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ON LE CAM TYPE BOUNDS IN GENERAL POISSON APPROXIMATION

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Abstract

The main goal of this note is to establish the Le Cam type bounds in general Poisson approximation for distributions of sums (and random sums) of independent, non-negative integer-valued random variables with respect to a probability distance based on a linear operator originated by Rényi.

1 Introduction

The Poisson approximation states that the distribution of a sum of independent Bernoulli random variables can be approximated by a Poisson distribution with the equal expectation if success probabilities are small and the number of random variables is large (see e.g. [1], [2], [3] and [13]). One of interesting results in Poisson approximation is a remarkable bound originated by Le Cam in ([11]). Le Cam's bound allows to estimate the total variation distance between the distribution of the sum of independent Bernoulli random variables and the Poisson distribution of the same mean. Specifically, for $n \geq 1$, let $X_{n,1}, X_{n,2}, \ldots$ be a sequence of independent Bernoulli random variables with success probabilities $P(X_{n,k} = 1) = 1 - P(X_{n,k} = 0) = p_{n,k} \in (0,1), k \in \{1, 2, \cdots, n\}$. For $n \geq 1$, set $S_n = \sum\limits_{k=1}^{n} X_{n,k}$ and write $\lambda_n = E(S_n) = \sum\limits_{k=1}^{n} p_{n,k}$. Assume that $\lim\limits_{n\to\infty} \lambda_n = \lambda \in (0, \infty)$. Let us denote by Z_λ the Poisson random variable with mean λ. Then, using the method of convolution operators, Le Cam ([11])

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obtained the following bound

$$\sum_{r=0}^{\infty} |P(S_n = r) - P(Z_\lambda = r)| \leq 2 \sum_{k=1}^{n} p_{n,k}^2. \tag{1}$$

(see [11], [14] and [15] for more details). It is to be noticed that the Le Cam's bound in (1) usually expressed via a probability distance as follows:

$$d_{TV}(S_n, Z_\lambda) \leq \sum_{k=1}^{n} p_{n,k}^2, \tag{2}$$

where $d_{TV}(S_n, Z_\lambda) := \frac{1}{2} \sum\limits_{r=0}^{\infty} \mid P(S_n = r) - P(Z_\lambda = r) \mid$ is the total variation distance between S_n and Z_λ (see e.g. [1], [2], [11], [9], [12], [15], [19] and [20]). Kerstan in [9] improved the bound in (2) to

$$d_{TV}(S_n, Z_\lambda) \leq 1.05 \times (\sum_{k=1}^{n} p_k^2) \Big/ (\sum_{k=1}^{n} p_k), \tag{3}$$

where $p_k \leq 1/4$. Barbour and Hall in [2] further improved the bound in (2) to

$$d_{TV}(S_n, Z_\lambda) \leq \min\{1, \lambda^{-1}\} \sum_{k=1}^{n} p_k^2. \tag{4}$$

And Chen (see [3]) showed the bound in following form

$$d_{TV}(S_n, Z_\lambda) \leq \left(\frac{1 - e^{-\sum\limits_{k=1}^{n} p_k}}{\sum\limits_{k=1}^{n} p_k} \right) \sum_{k=1}^{n} p_k^2. \tag{5}$$

Furthermore, let N be a non-negative, integer-valued random variable, which is independent of $X_{n,k}$. Then, the Le Cam type bound for random sum of independent Bernoulli random variables is given by Yannaros in [21] as follows:

$$d_{TV}(S_N, Z_\lambda) \leq E\left| \sum_{k=1}^{N} p_k - \lambda \right| + E \left(\frac{1 - e^{-\sum\limits_{k=1}^{N} p_k}}{\sum\limits_{k=1}^{N} p_k} \sum_{k=1}^{N} p_k^2 \right), \tag{6}$$

where $S_N = \sum\limits_{k=1}^{N} X_{n,k}$ and $S_0 = 0$.

During the last several decades various powerful mathematical tools for establishing and for improving the Le Cam's bound in forms of (1), (2), (3), (4), (5) and (6) have been demonstrated such as the coupling method, semigroup method, Stein-Chen method with some special approaches as local approach, size-bias coupling approach, Kerstan's method, Charlier-Parseval approach, operator-method approach, method of $\omega-$ functions, etc. Results of this nature may be found in [1], [2], [3], [11], [9], [14], [19], [20], [22], [12], [15], [4], and the references given there.

Recently, using the linear operator method introduced by Rényi in [13], the Le Cam type bound in form of (2) for independent Bernoulli random variables are obtained by Hung and Thao ([6]). Specially, for $f \in \mathbb{K}$

$$\| A_{S_n}(f) - A_{Z_{\lambda_n}}(f) \| \leq 2 \| f \| \sum_{k=1}^{n} p_{n,k}^2, \tag{7}$$

where $\mathbb{K}$ denoted the class of all real-valued bounded functions on set of all non-negative integers $\mathbb{Z}_+ := \{0, 1, 2, \ldots\}$. The norm of function $f \in \mathbb{K}$ is defined by $\|f\| = \sup_{x \in \mathbb{Z}_+} |f(x)|$. The operator A_X in (7) associated a random variable X is defined by

$$(A_X f)(x) := \sum_{r=0}^{\infty} f(x+r) P(X = r), \forall f \in \mathbb{K}, \forall x \in \mathbb{Z}_+. \tag{8}$$

(See [13] for more details). Additionally, let $\{N_n, n \geq 1\}$ be a sequence of non-negative, integer-valued random variables, independent of all $X_{n,k}, k \in \{1, 2, \cdots, n\}; n \geq 1$. Define a random sum as a sum of random number of $X_{n,1}, X_{n,2}, \cdots$ and it is denoted by $S_{N_n} := X_{n,1} + \ldots + X_{n,N_n}$. Such sums appear in a natural way with point processes, extreme value theory, and in various applied problems (see e. g. [5], [10] and [21]). Let us write $\lambda_{N_n} := E(S_{N_n})$. Then, Le Cam's bound for distribution of random sum S_{N_n} in Poisson approximation for independent Bernoulli random variables will be established as follows:

$$\| A_{S_{N_n}}(f) - A_{Z_{\lambda_{N_n}}}(f) \| \leq 2 \| f \| E\left(\sum_{k=1}^{N_n} p_{N_n,k}^2 \right), \tag{9}$$

for $f \in \mathbb{K}$, (see [6] for more details).

It makes sense to consider a more general framework in which a probability-distance approach will be applicable for providing the Le Cam type bounds in general Poisson approximation. For $n \geq 1$, let $X_{n,1}, X_{n,2}, \cdots$ be a sequence of independent, non-negative and integer-valued random variables. Put

$$P(X_{n,k} = r) = p_{n,k}(r), \quad r = 0, 1, 2, \cdots; k = 1, 2, \cdots, k_n; n = 1, 2, \cdots$$

and

$$R_{n,k} = \sum_{r=2}^{\infty} p_{n,k}(r),$$

where $k_n, n = 1, 2, \cdots$ be a sequence of positive integers, $k_n \to +\infty$, as $n \to +\infty$. Set $S_n := X_{n,1} + \cdots + X_{n,k_n}$ and write $\lambda_n := \sum_{k=1}^{k_n} p_{n,k}(1)$. Let Z_{λ_n} denote the Poisson distributed random variable with expectation λ_n. Then, the Le Cam type bound in general Poisson approximation will be stated as follows:

$$|P(S_n = r) - P(Z_{\lambda_n} = r)| \leq 2 \sum_{k=1}^{k_n} \left(p_{n,k}^2(1) + R_{n,k}\right). \tag{10}$$

Moreover, in addition let $N_n, n \geq 1$ be a sequence of non-negative, integer-valued random variables independent of all $X_{n,k}; k = 1, 2, \cdots, k_n; n \geq 1$. Then. the Le Cam type bound in general Poisson approximation for random sum is given by

$$\left|P(S_{N_n} = r) - P\left(Z_{\lambda_{N_n}} = r\right)\right| \leq 2E\left(\sum_{k=1}^{k_{N_n}} \left(p_{N_n,k}^2(1) + R_{N_n,k}\right)\right). \tag{11}$$

It is worth pointing out that the bounds in form of (10) and (11) are direct consequences of Theorem 3.1 and Theorem 3.2 in Section 3, (see Corollary 3.2 and Corollary 3.7).

In this paper, based on Rényi operator defined in (8), we wish to apply a probability-distance approach to general Poisson approximation for providing the Le Cam type bounds (10) and (11). Using this approach, two results concerning the Le Cam type bounds in general Poisson approximation for sums and random sums of independent, non-negative, integer-valued random variables are established (see Theorem 3.1 and Theorem 3.2). The present paper is a continuation of Hung and Thao in [6], Hung and Giang in [7] and [8]. The received results are also extensions of results of Le Cam in [11], Steele in [15], Teerapabolarn and Wongkasem in [16], Teerapabolarn in [17], [18], and Neammanee in [12].

It is to be noticed that the presented approach in this article can be viewed as a simplified version of Le Cam's operator–technique dating back to 1960's (see [11] for more details) and this technique has been used by Upadhye and Vellaisamy to provide some bounds in Poisson approximation via the total variation distance (see [19], [20], for more details).

The rest of this paper is organized as follows. We begin with the definitions of a probability distance based on Rényi operator in (8) with some main properties in Section 2. The Section 3 is devoted to two theorems with corollaries concerning the Le Cam type bounds for distributions of sums and random sums in general Poisson approximation.

2 Preliminaries

We now need a probability distance based on Rényi's operator ([13]) in form (8).

Definition 2.1. *A probability distance $d(X, Y; f)$ of two random variables X and Y with respect to function $f \in \mathbb{K}$ is defined by*

$$d(X, Y; f) := \sup_{x \in \mathbb{Z}_+} |Ef(X + x) - Ef(Y + x)|. \tag{12}$$

Based on the properties of Rényi's operator (see [13] for more details), the properties of probability distance d(X, Y; f) are summarized as follows.

1. It is easy to see that $d(X, Y; f)$ is a probability metric, i.e. for the random variables X, Y and Z the following properties are possessed

   (a) For every $f \in \mathbb{K}$, the distance $d(X, Y; f) = 0$ if $P(X = Y) = 1$.

   (b) $d(X, Y; f) = d(Y, X; f)$ for every $f \in \mathbb{K}$.

   (c) $d(X, Y; f) \leq d(X, Z; f) + d(Z, Y; f)$ for every $f \in \mathbb{K}$.

2. If $d(X, Y; f) = 0$ for every $f \in \mathbb{K}$, then $F_X \equiv F_Y$.

3. Let $\{X_n, n \geq 1\}$ be a sequence of random variables and let X be a random variable. The condition

   $$\lim_{n \to +\infty} d(X_n, X; f) = 0, \quad \text{for all} \quad f \in \mathbb{K},$$

   implies that $X_n \xrightarrow{D} X$ as $n \to \infty$, where $\xrightarrow{D}$ denotes convergence in distribution.

4. Suppose that $X_1, \ldots X_n; Y_1, \ldots Y_n$ are independent random variables (in each group). Then, for every $f \in \mathbb{K}$,

   $$d\left(\sum_{j=1}^{n} X_j, \sum_{j=1}^{n} Y_j; f\right) \leq \sum_{j=1}^{n} d(X_j, Y_j; f).$$

   Moreover, if the random variables are identically (in each group), then we have

   $$d\left(\sum_{j=1}^{n} X_j, \sum_{j=1}^{n} Y_j; f\right) \leq nd(X_1, Y_1; f).$$

5. Suppose that $X_1, \ldots X_n; Y_1, \ldots Y_n$ are independent random variables (in each group). Additionally, let $\{N_n, n \geq 1\}$ be a sequence of positive

integer-valued random variables that independent of $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_n$. Then, for every $f \in \mathbb{K}$,

$$d\left(\sum_{j=1}^{N_n} X_j, \sum_{j=1}^{N_n} Y_j; f\right) \leq \sum_{k=1}^{\infty} P(N_n = k) \sum_{j=1}^{k} d(X_j, Y_j; f).$$

6. Suppose that $X_1, \ldots X_n; Y_1, \ldots Y_n$ are independent, identically distributed random variables (in each group). Let $\{N_n, n \geq 1\}$ be a sequence of positive integer-valued random variables that independent of $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_n$. Moreover, suppose that $E(N_n) < +\infty, n \geq 1$. Then, for every $f \in \mathbb{K}$,
$$d\left(\sum_{j=1}^{N_n} X_j, \sum_{j=1}^{N_n} Y_j; f\right) \leq E(N_n) \times d(X_1, Y_1; f).$$

7. Let $d_{TV}(X, Y)$ be a total variation distance of random variables X and Y. Then, for $A \subset \mathbb{Z}_+$, and for case of χ_A is an indicator function of set A, one has
$$d(X, Y, \chi_A) \leq d_{TV}(X, Y).$$
However, it is true that
$$\sup_{A \subset \mathbb{Z}_+} d(X, Y, \chi_A) = d_{TV}(X, Y).$$

3 Main results

The next theorem will be fundamental in this paper and it is a generalization of known results related to Le Cam's bound ([11]) in Poisson approximation.

Theorem 3.1. *(Le Cam type bound in general Poisson approximation) For $n \geq 1$, let $X_{n,1}, X_{n,2}, \cdots$ be a sequence of independent, non-negative integer-valued random variables such that*

$$P(X_{n,k} = r) = p_{n,k}(r); r = 0, 1, 2, \cdots; k = 1, 2, \cdots, k_n; n = 1, 2, \cdots$$

where $k_1, k_2, \cdots$ be a sequence of positive integers and $k_n \to \infty$ as $n \to \infty$. Set $S_n := X_{n,1} + \cdots + X_{n,k_n}$ and write $\lambda_n := \sum_{k=1}^{k_n} p_{n,k}(1)$. Let Z_{λ_n} denote the Poisson distributed random variable with expectation λ_n. Then, for every $f \in \mathbb{K}$

$$d(S_n, Z_{\lambda_n}, f) \leq 2 \parallel f \parallel \sum_{k=1}^{k_n} \left(p_{n,k}^2(1) + R_{n,k}\right),$$

where

$$R_{nk} = \sum_{r=2}^{\infty} p_{n,k}(r).$$

Proof. We shall begin with showing that

$$Z_{\lambda_n} \stackrel{D}{=} \sum_{k=1}^{k_n} Z_{p_{n,k}(1)},$$

where the $Z_{p_{n,k}(1)}$ are independent, Poisson distributed random variables with expectations $p_{n,k}(1), k = 1, 2, \cdots, k_n$. Here and from now on, the notation $\stackrel{D}{=}$ means that the equal in distribution. The theorem will be proved if for every $f \in \mathbb{K}$,

$$d(X_{n,k}, Z_{p_{n,k}(1)}, f) \leq 2 \parallel f \parallel (p_{n,k}^2(1) + R_{n,k}).$$

Let us first compute for every $f \in \mathbb{K}, x \in \mathbb{Z}_+$,

$$\begin{aligned} Ef(X_{n,k} + x) &= \sum_{r=0}^{\infty} f(x+r) p_{n,k}(r) \\ &= f(x)p_{n,k}(0) + f(x+1)p_{n,k}(1) + \sum_{r=2}^{\infty} f(x+r)p_{n,k}(r), \end{aligned} \tag{13}$$

and

$$\begin{aligned} &Ef(Z_{p_{nk}(1)} + x) = \sum_{r=0}^{\infty} f(x+r) \frac{e^{-p_{n,k}(1)}}{r!} p_{n,k}^r(1) \\ &= f(x)e^{-p_{n,k}(1)} + f(x+1)p_{n,k}(1)e^{-p_{n,k}(1)} + \sum_{r=2}^{\infty} f(x+r)\frac{e^{-p_{n,k}(1)}}{r!} p_{n,k}^r(1). \end{aligned} \tag{14}$$

Combining (13) with (14), we obtain, for every $f \in \mathbb{K}$,

$$\begin{aligned} &Ef(X_{n,k} + x) - Ef(Z_{p_{n,k}(1)}) = f(x)\left(p_{n,k}(0) - e^{-p_{n,k}(1)}\right) \\ &+ f(x+1)\left(p_{n,k}(1) - p_{n,k}(1)e^{-p_{n,k}(1)}\right) + \sum_{r=2}^{\infty} f(x+r)p_{n,k}(1) \\ &- \sum_{r=2}^{\infty} f(x+r)\frac{e^{-p_{n,k}(1)}}{r!} p_{n,k}^r(1). \end{aligned}$$

Using the probability distance defined in (12), for every $f \in \mathbb{K}$, we obtain

$$\begin{aligned} d(X_{n,k}, Z_{p_{n,k}(1)}, f) \leq& \parallel f \parallel \left(e^{-p_{n,k}(1)} - p_{n,k}(0) + p_{n,k}(1)(1 - e^{-p_{n,k}(1)})\right) \\ &+ \parallel f \parallel R_{n,k} + \parallel f \parallel \left(1 - e^{-p_{n,k}(1)} - p_{n,k}(1)e^{-p_{n,k}(1)}\right). \end{aligned} \tag{15}$$

Since

$$p_{n,k}(1)\left(1-e^{-p_{n,k}(1)}\right) \leq p_{n,k}^2(1),$$

and

$$1-p_{n,k}(0) = p_{n,k}(1) + R_{n,k}.$$

From (15) it may be concluded that

$$d(X_{n,k}, Z_{p_{n,k}(1)}, f) \leq 2 \parallel f \parallel \left(p_{n,k}^2(1) + R_{n,k}\right).$$

Consequently,

$$d(S_n, Z_{\lambda_n}, f) \leq 2 \parallel f \parallel \sum_{k=1}^{k_n} \left(p_{n,k}^2(1) + R_{n,k}\right).$$

This finishes the proof. □

Corollary 3.1. *(General Poisson Limit Theorem) Under the assumptions of Theorem 3.1, for $r = 0, 1, 2, \cdots$, the Le Cam type bound is stated as in (10)*

$$|P(S_n = r) - P(Z_{\lambda_n} = r)| \leq 2 \sum_{k=1}^{k_n} \left(p_{n,k}^2(1) + R_{n,k}\right).$$

As an immediate consequence of Theorem 3.1, the general Poisson limit theorem (see [13] for more details) can be restated as following corollary.

Corollary 3.2. *Under hypotheses of Theorem 3.1 and assume that the following conditions are satisfied*

1. $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{k_n} p_{n,k}(1) = \lambda, \quad (0 < \lambda < +\infty),$

2. $\lim\limits_{n\to+\infty} \max\limits_{1\leq k\leq k_n} (1 - p_{n,k}(0)) = 0$

3. $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{k_n} R_{n,k} = 0.$

Then, $S_n \xrightarrow{D} Z_\lambda$ as $n \to \infty$.

Throughout the forthcoming, let $\{N_n, n \geq 1\}$ be a sequence of non-negative integer-valued random variables independent of $X_{n,k}; k = 1, 2, \cdots, k_n; n = 1, 2, \cdots$. Then, in the same way as in proof of Theorems 3.1, the Le Cam type bounds in general Poisson limit theorems for distributions of random sums will be established via probability distance defined in (12) as follows:

Theorem 3.2. *(Le Cam type in General Poisson approximation for random sum) For $n \geq 1$, let $X_{n,1}, X_{n,2}, \cdots$ be a sequence of independent, non-negative integer-valued random variables such that*

$$P(X_{n,k} = r) = p_{n,k}(r); r = 0, 1, 2, \cdots; k = 1, 2, \cdots, k_n; n = 1, 2, \cdots$$

Let $\{N_n, n \geq 1\}$ be a sequence of non-negative integer-valued random variables independent of all $X_{n,k}; k = 1, 2, \cdots, k_n; n = 1, 2, \cdots$. Set $S_{N_n} := \sum_{k=1}^{k_{N_n}} X_{n,k}$, and write $\lambda_{N_n} = \sum_{k=1}^{k_{N_n}} p_{n,k}(1)$ with $S_0 = 0$ and $\lambda_0 = 0$. Then, for all function $f \in \mathbb{K}$,

$$d(S_{N_n}, Z_{\lambda_{N_n}}, f) \leq 2 \parallel f \parallel E\left(\sum_{k=1}^{k_{N_n}} (p_{N_n,k}^2(1) + R_{N_n,k})\right),$$

where $Z_{\lambda_{N_n}}$ are Poisson distributed random variables with expectations λ_{N_n}, and $R_{N_n,k}(r) = \sum_{r=2}^{\infty} p_{N_n,k}(r)$.

Proof. The proof is immediate from Theorem 3.1

$$\begin{aligned} d\left(S_{N_n}, Z_{\lambda_{N_n}}; f\right) &\leq \sum_{k_n \in Z_+} P\left(k_{N_n} = k_n\right) \sum_{k=1}^{k_n} d\left(X_{n,k}, Z_{p_{n,k}(1)}; f\right) \\ &\leq 2 \|f\| \sum_{k_n \in Z_+} P\left(k_{N_n} = k_n\right) \sum_{k=1}^{k_n} \left(p_{n,k}^2\left(1\right) + R_{n,k}\right) \\ &\leq 2 \|f\| E\left(\sum_{k=1}^{k_{N_n}} \left(p_{N_n,k}^2\left(1\right) + R_{N_n,k}\right)\right). \end{aligned}$$

The proof is complete. □

Corollary 3.3. *As an immediate consequence of Theorem 3.2, for $r = 0, 1, \cdots,$ the Le Cam type bound for random sum in general Poisson approximation is given by*

$$\left|P\left(S_{N_n} = r\right) - P\left(Z_{\lambda_{N_n}} = r\right)\right| \leq 2E\left(\sum_{k=1}^{k_{N_n}} \left(p_{N_n,k}^2\left(1\right) + R_{N_n,k}\right)\right).$$

It is worth pointing that by an argument analogous to that used for the proof of above Theorems 3.1 and 3.2, we can establish some Le Cam type bounds in Poisson approximation for sums and random sums of independent Bernoulli random variables like results in [6] via probability distance defined in (12). However, we can also consider the results in [6] as the direct consequence of general Poisson approximation theorems 3.1 and 3.2.

Corollary 3.4. *(see [6], Theorem 3.1) For* $n \geq 1$, *let* $X_{n,1}, X_{n,2}, \ldots; n = 1, 2, \ldots$ *be a independent, Bernoulli distributed random variables with success probabilities*

$$P\left(X_{n,k} = 1\right) = 1 - P\left(X_{n,k} = 0\right) = p_{n,k} \in (0, 1); k = 1, 2, \cdots, k_n; n = 1, 2, \cdots.$$

Write $S_n = \sum\limits_{k=1}^{k_n} X_{n,k}$. *Let us denote by* Z_{λ_n} *the Poisson distributed random variable with mean* $\lambda_n := E(S_n) = \sum\limits_{k=1}^{k_n} p_{n,k}$. *Then, for all functions* $f \in \mathbb{K}$,

$$d\left(S_n, Z_{\lambda_n}, f\right) \leq 2 \|f\| \sum_{k=1}^{k_n} p_{n,k}^2.$$

Corollary 3.5. *Under the above assumptions of Corollary 3.4, for all* $r = 0, 1, 2, \cdots$, *we have*

$$\left|P\left(S_n = r\right) - P\left(Z_{\lambda_n} = r\right)\right| \leq 2 \sum_{k=1}^{k_n} p_{n,k}^2.$$

Corollary 3.6. *Under above assumptions of Theorem 3.2 on* $X_{n,k}, k = 1, 2, \cdots, k_n; n = 1, 2, \cdots$ *and let* $N_n, n = 1, 2, \cdots$ *be a sequence of non-negative integer-valued random variables. Assume that* $N_n, n = 1, 2, \ldots$ *are independent of all* $X_{n,k}; k = 1, 2, \cdots, k_n; n = 1, 2, \cdots$. *Then, for all functions* $f \in \mathbb{K}$,

$$d(S_{N_n}, Z_{\lambda_{N_n}}, f) \leq 2 \parallel f \parallel E\left(\sum_{k=1}^{k_{N_n}} p_{N_n,k}^2\right).$$

Corollary 3.7. *Under the above assumptions of Corollary 3.6, for all* $k = 0, 1, \cdots, k_n$, *the Le Cam type bound in general Poisson approximation for random sum in form of (11) will be stated as follows:*

$$\left|P\left(S_{N_n} = r\right) - P\left(Z_{\lambda_{N_n}} = r\right)\right| \leq 2E\left(\sum_{k=1}^{k_{N_n}} p_{N_n,k}^2\right).$$

Concluding remark

We concluded this paper with the following comments. The probability distance approach in this paper has been presented the simple and efficient technique for establishing the Le Cam type bounds in Poisson approximation for sums and random sums of independent discrete random variables. This approach will certainly be more effective in establishing the Le Cam bounds in multivariate Poisson approximation.

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References

[1] R. Arratia, L. Goldstein and L. Gordon. *Poisson approximation and the Chen-Stein method,* Statist. Sci., **5** (1990), No. 4, 403–434.

[2] A. D. Barbour, L. Holst and S. Janson. "Poisson approximation", Oxford Univ. Press, New York, 1992.

[3] Louis H. Y. Chen, *On the convergence of Poisson binomial to Poisson distributions,* Ann. Probability, **2**(1) (1974), 178–180.

[4] P. Deheuvels, A. Karr, D. Pfeifer and R. Serfling, *Poisson approximations in selected metrics by coupling and semi-group methods with applications,* J. Statist. Plann. Inference **20** (1988), No. 1, 1–22.

[5] Boris V. Gnedenko and Victor Yu. Korolev, "Random summation. Limit theorems and applications", CRC Press, Boca Raton, FL, 1996.

[6] Tran Loc Hung and Vu Thi Thao, *Bounds for the approximation of Poisson-binomial distribution by Poisson distribution,* J. Ineq. Appl. (2013), 2013:30, 10 pp, http://www.journalofinequalitiesandapplications.com/content/2013/1/30.

[7] Tran Loc Hung and Le Truong Giang, *On bounds in Poisson approximation for integer-valued independent random variables,* J. Ineq. Appl. (2014), 2014:291, 11 pp, http://www.journalofinequalitiesandapplications.com/content/2014/1/291.

[8] Tran Loc Hung and Le Truong Giang, *On the bounds in Poisson approximation for independent geometric distributed random variables,* Bulletin of the Irannian Mathematical Society, **42**(5)(2016), 1087–1096.

[9] Johannes Kerstan, *Verallgemeinerung eines Satzes von Prochorow und Le Cam,* (German) Z. Wahrscheinlichkeitstheorie und Verw. Gebiete **2** (1964), 173–179.

[10] V. M. Kruglov and V. Yu. Korolev, "Limit theorems for random sums" (Russian), Moskov. Gos. Univ., Moscow, 1990.

[11] L. Le Cam, *An approximation theorem for the Poisson binomial distribution,* Pacific J. Math. **10** (1960), 1181–1197.

[12] K. Neammanee, *A nonuniform bound for the approximation of Poisson binomial by Poisson distribution,* Int. J. Math. Sci. (2003), No. 48, 3041–3046.

[13] A. Renyi, "Probability theory", Translated by L. Vekerdi, North-Holland Series in Applied Mathematics and Mechanics, Vol. 10. North-Holland Publishing Co., Amsterdam-London, American Elsevier Publishing Co., Inc., New York, 1970.

[14] R. J. Serfling, *A general Poisson approximation theorem,* Ann. Probability **3** (1975), No. 4, 726–731.

[15] J. Michael Steele, *Le Cam's inequality and Poisson approximations,* Amer. Math. Monthly, **101**(1) (1994), 48–54.

[16] K. Teerapabolarn K and P. Wongkasem, *Poisson approximation for independent geometric random variables,* Int. Math. Forum, **2** (2007), No. 65-68, 3211–3218.

[17] K. Teerapabolarn, *A note on Poisson approximation for independent geometric random variables,* Int. Math. Forum 4 (2009), no. 9-12, 531–535.

[18] K. Teerapabolarn, *A pointwise approximation of generalized binomial by Poisson distribution,* Appl. Math. Sci. (Ruse), 6, (2012), No. 21-24, 1095–1104.

[19] N. S. Upadhye and P. Vellaisamy, *Improved bounds for approximations to compound distributions,* Statist. Probab. Lett. **83** (2013), No. 2, 467–473.

[20] N. S. Upadhye and P. Vellaisamy, *Compound Poisson approximation to convolutions of compound negative binomial variables,* Methodol. Comput. Appl. Probab. **16**(4) (2014), 951–968.

[21] Nikos Yannaros, *Poisson approximation for random sums of Bernoulli random variables,* Statistics and Probability Letters **11** (1991), 161–165.

[22] V. Zacharovas and H. Hwang, *A Charlier-Parseval approach to Poisson approximation and its applications,* Lith. Math. J., **50**)1) (2010), 88–119.