular semirings are obtained by means of uni-soft ideals (bi-ideals, quasi-ideals).

1 Introduction

The traditional classical models often fail to overcome the complexities arising in the modeling of uncertain data in many fields like economics, engineering, environmental science, sociology, medical science etc. In order to overcome these difficulties, Molodtsov [18] introduced the concept of soft sets as a new mathematical tool for dealing with uncertainties. Maji et al. [16] discussed further soft set theory. Ali et al. [2] proposed some new operations on soft sets. As a continuation, several authors (for example, see [4, 5, 17]) described the application of soft set theory to decision making problems. Recently, the algebraic structures of soft sets [6, 22] have been studied increasingly, such as soft groups [20, 24], soft ideals in ordered semigroup [10], uni-soft semigroups [12, 13], soft rings [1], soft semirings [7, 15, 26, 27], soft near-rings [23, 25] and so on.
Semirings which are regarded as a generalization of rings have been found useful for dealing with problems in different areas of applied mathematics and information sciences, as the semiring structure provides an algebraic framework

Key words: Semiring, Soft set, Uni-Soft ideals (bi-ideals, quasi-ideals), Regular, Intra-regular.
2010 AMS Mathematics classification: 16Y60, 06D72.

for modeling and investigating the key factors in these problems. We know that ideals of the semiring [8, 11] plays a central role in the structure theory and useful for many purposes and they do not in general coincide with the usual ring ideals and so many results in ring theory have no analogues in semirings using only ideals. In [19], Nobusawa introduced the notion of Γ-ring as more general than ring. After this many mathematicians, for example [14, 21], made works on this subject using the theory of soft sets.
As a continuation of this, in this paper, we have investigated the ideal theory of semirings [8] based on uni-soft sets and obtain some related properties. Finally, we describe some characterizations of regular and intra-regular semirings by means of uni-soft ideals.

2 Preliminaries

We recall the following definitions for subsequent use.

Definition 2.1. [8] A hemiring [respectively, semiring] is a nonempty set S on which operations addition and multiplication have been defined such that the following conditions are satisfied:
(i) $(S,+)$ is a commutative monoid with identity 0.
(ii) $(S,.)$ is a semigroup [respectively, monoid with identity 1_S].
(iii) Multiplication distributes over addition from either side.
(iv) $0s = 0 = s0$ for all $s \in S$.
(v) $1_S \neq 0$

Definition 2.2. A left ideal I of semiring S is a nonempty subset of S satisfying the following conditions:

(i) If $a, b \in I$ then $a + b \in I$

(ii) If $a \in I$ and $s \in S$ then $sa \in I$

(iii) $I \neq S$.

A right ideal of S is defined in an analogous manner and an ideal of S is a nonempty subset which is both a left ideal and a right ideal of S.

Definition 2.3. [18] Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denotes the power set of U. Consider a nonempty set A, $A \subseteq E$. A pair (f, A) is called a soft set over U, where f is a mapping given by $f : A \to P(U)$.
It is clear to see that a soft set is a parameterized family of subsets of the set U. Note that the set of all soft sets over U will be denoted by $S(U)$.

Definition 2.4. [1] Let (f, A) be a soft set. The set $Supp(f, A) = \{x \in A | f(x) \neq \phi\}$ is called the support of the soft set (f, A). The soft set is said to be non-empty if its support is not equal to the empty set.

Definition 2.5. [5] Let $f, g \in S(U)$, then

(i) The union of f and g, denoted by $f \cup g$, is defined as $(f \cup g)(x) = f(x) \cup g(x)$ for all $x \in E$.

(ii) The $\vee$-product of f and g, denoted by $f \vee g$, is defined as $(f \vee g)(x, y) = f(x) \cup g(y)$ for all $x,\ y \in E$.

Definition 2.6. [13] Let $A \subseteq E$, the set of parameters and U be the universe. We denote χ_A, the uni-soft characteristic function of A and define as

$$\begin{aligned} \chi_A(x) &= \phi \text{ if } x \in A \\ &= U \text{ if } x \notin A. \end{aligned}$$

3 Uni-soft ideals with some operations

Throughout this paper unless otherwise mentioned S denotes the semiring, χ_S denotes its characteristic function.

Definition 3.1. Let f and g be two soft sets of a semiring S over U. We define composition of f and g as follows:

$$\begin{aligned} fog(x) &= \cap[\underset{i}{\cup}\{\cup\{f(a_i), f(c_i), g(b_i), g(d_i)\}\}] \\ &\qquad {\scriptstyle x+}\sum_{i=1}^{n} a_i b_i = \sum_{i=1}^{n} c_i d_i \\ &= 0, \text{ if } x \text{ cannot be expressed as above} \\ &\quad \text{where } x, z, a_i, b_i, c_i, d_i \in S. \end{aligned}$$

Definition 3.2. Let f be a non-empty soft set of an semiring S (i.e. $f(x) \neq \phi$ for some $x \in S$) over U. Then f is called a uni-soft left ideal [resp. uni-soft right ideal] of S over U if

(i) $f(x + y) \subseteq f(x) \cup f(y)$

(ii) $f(xy) \subseteq f(y)$ [respectively, $f(xy) \subseteq f(x)$]

for all $x, y \in S$.
Note that for uni-soft k-ideal the following additional relation must holds:
For $x, a, b \in S$ with $x + a = b \Rightarrow f(x) \subseteq f(a) \cup f(b)$.

***Example* 3.3.** Let $U = S = Z_4 = \{0, 1, 2, 3\}$. Then S forms a semiring with respect to usual addition and multiplication of integers. Define $f(1) = f(3) = \{0, 1, 2, 3\}$ and $f(0) = f(2) = \{0, 2\}$. Then f is a uni-soft ideal of S over U.

***Example* 3.4.** Let $U = Z^+$, $S = Z_4 = \{0, 1, 2, 3\}$. Define soft set f of S over U by $f(0) = \{4n|n \in Z^+\}$, $f(1) = f(3) = \{n|n \in Z^+\}$ and $f(2) = \{2n|n \in Z^+\}$. Then we can easily check that f is a uni-soft k-ideal of S over U.

Proposition 3.5. Union of a non-empty collection of uni-soft left (respectively, right) ideals is also a uni-soft left (respectively, right) ideal of S over U.

Proof. Let $\{f_i : i \in I\}$ be a non-empty family of uni-soft left ideals of S over U and $x, y \in S$.
Then

$$\begin{aligned}(\underset{i\in I}{\cup} f_i)(x+y) &= \underset{i\in I}{\cup}\{f_i(x+y)\} \subseteq \underset{i\in I}{\cup}\{f_i(x) \cup f_i(y)\} \\ &= \cup\{\underset{i\in I}{\cup} f_i(x), \underset{i\in I}{\cup} f_i(y)\} = (\underset{i\in I}{\cup} f_i)(x) \cup (\underset{i\in I}{\cup} f_i)(y).\end{aligned}$$

Again

$$(\underset{i\in I}{\cup} f_i)(xy) = \underset{i\in I}{\cup}\{f_i(xy)\} \subseteq \underset{i\in I}{\cup} f_i(y) = (\underset{i\in I}{\cup} f_i)(y).$$

Hence $\underset{i\in I}{\cup} f_i$ is a uni- soft left ideal of S.
Similarly we can prove the result for uni-soft right ideal also. □

Theorem 3.6. *Let f and g be uni-soft left ideals of an semiring S over U. Then $f \vee g$ is a uni-soft left ideal of $S \times S$ over U.*

Proof. Let $(x_1, x_2), (y_1, y_2) \in S \times S$. Then

$$\begin{aligned}(f \vee g)((x_1, x_2) + (y_1, y_2)) &= (f \vee g)(x_1 + y_1, x_2 + y_2) \\ &= (f(x_1 + y_1) \cup g(x_2 + y_2)) \\ &= (f(x_1 + y_1)) \cup (g(x_2 + y_2)) \\ &\subseteq (f(x_1) \cup f(y_1)) \cup (g(x_2) \cup g(y_2)) \\ &= (f \vee g)(x_1, x_2) \cup (f \vee g)(y_1, y_2)\end{aligned}$$

and

$$\begin{aligned}(f \vee g)((x_1, x_2)(y_1, y_2)) &= (f \vee g)(x_1y_1, x_2y_2) \\ &= (f(x_1y_1) \cup g(x_2y_2)) \\ &= (f(x_1y_1)) \cup (g(x_2y_2)) \\ &\subseteq f(y_1) \cup g(y_2) = (f \vee g)(y_1, y_2).\end{aligned}$$

Therefore $f \vee g$ is a uni-soft left ideal of $S \times S$ over U. □

Theorem 3.7. *Let f be a soft set of an semiring S over U. Then f is a uni-soft left ideal of S if and only if $f \vee f$ is a uni-soft left ideal of $S \times S$ over U.*

Proof. Assume that f is a uni-soft left ideal of S over U. Then by Theorem 3.6, $f \vee f$ is a uni-soft left ideal of $S \times S$ over U.
Conversely, suppose that $f \vee f$ is a uni-soft left ideal of $S \times S$ over U. Let $x_1, x_2, y_1, y_2 \in S$ and $\gamma \in \Gamma$. Then

$$\begin{aligned}(f(x_1 + y_1) \cup f(x_2 + y_2)) &= (f \vee f)(x_1 + y_1, x_2 + y_2) \\ &= (f \vee f)((x_1, x_2) + (y_1, y_2)) \\ &\subseteq (f \vee f)(x_1, x_2) \cup (f \vee f)(y_1, y_2) \\ &= (f(x_1) \cup f(x_2)) \cup (f(y_1) \cup f(y_2)).\end{aligned}$$

Now, putting $x_1 = x,\ x_2 = 0,\ y_1 = y$ and $y_2 = 0$, in this inequality and noting that $f(0) \subseteq f(x)$ for all $x \in S$, we obtain $f(x+y) \subseteq f(x) \cup f(y)$.
Next, we have

$$\begin{aligned}(f(x_1y_1) \cup f(x_2y_2)) &= (f \vee f)(x_1y_1, x_2y_2) \\ &= (f \vee f)((x_1, x_2)(y_1, y_2)) \\ &\subseteq (f \vee f)(y_1, y_2) \\ &= f(y_1) \cup f(y_2).\end{aligned}$$

Taking $x_1 = x,\ y_1 = y$ and $y_2 = 0$, we obtain $f(xy) \subseteq f(y)$.
Hence f is a uni-soft left ideal of S over U. □

Theorem 3.8. *Let $f_1,\ f_2$ be any two uni-soft k-ideals of S over U. Then $f_1 o f_2$ is a uni-soft ideal of S over U.*

Proof. Let $f_1,\ f_2$ be any two uni-soft k-ideals of S over U and $x,\ y \in S$. Then

$$\begin{aligned}&(f_1 o f_2)(x+y) \\ &= \underset{x+y+\sum a_ib_i=\sum c_id_i}{\cap}\{\underset{i}{\cup}\{f_1(a_i), f_2(b_i), f_1(c_i), f_2(d_i)\}\} \\ &\subseteq \underset{x+y+\sum x_{1i}x_{2i}+\sum y_{1i}y_{2i}=\sum x_{3i}x_{4i}+\sum y_{3i}y_{4i}}{\cap}\{\underset{i}{\cup}\{f_1(x_{1i}), f_1(x_{3i}), f_1(y_{1i}), f_1(y_{3i}),\ f_2(x_{2i}), f_2(x_{4i}), f_2(y_{2i}), f_2(y_{4i})\}\} \\ &\subseteq \cup\{\underset{x+\sum x_{1i}x_{2i}=\sum x_{3i}x_{4i}}{\cap}\{\underset{i}{\cup}\{f_1(x_{1i}), f_1(x_{3i}), f_2(x_{2i}), f_2(x_{4i})\}\}, \underset{y+\sum y_{1i}y_{2i}=\sum y_{3i}y_{4i}}{\cap}\{\underset{i}{\cup}\{f_1(y_{1i}), f_1(y_{3i}), f_2(y_{2i}), f_2(y_{4i})\}\}\} \\ &= (f_1 o f_2)(x) \cup (f_1 o f_2)(y).\end{aligned}$$

Now assuming $f_1,\ f_2$ are as uni-soft right ideals we have

$$\begin{aligned}(f_1 o f_2)(xy) &= \underset{xy+\sum a_ib_i=\sum c_id_i}{\cap}\{\underset{i}{\cup}\{f_1(a_i), f_2(b_i), f_1(c_i), f_2(d_i)\}\} \\ &\subseteq \underset{xy+\sum x_{1i}x_{2i}y=\sum x_{3i}x_{4i}y}{\cap}\{\underset{i}{\cup}\{f_1(x_{1i}), f_1(x_{3i}), f_2(x_{2i}y), f_2(x_{4i}y)\}\} \\ &\subseteq \underset{x+\sum x_{1i}x_{2i}=\sum x_{3i}x_{4i}}{\cap}\{\underset{i}{\cup}\{f_1(x_{1i}), f_1(x_{3i}), f_2(x_{2i}), f_2(x_{4i})\}\} \\ &= (f_1 o f_2)(x).\end{aligned}$$

Similarly, assuming $f_1,\ f_2$ are as uni-soft left k-ideals we can show that $(f_1 o f_2)(xy) \subseteq (f_1 o f_2)(y)$.
Hence $f_1 o f_2$ is a uni-soft ideal of S over U. □

4 Uni-soft ideals of regular semiring

Definition 4.1. A soft set f of a semiring S over U is called uni-soft bi-ideal if for all $x, y, z \in S$ we have

(i) $f(x+y) \subseteq f(x) \cup f(y)$

(ii) $f(xy) \subseteq f(x) \cup f(y)$

(iii) $f(xyz) \subseteq f(x) \cup f(z)$

Definition 4.2. A soft set f of a semiring S over U is called uni-soft quasi-ideal if for all $x, y \in S$ we have

(i) $f(x+y) \subseteq f(x) \cup f(y)$

(ii) $((fo\chi_S) \cup (\chi_S of))(x) \subseteq f(x)$

Proposition 4.3. Intersection of a non-empty collection of uni-soft bi-ideals of S over U is also a uni-soft bi-ideal of S over U.

Proof. The proof follows by routine verifications. □

Proposition 4.4. Let $\{f_i : i \in I\}$ be a family of uni-soft bi-ideals of S over U such that $f_i \subseteq f_j$ or $f_j \subseteq f_i$ for $i, j \in I$. Then $\underset{i\in I}{\cup} f_i$ is a uni-soft bi-ideal of S over U.

Proof. Straightforward. □

Lemma 4.5. In a semiring every uni-soft quasi ideals are uni-soft bi-ideals.

Proof. Let f be a uni-soft quasi ideal of S over U. It is sufficient to prove that $f(xyz) \subseteq f(x) \cup f(z)$ for all $x,\ y,\ z \in S$. Since f is a uni-soft quasi ideal of S over U, we have

$$\begin{aligned} f(xyz) \quad & \subseteq ((fo\chi_S) \cup (\chi_S o\mu))(xyz) \\ & = \{(fo\chi_S)(xyz) \cup (\chi_S of)(xyz)\} \\ & = \cup\{ \underset{xyz+\sum_{i=1}^{n} a_ib_i = \sum_{i=1}^{n} c_id_i}{\cap(\underset{i}{\cup}(f(a_i) \cup f(c_i)))} \ , \ \underset{xyz+\sum_{i=1}^{n} a_ib_i = \sum_{i=1}^{n} c_id_i}{\cap(\underset{i}{\cup}(f(b_i) \cup f(d_i)))} \ \} \\ & \subseteq f(0) \cup f(x) \cup f(0) \cup f(z))(\text{since } xyz + 00 + 0 = xyz + 0) \\ & = f(x) \cup f(z). \end{aligned}$$

Similarly, we can show that $f(xy) \subseteq f(x) \cup f(y)$ for all $x, y \in S$.

***Example* 4.6.** Let $U = Z$, $S = \{0, a, b\}$. Then S forms a semiring with respect to addition and multiplication defined as:

+	0	a	b
0	0	a	b
a	a	a	a
b	b	a	b

and

.	0	a	b
0	0	0	0
a	0	a	a
b	0	a	a

Now define soft set f on S over U as: $f(0) = 4Z,\ f(a) = 2Z,\ f(b) = Z$. Then f is a uni-soft bi-ideal of S over U which is not a uni-soft quasi-ideal.

□

Definition 4.7. A semiring S is said to be k-regular if for each $x \in S$, there exist $a, b \in S$ such that $x + xax = xbx$.

Definition 4.8. A semiring S is said to be k-intra-regular if for each $x \in S$, there exist $z, a_i, a_i^{'}, b_i, b_i^{'} \in S$, $i \in \mathbf{N}$, the set of natural numbers, such that $x + \sum\limits_{i=1}^{n} a_i x^2 a_i^{'} = \sum\limits_{i=1}^{n} b_i x^2 b_i^{'}$.

Theorem 4.9. *Let S be a k-regular semiring and $x \in S$. Then*

(i) $f(x) \subseteq (fo\chi_S of)(x)$ for every uni-soft k-bi-ideal f of S over U.

(ii) $f(x) \subseteq (fo\chi_S of)(x)$ for every uni-soft k-quasi-ideal f of S over U.

Proof. Let S be a k-regular semiring and x be any element of S. Suppose f be any uni-soft k-bi-ideal of S. Since S is k-regular there exist $a, b \in S$ such that $x + xax = xbx$. Now

$$
\begin{aligned}
&(fo\chi_S of)(x) \\
&= \underset{x+\sum\limits_{i=1}^{n} a_i b_i = \sum\limits_{i=1}^{n} c_i d_i}{\cap}\left(\bigcup_i \{(fo\chi_S)(a_i), (fo\chi_S)(c_i), f(b_i), f(d_i)\}\right) \\
&\subseteq \cap\{(fo\chi_S)(xa), (fo\chi_S)(xb), f(x)\} \\
&\qquad [\text{since } x + xax = xbx] \\
&= \cap\Big\{ \underset{xa+\sum\limits_{i=1}^{n} a_i b_i = \sum\limits_{i=1}^{n} c_i d_i}{\cap}\left(\bigcup_i \{(f(a_i), f(c_i))\}\right),\ \underset{xb+\sum\limits_{i=1}^{n} a_i b_i = \sum\limits_{i=1}^{n} c_i d_i}{\cap}\left(\bigcup_i \{(f(a_i), f(c_i))\}\right),\ f(x)\Big\} \\
&\subseteq \cap\{f(x), f(x), f(x)\} \\
&\qquad [\text{since } xa + xaxa = xbxa \text{ and } xb + xaxb = xbxb] \\
&= f(x).
\end{aligned}
$$

This implies that $f(x) \subseteq (fo\chi_S of)(x)$.
$(i) \Rightarrow (ii)$ is straightforward from Lemma 4.5. □

Theorem 4.10. *Let S be a k-regular semiring and $x \in S$. Then*

(i) $(f \cup g)(x) \subseteq (fogof)(x)$ for every uni-soft k-bi-ideal f and every uni-soft k-ideal g of S over U .

(ii) $(f \cup g)(x) \subseteq (fogof)(x)$ for every uni-soft k-quasi-ideal f and every uni-soft k-ideal g of S over U .

Proof. Assume that S is a k-regular semiring. Let f and g be any uni-soft k-bi-ideal and uni-soft k-ideal of S over U, respectively and x be any element of S. Since S is k-regular, there exist $a, b \in S$ such that $x + xax = xbx$. Then

$$\begin{aligned}
&(fogof)(x)\\
&= \cap\Big(\underset{i}{\cup}\{(fog)(a_i), (fog)(c_i), f(b_i), f(d_i)\}\Big)\\
&\qquad\qquad {\scriptstyle x+\sum_{i=1}^{n} a_i b_i = \sum_{i=1}^{n} c_i d_i}\\
&\subseteq \cap\{(fog)(xa), (fog)(xb), f(x)\}\\
&\qquad [\text{since } x + xax = xbx]\\
&= \cap\{\cap(\underset{i}{\cup}\{(f(a_i), f(c_i), g(b_i), g(d_i))\}), \cap(\underset{i}{\cup}\{(f(a_i), f(c_i), g(b_i), g(d_i))\}), f(x)\}\\
&\qquad\qquad {\scriptstyle xa+\sum_{i=1}^{n} a_i b_i = \sum_{i=1}^{n} c_i d_i} \qquad\qquad {\scriptstyle xb+\sum_{i=1}^{n} a_i b_i = \sum_{i=1}^{n} c_i d_i}\\
&\subseteq \cup\{\cup\{f(x), f(axa), g(bxa)\}, \cup\{f(x), g(axb), g(bxb)\}, f(x)\}\\
&\qquad [\text{since } xa + xaxa = xbxa \text{ and } xb + xaxb = xbxb]\\
&\subseteq f(x) \cup g(x) = (f \cup g)(x).
\end{aligned}$$

(i)⇒(ii) is straightforward from Lemma 4.5. □

Theorem 4.11. *Let S is both k-regular and k-intra-regular semiring and $x \in S$. Then*

(i) $f(x) = (fof)(x)$ for every uni-soft k-bi-ideal f of S over U.

(ii) $f(x) = (fof)(x)$ for every uni-soft k-quasi-ideal f of S over U.

Proof. Suppose S is both k-regular and k-intra-regular semiring. Let $x \in S$ and f be any uni-soft k-bi-ideal of S over U. Since S is both k-regular and k-intra-regular there exist $a_i, b_i, c_i, d_i \in S$, $i \in \mathbf{N}$ such that $x + \sum_{i=1}^{n} xa_i x^2 b_i x = \sum_{i=1}^{n} xc_i x^2 d_i x$. Therefore

$$\begin{aligned}
(fof)(x) &= \cap\Big[\underset{i}{\cup}\{f(a_i), f(c_i), f(b_i), f(d_i)\}\Big]\\
&\qquad {\scriptstyle x+\sum_{i=1}^{n} a_i b_i \subseteq \sum_{i=1}^{n} c_i d_i}\\
&\subseteq \cap\Big[\underset{i}{\cup}\{f(xa_i x), f(xb_i x), f(xc_i x), f(xd_i x)\}\Big]\\
&\qquad\quad {\scriptstyle x+\sum_{i=1}^{n} xa_i x^2 b_i x = \sum_{i=1}^{n} xc_i x^2 d_i x}\\
&\subseteq f(x).
\end{aligned}$$

Now $(fof)(x) \subseteq (fo\chi_S)(x) \subseteq f(x)$. Hence $f(x) = (fof)(x)$ for every uni-soft k-bi-ideal f of S.
$(i) \Rightarrow (ii)$ is straightforward from the Lemma 4.5. □

Theorem 4.12. *Let S is both k-regular and k-intra-regular semiring and $x \in S$. Then*

(i) $(f \cup g)(x) \subseteq (fog)(x)$ *for all uni-soft k-bi-ideals f and g of S over U.*

(ii) $(f \cup g)(x) \subseteq (fog)(x)$ *for every uni-soft k-bi-ideals f and every uni-soft k-quasi-ideal g of S over U.*

(iii) $(f \cup g)(x) \subseteq (fog)(x)$ *for every uni-soft k-quasi-ideals f and every uni-soft k-bi-ideal g of S over U.*

(iv) $(f \cup g)(x) \subseteq (fog)(x)$ *for all uni-soft k-quasi-ideals f and g of S over U.*

Proof. Assume that S is both k-regular and k-intra-regular semiring. Let $x \in S$ and f, g be any uni-soft k-bi-ideals of S over U. Since S is both k-regular and k-intra-regular there exist $a_i, b_i, c_i, d_i \in S$, $i \in \mathbf{N}$ such that $x + \sum_{i=1}^{n} xa_ix^2b_ix \subseteq \sum_{i=1}^{n} xc_ix^2d_ix$. Therefore

$$\begin{aligned}(fog)(x) &= \bigcap_{x+\sum_{i=1}^{n} a_ib_i = \sum_{i=1}^{n} c_id_i} [\bigcup_i \{f(a_i), f(c_i), g(b_i), g(d_i)\}] \\ &\subseteq \cap[\bigcup_i \{f(xa_ix), g(xb_ix), f(xc_ix), g(xd_ix\}] \\ &\subseteq f(x) \cup g(x) = (f \cup g)(x).\end{aligned}$$

$(i) \Rightarrow (ii) \Rightarrow (iv)$ and $(i) \Rightarrow (iii) \Rightarrow (iv)$ are obvious from Lemma 4.5. □

References

[1] Acar, U., Koyuncu, F., Tanay, B. , Soft sets and soft rings, Comput. Math. Appl., 59 (2010), 3458-3463.

[2] Ali, M.I., Feng, F., Liu, X., Min, W.K., Shabir, M. , On some new operations in soft set theory, Comput. Math. Appl., 57 (2009), 1547-1553.

[3] Cagman, N., Citak, F., Aktas, H. , Soft-int group and its applications to group theory, Neural Comput. Appl., 21 (2012), (Suppl1), s151-s158.

[4] Cagman, N., Enginoglu, S., Soft matrix theory and its decision making, Comput. Math. Appl., 59 (2010), 3308-3314.

[5] Cagman, N., Enginoglu, S., Soft set theory and uni-int decision making, Eur. J. Oper. Res., 207 (2) (2010), 848-855.

[6] Eraslan, S., A Decision Making Method via TOPSIS on Soft Sets, Journal of New results in Science, 8(2015), 57-71.

[7] Feng, F., Jun, Y.B., Zhao, X. , Soft semirings, Comput. Math. Appl., 56 (2008), 2621-2628.

[8] Golan, J.S., Semirings and their applications, Kluwer Academic Publishers, Dodrecht, 1999.

[9] Gulistan, M., Shahzad, M., Ahmed, S., Ilyas, M., Characterization of Gamma Hemirings by Generalized Fuzzy Gamma Ideals, Applications and Applied Mathematics, 10(1) (2015), 495-520.

[10] Hamouda, E.H., Soft ideals in ordered semigroup, Revista de la Union Matematica Argentina, Vol. 58, No. 1, 2017, 85-94

[11] Hebisch,U., Weinert, J., Semirings algebraic theory and application in computer science, World Scientific, 1998.

[12] Khan, A., Sarwar, M., Uni-Soft Ideals of Ternary Semigroups, International Journal of Mathematics and Scientific Computing, Vol. 5, No. 1 (2015) 60-65

[13] Kim,C.S., Kang, J.G., Kim, J. S., Uni-soft (Quasi) Ideals of Semigroups, Applied Mathematical Sciences, Vol. 7, 2013, no. 50, 2455 - 2468

[14] Liu, X. Some kind of idealistic soft Γ-rings induced by fuzzy ideals, Annals of Fuzzy mathematics and Informatics, 4(1) (2012) 197-206.

[15] Ma, X., Zahn, J., Fuzzy soft Γ-hemirings, Kuwait J. Sci. 41(2)(2014), 15-33.

[16] Maji, P.K., Biswas, R., Roy, A.R. , Soft set theory, Comput. Math. Appl., 45 (2003), 555-562.

[17] Maji, P.K., Roy, A.R., Biswas, R. , An application of soft sets in a decision making problem, Comput. Math. Appl., 44 (2002), 1077-1083.

[18] Molodtsov, D. , Soft set theory-First results, Comput. Math. Appl., 37(1999), 19-31.

[19] Nobusawa, N., On generalization of ring theory, Osaka J. Math. 1(1978) 185-190.

[20] Nazmul, Sk., Samanta, S.K., On soft multigroups, Annals of Fuzzy Mathematics and Informatics 10(2), (2015), 271-285.

[21] Ozturk, M.A., Inan, E., Soft Γ-rings and idealistic soft Γ-rings, Annals of Fuzzy mathematics and Informatics, 1(1) (2011) 71-80.

[22] Roy, S., Samanta, T.K., Balanced and Absorbing Soft Sets, Journal of New results in Science, 8(2015), 36-46.

[23] Sezgin, A., Atagun, A.O., Caggman, N., Soft intersection near-rings with its applications, Neural Comput. Appl., 21 (2012), (Suppl1), s221-s229.

[24] Sezer, S., Caggman, N., Atagun, A.O., Uni-soft substructures of groups, Annals of Fuzzy Mathematics and Informatics, 9(2)(2015), 235-246.

[25] Shabir, M., Kanwal, R.S., On L-fuzzy soft ideals of nearrings, Annals of Fuzzy Mathematics and Informatics, 10(1)(2015), 161-180.

[26] Wu, H., Zhan J., The characterizations of some kind of soft hemirings, Ann. Fuzzy Math Inform., 3(2) (2012) 267-280.

[27] Zhan, J., Maji, P. K., Applications of soft union sets to h-semisimple and h-quasi-hemiregular hemirings, Hacettepe Journal of Mathematics and Statistics, 43 (5) (2014), 815-825.